بسم الله الرحمن الرحیم

Principles of Mechatronic Systems

مبانی سیستم های مکاترونیکی (جلسه دوازدهم)

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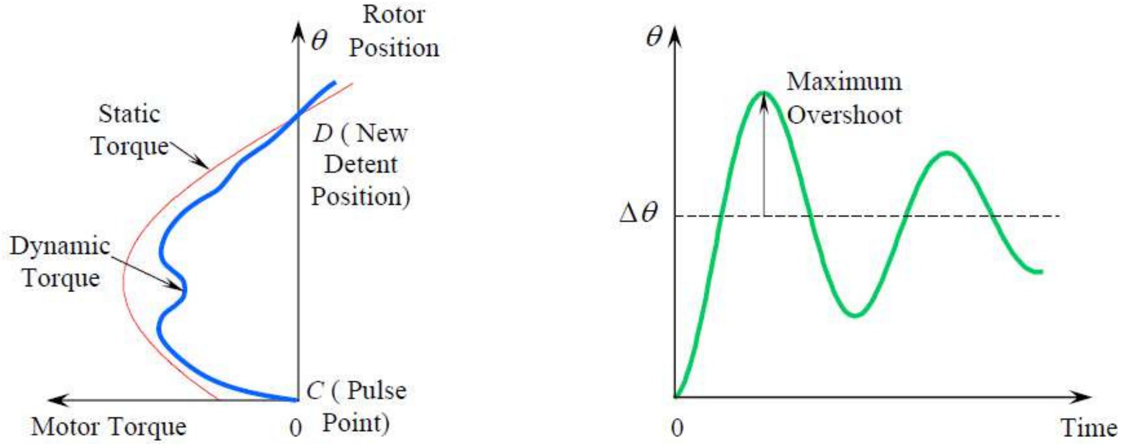
عملگرهای گامی

مشخصه های گشتاور موتور:

- Consider a response of a stepper motor to a single pulse. Ideally the stepper motor should instantaneously start, rotate, and instantaneously come to a stop



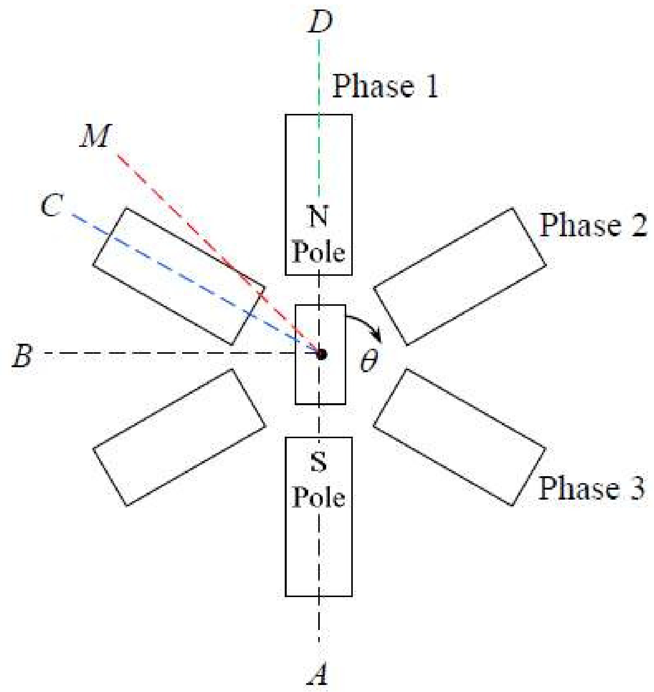
- Pulse is applied at C and the corresponding winding is energized (single phase). Torque is generated, rotor turns to the detent position (minimum reluctance)
- This is the static torque – In normal operations, a finite time is needed for current to build up - mutual and self inductions, eddy currents – Dynamic Torque
- There are oscillations at D due to its kinetic energy (velocity) at that position
- For stepper analysis we assume the static torque curve

مشخصه های گشتاور موتور:

Suppose that we turn the rotor clockwise from this stable equilibrium position, using an external rotating mechanism (e.g., by hand). At position C, which is the previous detent position where Phase 1 would have been energized under normal operation, there is a positive torque that tries to turn the rotor to its present detent position D.

At position B, the static torque is zero, because the force from the N pole of Phase 1 exactly balances that from the S pole. This point, however, is an unstable equilibrium position.

The maximum static torque occurs at position M, which is located approximately halfway between positions B and D.

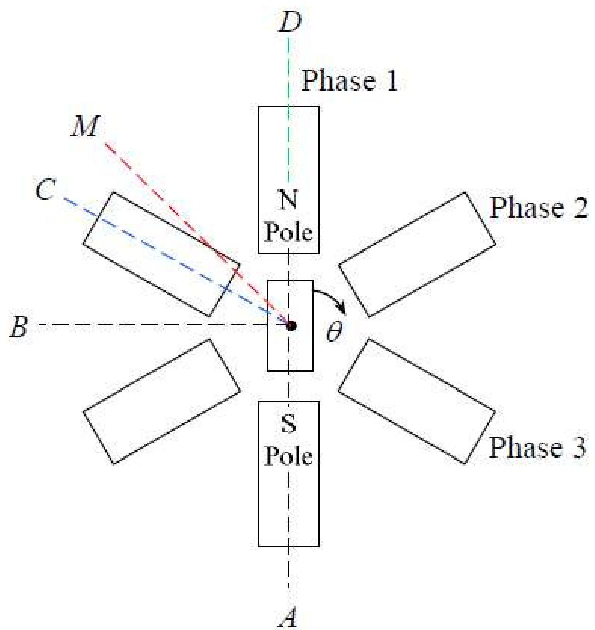


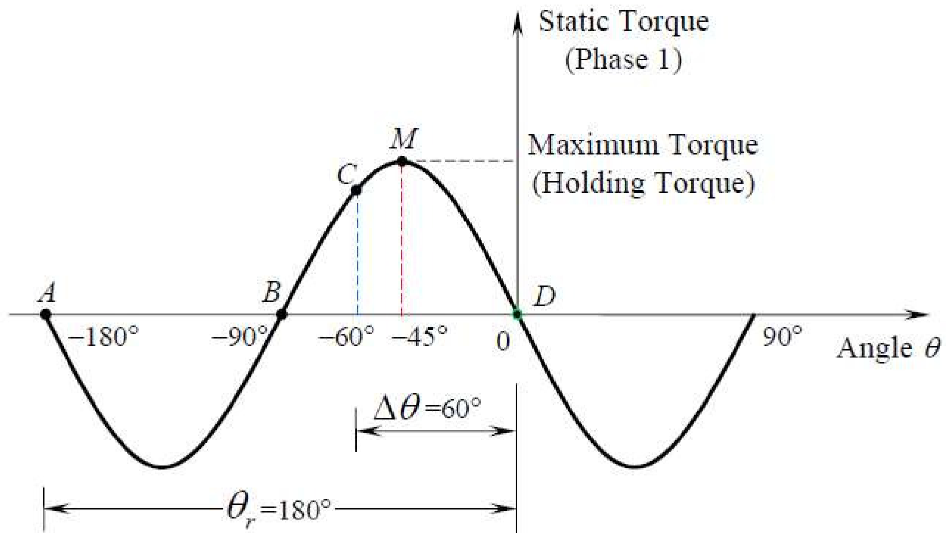
عملگرهای گامی

مشخصه های گشتاور موتور:

- The torque curve can be considered sinusoidal

$$T_1 = -T_{max} \sin n_r \theta$$





Mechatronics: An integrated approach, D Silva

عملگرهای گامی

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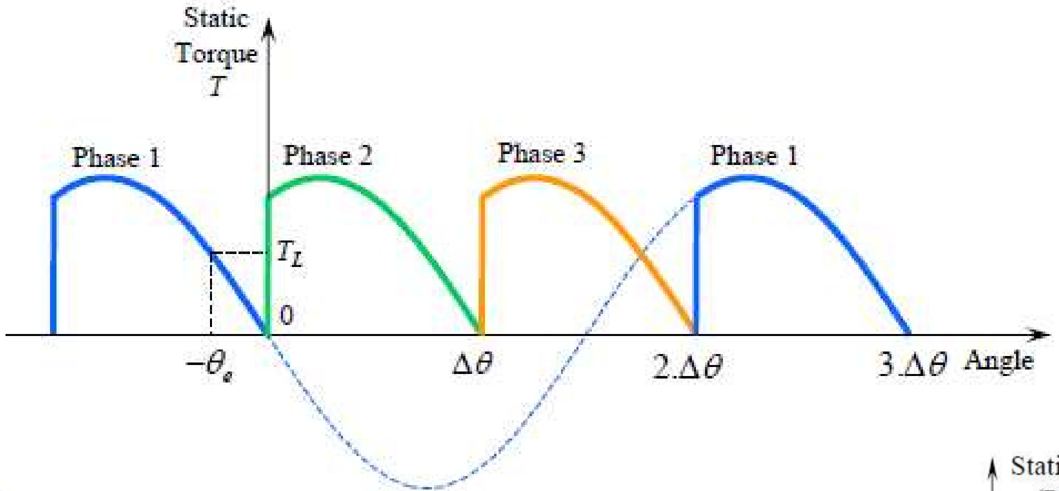

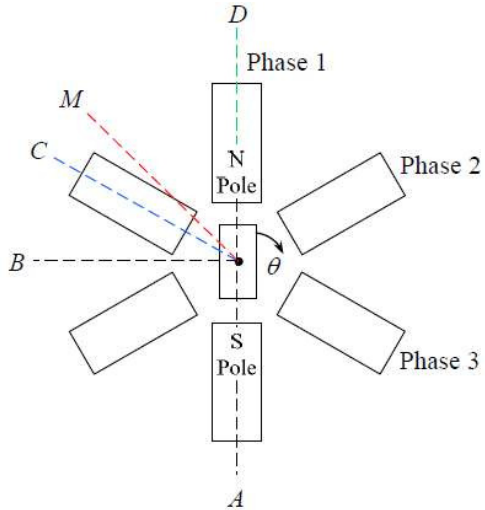

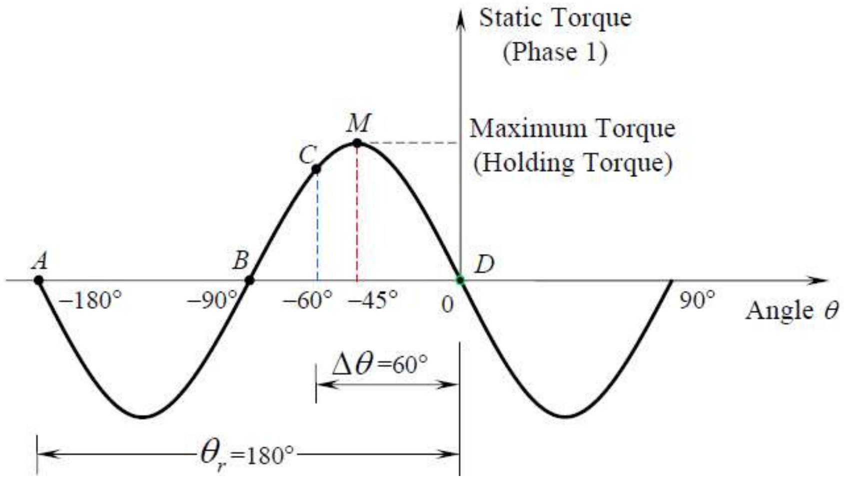

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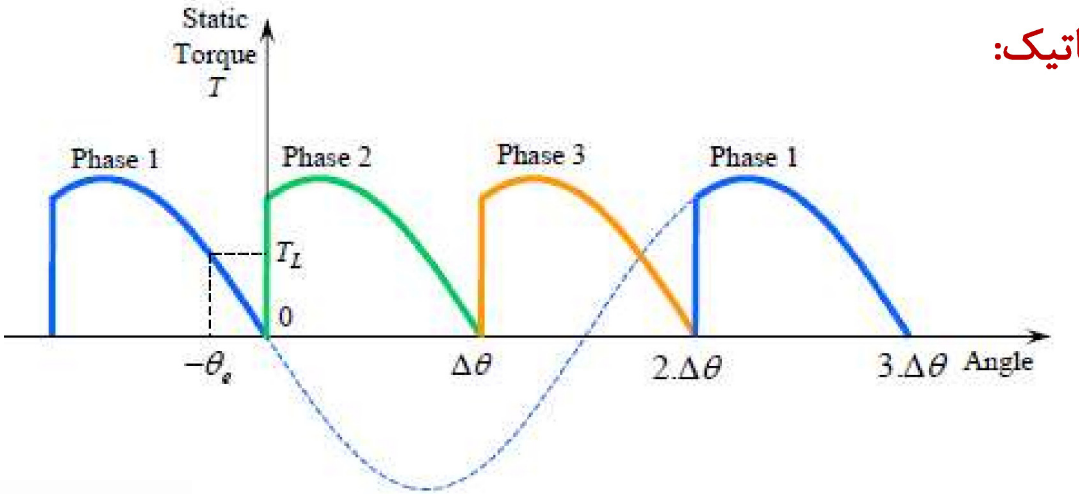
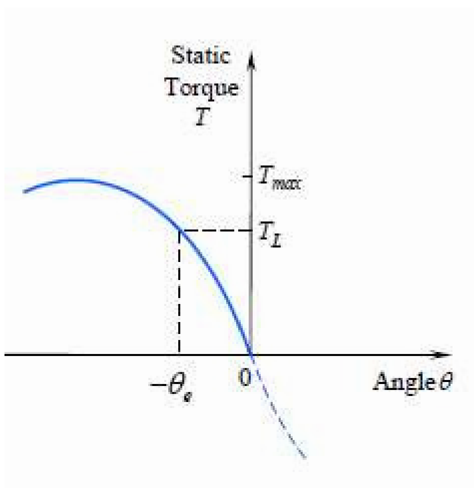
خطای موقعیت استاتیک:





$$T = -T_{\max} \sin\left(\frac{2\pi\theta}{p \cdot \Delta\theta}\right)$$

$$T_L = -T_{\max} \sin\left[\frac{2\pi(-\theta_e)}{p \cdot \Delta\theta}\right]$$

$$\theta_e = \frac{p \cdot \Delta\theta}{2\pi} \sin^{-1}\left(\frac{T_L}{T_{\max}}\right)$$

$$\theta_e = \frac{p}{n} \sin^{-1}\left(\frac{T_L}{T_{\max}}\right)$$

مثال:

Consider a three-phase stepping motor with seventy-two steps per revolution. If the static load torque is 10% of the maximum static torque of the motor, determine the static position error.

$$\frac{T_L}{T_{max}} = 0.1, \quad p = 3, \quad n = 72$$

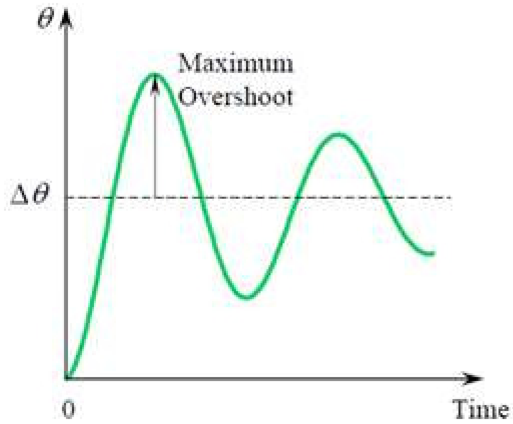
$$\theta_e = \frac{3}{72} \sin^{-1} 0.1 = 0.0042 \text{ rad} = 0.24°$$

Note that this is less than 5% of the step angle.

عملگرهای گامی

استهلاک در موتورهای گامی:

- Damping of stepper motors to suppress overshoot and reduce settling time can be achieved in several ways.
- Straightforward conventional techniques are mechanical and electrical damping
  - Mechanical damping – torsional damper attached to the motor shaft
  - Electrical damping
- Electronic damping can overcome the shortcomings of above techniques – heat generation and reduction in output torque



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عملگرهای گامی

استهلاک مکانیکی در موتورهای گامی

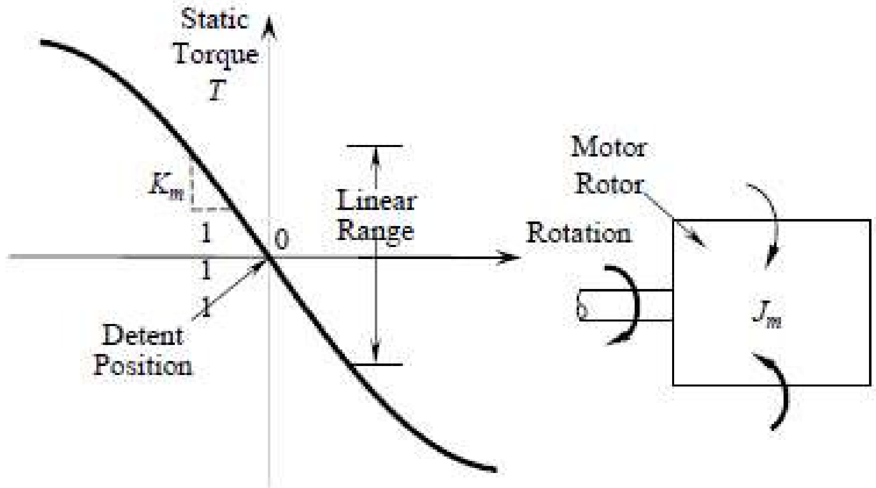
معادله حاکم بر موتور بدون مستهلک کننده خارجی:

$$T = -K_m\theta$$

$$J_m\frac{d\omega}{dt} = -C_m\omega - K_m\theta$$

$$J_m\ddot{\theta} + C_m\dot{\theta} + K_m\theta = 0$$

$$\omega_n = \sqrt{\frac{K_m}{J_m}} \qquad \zeta = \frac{C_m}{2\sqrt{K_m J_m}}$$

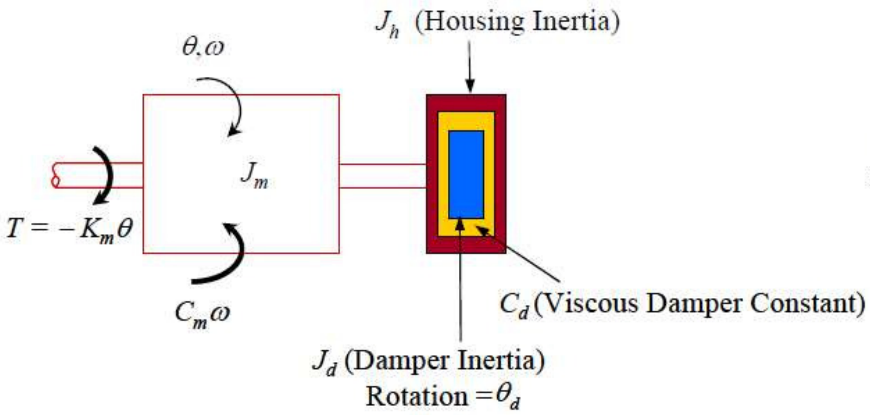


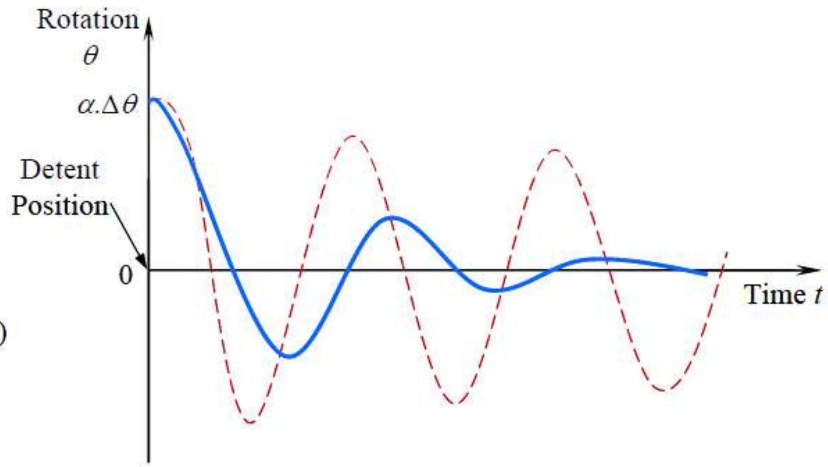
عملگرهای گامی

استهلاک مکانیکی در موتورهای گامی

معادله حاکم بر موتور با اضافه شدن مستهلک کننده خارجی:

A stepper motor with a Houdaille damper:





$$(J_m + J_h)\ddot{\theta} = -C_m\dot{\theta} - K_m\theta - C_d(\dot{\theta} - \dot{\theta}_d)$$

$$J_d\ddot{\theta}_d = C_d(\dot{\theta} - \dot{\theta}_d)$$

A Lanchester damper is similar to a Houdaille damper except that the former depends on nonlinear (Coulomb) friction instead of viscous damping.

استهلاک الکتریکی در موتورهای گامی

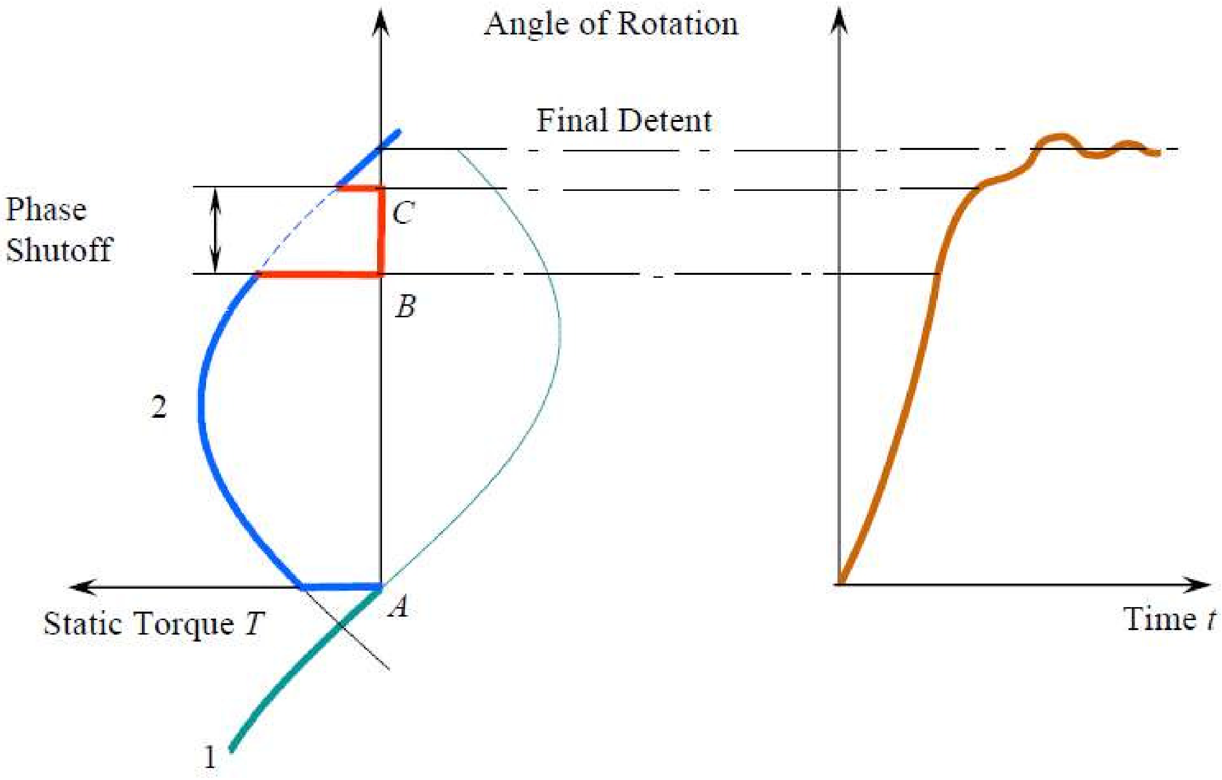
These direct techniques of damping have undesirable side effects, such as excessive **heat generation**, **reduction of the net output torque** of the motor, and **decreased speed of response**. **Electronic damping** methods have been developed to overcome such shortcomings.

Three common methods:

1. The pulse turn-off method: Turn off the motor (all phases) for a short time.
2. The pulse reversal method: Apply a pulse in the opposite direction (i.e., energize the reverse phase) for a short time.
3. The pulse delay method: Maintain the present phase beyond its detent position for a short time.

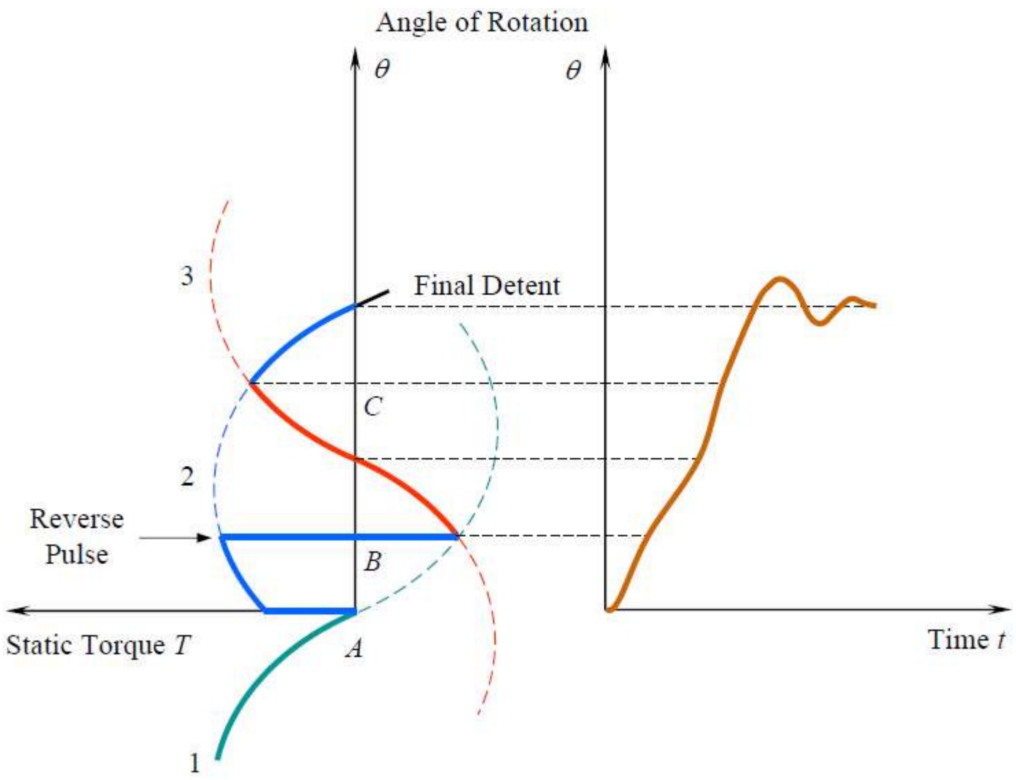
استهلاک الکتریکی در موتورهای گامی

1.The pulse turn-off method: Turn off the motor (all phases) for a short time.



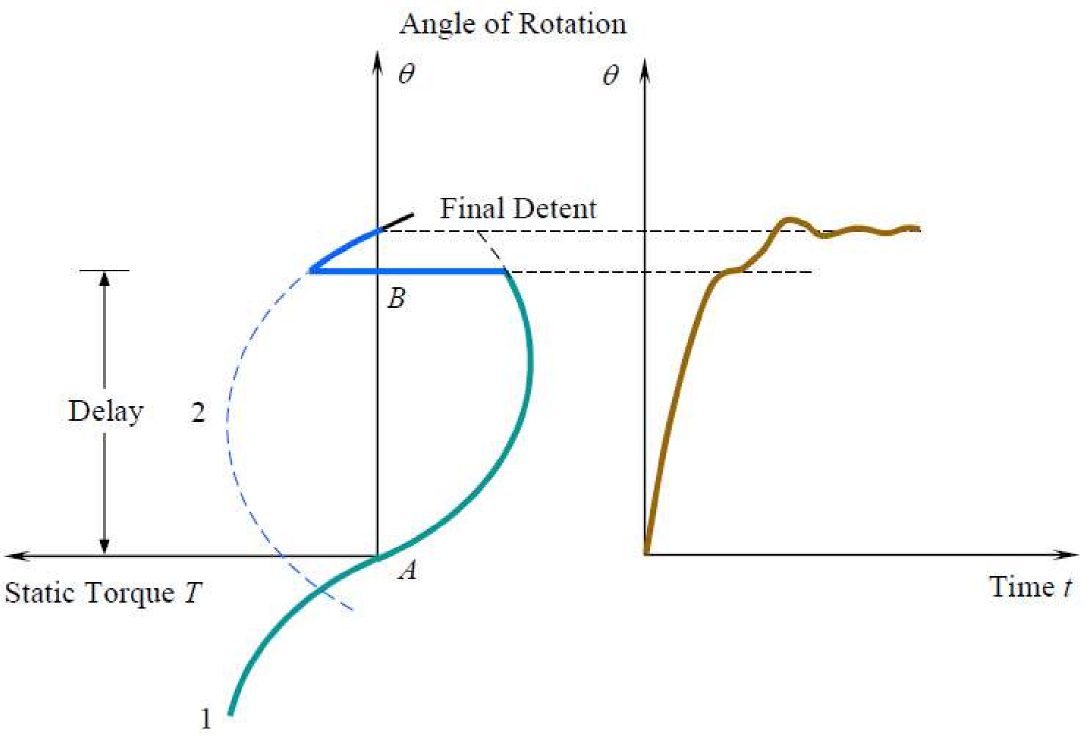
استهلاک الکتریکی در موتورهای گامی

2.The pulse reversal method: Apply a pulse in the opposite direction (i.e., energize the reverse phase) for a short time.



استهلاک الکتریکی در موتورهای گامی

3.The pulse delay method: Maintain the present phase beyond its detent position for a short time.



انتخاب موتورهای گامی

- Selection of a stepper motor for a particular application depends on the torque and speed consideration and geometric properties.

- *Step 1:* List the main requirements for the particular application – speed, acceleration, accuracy, resolution, load characteristics
- *Step 2:* Compute the operating torque and stepping rate requirements for the particular application

$$T = T_R + J_{eq}\frac{\omega_{\max}}{\Delta t}$$

T_R = net resistance torque
J_{eq} = equivalent moment of inertia (including rotor, load, gearing, dampers, etc.)
$\omega_{\max}$= maximum operating speed
Δt = time taken to accelerate the load to the maximum speed, starting from rest

انتخاب موتورهای گامی

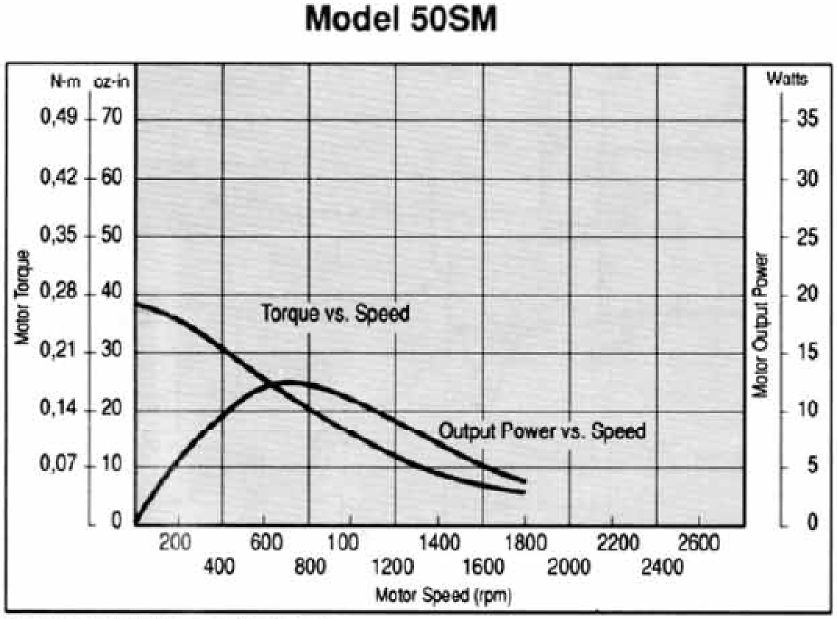
- ***Step 3**:* Using the torque versus stepping rate curves (pull-out curves) for a group of commercially available stepper motors, select a suitable stepper motor.
- ***Step 4**:* If a stepper motor that meets the requirements is not available, modify the basic design.

- The most important information in selecting a stepper motor is the torque versus stepping rate curve.

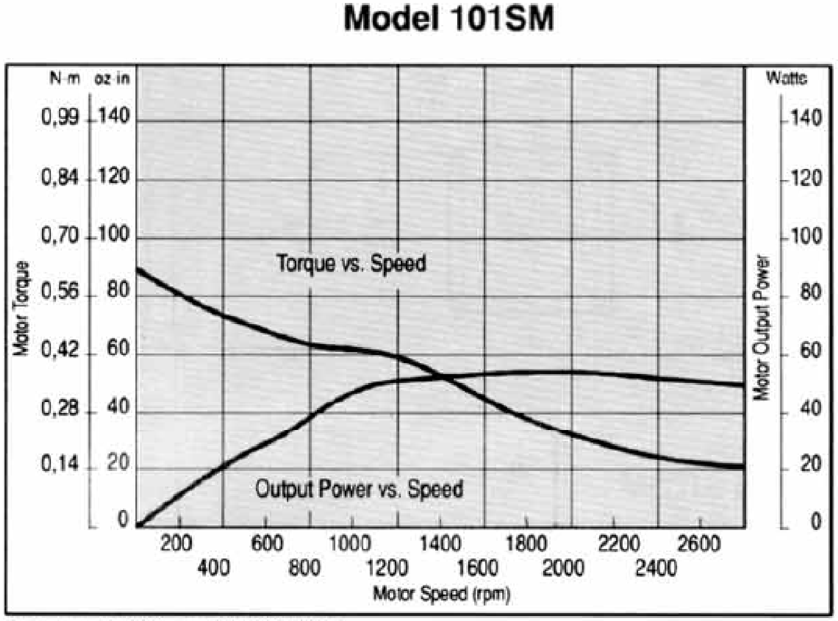
عملگرهای گامی

انتخاب موتورهای گامی

- The most important information in selecting a stepper motor is the torque versus stepping rate curve.



Recommended Drivers : DM4001, U1A

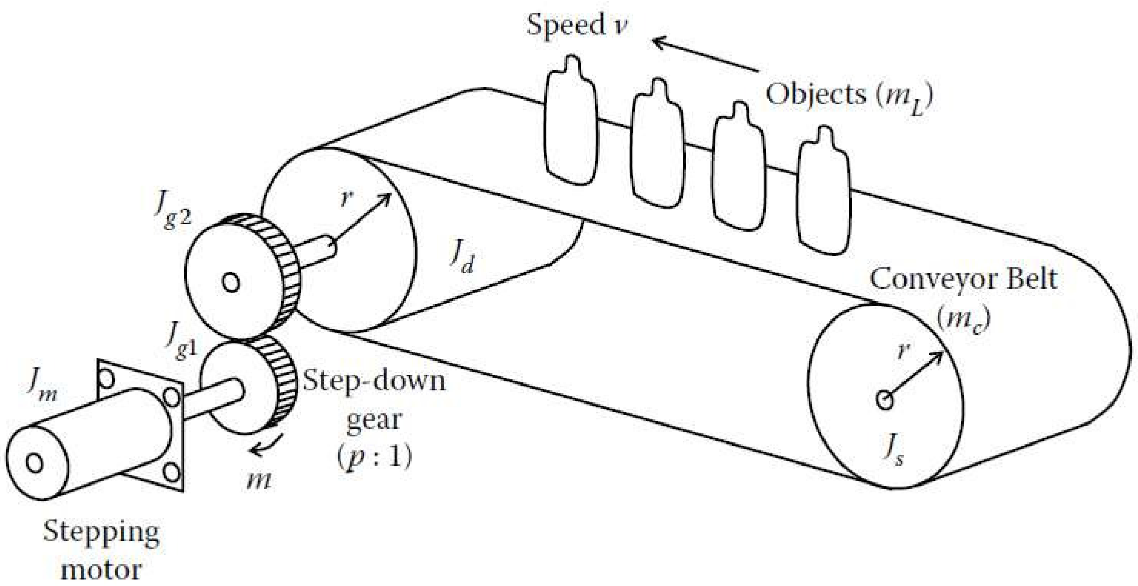


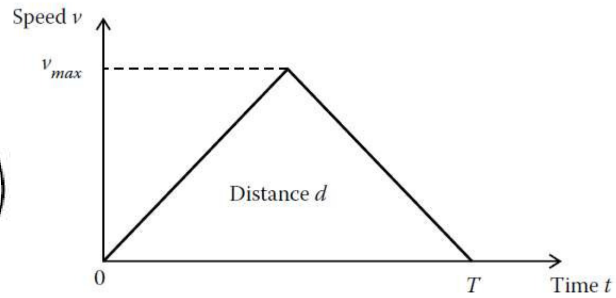
Recommended Drivers: DM4005, U1B

عملگرهای گامی

مثال:

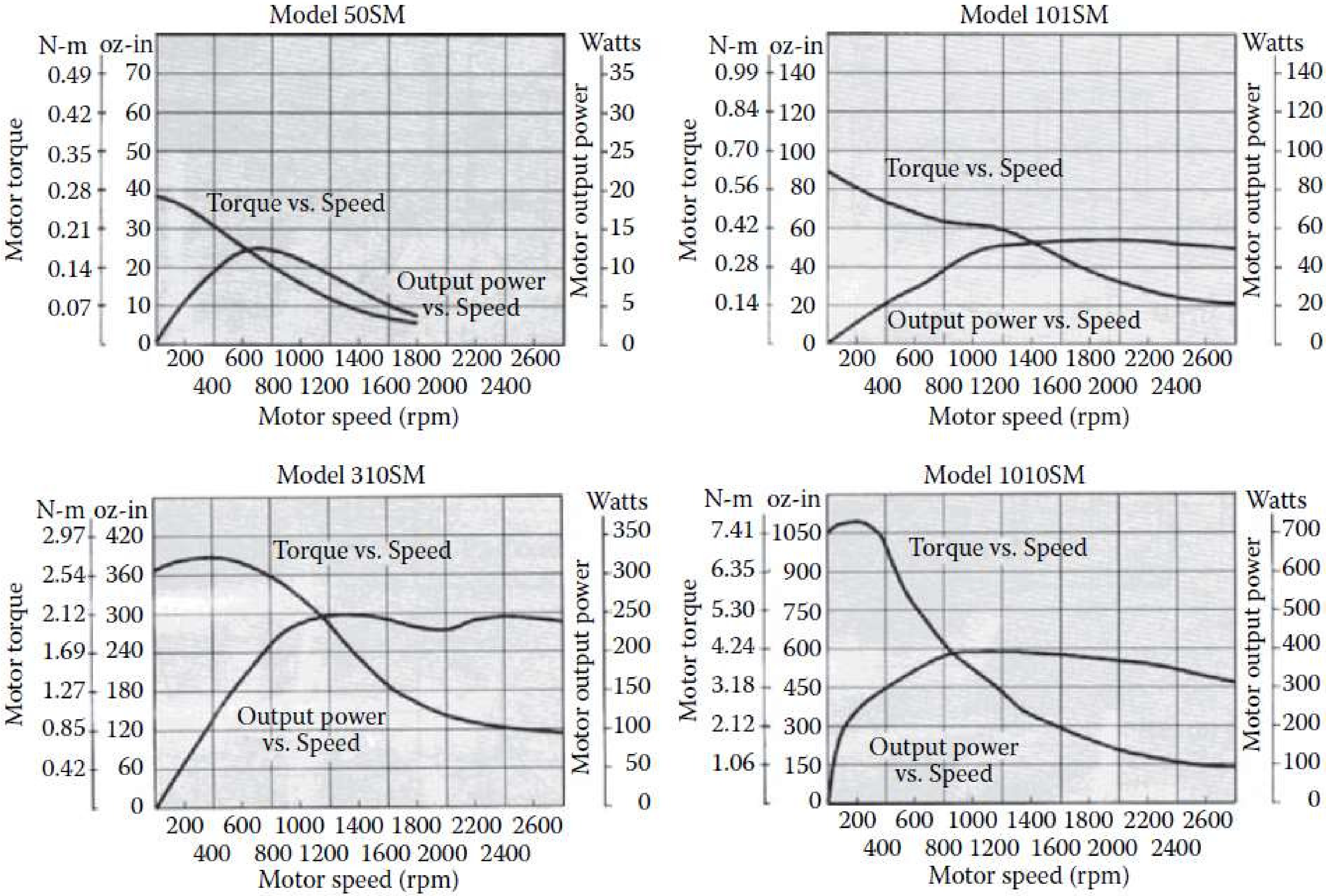
A schematic diagram of an industrial conveyor unit is shown in the figure. In this application, the conveyor moves intermittently at a fixed rate, thereby indexing the objects on the conveyor through a fixed distance d in each time period T. A triangular speed profile is used for each motion interval, having an acceleration and a deceleration that are equal in magnitude. The conveyor is driven by a stepper motor. A gear unit with step-down speed ratio p:1, where p >1, may be used if necessary, as shown in the figure.





عملگرهای گامی

مثال:



Mechatronics: An integrated approach, D Silva

عملگرهای گامی

مثال:

Stepper Motor Data

Model		50SM	101SM	310SM	1010SM
NEMA Motor frame size		23		34	42
Full step angle	degrees	1.8			
Accuracy	percent	±3 (noncumulative)			
Holding torque	oz-in	38	90	370	1050
	N-m	0.27	0.64	2.61	7.42
Detent torque	oz-in	6	18	25	20
	N-m	0.04	0.13	0.18	0.14
Rated phase current	Amps	1	5	6	8.6
Rotor inertia	oz-in-sec^2	1.66×10^{-3}	5×10^{-3}	26.5×10^{-3}	114×10^{-3}
	kg-m^2	11.8×10^{-6}	35×10^{-6}	187×10^{-6}	805×10^{-6}
Maximum radial load	lb	15		35	40
	N	67		156	178
Maximum thrust load	lb	25		60	125
	N	111		267	556
Weight	lb	1.4	2.8	7.8	20
	kg	0.6	1.3	3.5	9.1
Operating temperature	°C	–55 to +50			
Storage temperature	°C	–55 to +130			

Source: Aerotech, Inc. With permission.

Mechatronics: An integrated approach, D Silva

مثال:

a. Explain why the equivalent moment of inertia J_e at the motor shaft, for the overall system, is given by:

$$J_e = J_m + J_{g1} + (1/p^2)(J_{g2} + J_d + J_s) + (r^2/p^2)(m_c + m_L)$$

where J_m, J_{g1}, J_{g2}, J_d, and J_s are the moments of inertia of the motor rotor, drive gear, driven gear, drive cylinder of the conveyor, and the driven cylinder of the conveyor, respectively; m_c and m_L are the overall masses of the conveyor belt and the moved objects (load), respectively; and r is the radius of each of the two conveyor cylinders.

b. Four models of stepping motor are available for the application. Their specifications are given in Table 12.2 and the corresponding performance curves are given in Figure 12.16. The following values are known for the system:

$$d = 10 \text{ cm},\ T = 0.2 \text{ seconds},\ r = 10 \text{ cm},\ m_c = 5 \text{ kg},\ m_L = 5 \text{ kg},\ J_d = J_s = 2.0 \times 10^{-3} \text{ kg m}^2.$$

Also two gear units with $p = 2$ and 3 are available, and for each unit $J_{g1} = 50 \times 10^{-6}$ kg m^2 and $J_{g2} = 200 \times 10^{-6}$ kg m^2.

Indicating all calculations and procedures, select a suitable motor unit for this application. You must not use a gear unit unless it is necessary to have one with the available motors.

What is the positioning resolution of the conveyor (rectilinear) for the final system?

Note: Assume an overall system efficiency of 80% regardless of whether a gear unit is used.

حل مثال:

Angular speed of the motor and drive gear $= \omega_m$.
Angular speed of the driven gear and conveyor cylinders $= (\omega_m/p)$.
Rectilinear speed of the conveyor and objects $v = (r\omega_m/p)$.

$$KE = \frac{1}{2}(J_m + J_{g1})\omega_m^2 + \frac{1}{2}(J_{g2} + J_d + J_s)\left(\frac{\omega_m}{p}\right)^2 + \frac{1}{2}(m_c + m_L)\left(\frac{r\omega_m}{p}\right)^2$$

$$= \frac{1}{2}[J_m + J_{g1} + \frac{1}{p^2}(J_{g2} + J_d + J_s) + \frac{r^2}{p^2}(m_c + m_L)]\omega_m^2$$

$$= \frac{1}{2}J_e\omega_m^2$$

$$J_e = J_m + J_{g1} + \frac{1}{p^2}(J_{g2} + J_d + J_s) + \frac{r^2}{p^2}(m_c + m_L)$$

عملگرهای گامی

مثال:

Area of the speed profile is equal to the distance travelled. Hence:

$$d = \frac{1}{2} v_{\max} T$$

Substitute numerical values: $0.1 = \frac{1}{2} v_{\max} 0.2 \rightarrow v_{\max} = 1.0 \text{ m/s}$

The acceleration/deceleration of the system: $a = \frac{v_{\max}}{T/2} = \frac{1.0}{0.2/2} \text{ m/s}^2 = 10.0 \text{ m/s}^2$

Corresponding angular acceleration/deceleration of the motor:

$$\alpha = \frac{pa}{r}$$

مثال:

With an overall system efficiency of η, the motor torque T_m that is needed to accelerate/decelerate the system is given by:

$$\eta T_m = J_e \alpha = J_e \frac{pa}{r} = [J_m + J_{g1} + \frac{1}{p^2}(J_{g2} + J_d + J_s) + \frac{r^2}{p^2}(m_c + m_L)]\frac{pa}{r}$$

Maximum speed of the motor: $\omega_{\max} = \frac{pv_{\max}}{r}$

Without gears ($p=1$) we have

$$\eta T_m = [J_m + J_d + J_s + r^2(m_c + m_L)]\frac{a}{r}$$

$$\omega_{\max} = \frac{v_{\max}}{r}$$

حل مثال:

Case 1: Without Gears

$$0.8T_m = [J_m + 2\times10^{-3} + 2\times10^{-3} + 0.1^2(5+5)]\frac{10}{0.1} \text{ N.m}$$

$$\text{Or: } T_m = 125.0\,[J_m + 0.104] \text{ N.m}$$

$$\omega_{max} = \frac{1.0}{0.1} \text{ rad/s} = 10\times\frac{60}{2\pi} \text{ rpm} = 95.5 \text{ rpm}$$

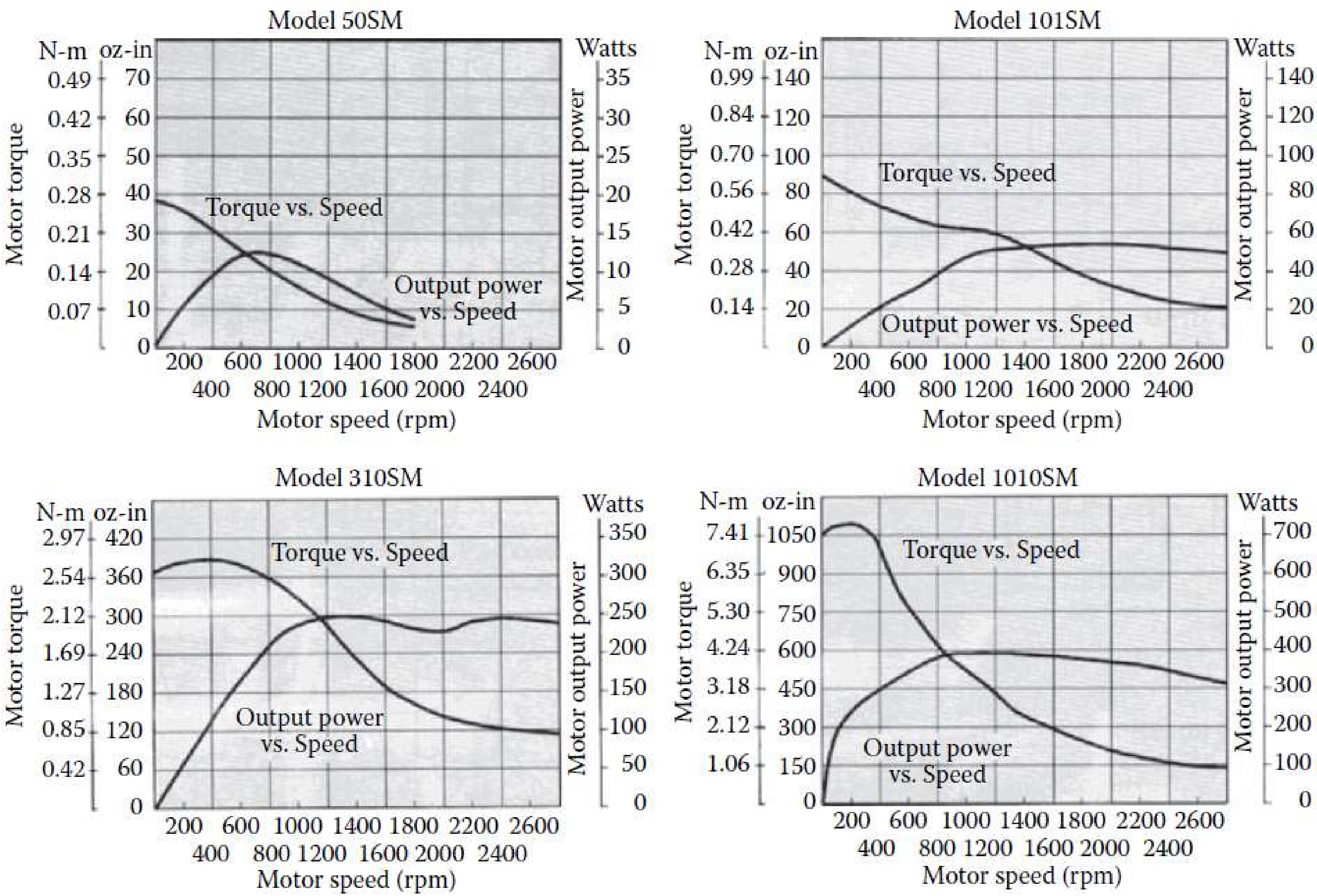
Data for Selecting a Motor without a Gear Unit

Motor Model	Available Torque at ω_{max} (N.m)	Motor Rotor Inertia (kg.m²)	Required Torque (N.m)
50 SM	0.26	11.8×10^{-6}	13.0
101 SM	0.60	35.0×10^{-6}	13.0
310 SM	2.58	187.0×10^{-6}	13.0
1010 SM	7.41	805.0×10^{-6}	13.1

عملگرهای گامی

Data for Selecting a Motor without a Gear Unit

Motor Model	Available Torque at ω_{max} (N.m)	Motor Rotor Inertia (kg.m²)	Required Torque (N.m)
50 SM	0.26	11.8×10^{-6}	13.0
101 SM	0.60	35.0×10^{-6}	13.0
310 SM	2.58	187.0×10^{-6}	13.0
1010 SM	7.41	805.0×10^{-6}	13.1



مثال:

Case 2: With Gears

$$0.8T_m = \left[J_m + 50\times10^{-6} + \frac{1}{p^2}(200\times10^{-6} + 2\times10^{-3} + 2\times10^{-3}) + \frac{0.1^2}{p^2}(5+5) \right] p \times \frac{10}{0.1} \text{ N.m}$$

$$\Rightarrow T_m = 125.0\left[J_m + 50\times10^{-6} + \frac{1}{p^2}\times104.2\times10^{-3} \right] p \text{ N.m}$$

$$\omega_{\max} = \frac{1.0p}{0.1} \text{ rad/s} = 10p\times\frac{60}{2\pi} \text{ rpm} \Rightarrow \omega_{\max} = 95.5p \text{ rpm}$$

Data for Selecting a Motor with a Gear Unit

Motor Model	Available Torque at $\omega_{\max}$ (N.m)	Motor Rotor Inertia (kg.m²)	Required Torque (N.m)
50 SM	0.25	11.8×10^{-6}	6.53
101 SM	0.58	35.0×10^{-6}	6.53
310 SM	2.63	187.0×10^{-6}	6.57
1010 SM	7.41	805.0×10^{-6}	6.73

عملگرهای گامی

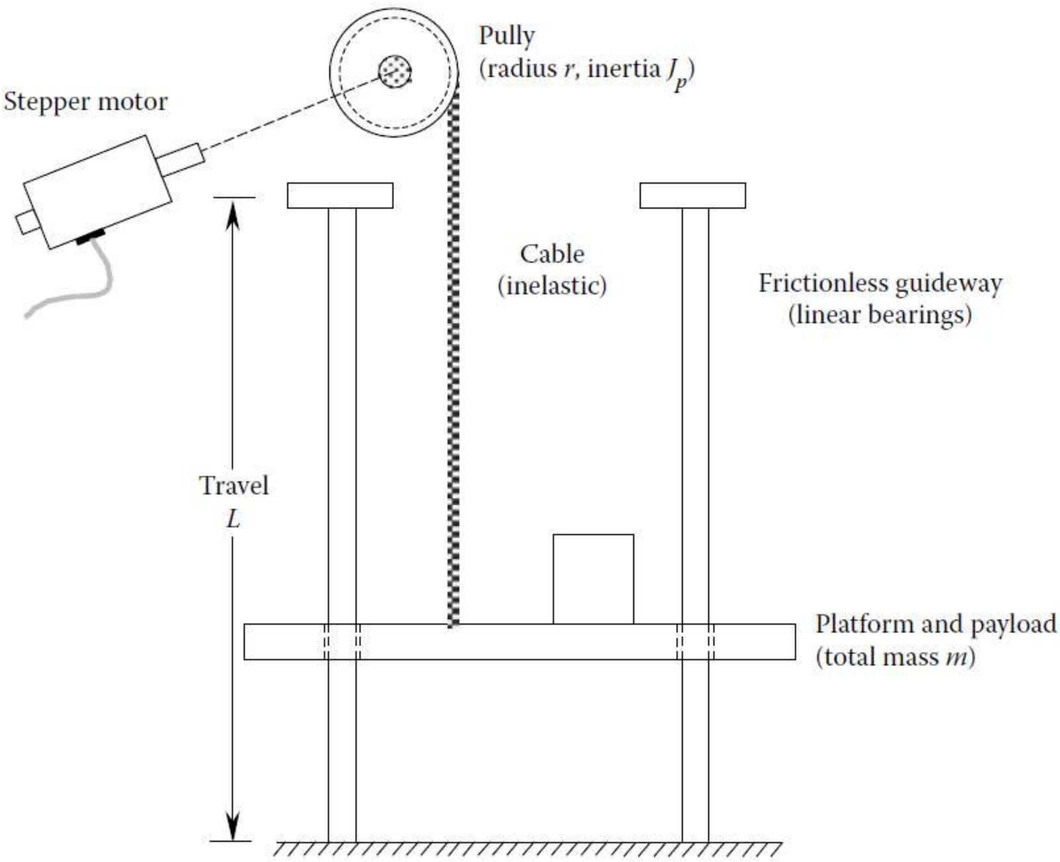
مثال:

With full stepping, step angle of the rotor = 1.8°. Corresponding step in the conveyor motion is the positioning resolution.

With $p = 2$ and $r = 0.1$ m, the position resolution is $\frac{1.8^\circ}{2} \times \frac{\pi}{180^\circ} \times 0.1 = 1.57 \times 10^{-3}$ m.

- ✓ تکلیف

A stepper-motor-driven positioning platform is schematically shown in the figure. Suppose that the maximum travel of the platform is L and this is accomplished in a time period of Δt. A trapezoidal velocity profile is used with a region of constant speed V in between an initial region of constant acceleration from rest and a final region of constant deceleration to rest, in a symmetric manner.



(i) Show that the acceleration is given by:

$$a = V^2/(V \cdot \Delta t - L)$$

The platform is moved using a mechanism of light, inextensible cable and a pulley, which is directly (without gears) driven by a stepper motor. The platform moves on a pair of vertical guideways that use linear bearings and, for design purposes, the associated frictional resistance to platform motion may be neglected. The frictional torque at the bearings of the pulley is not negligible, however. Suppose that:

باسمه تعالی

تکلیف بخش موتورهای پله‌ای درس مکاترونیک   موعد تحویل: ۹۵/۱/۳۱

$$\frac{\text{Frictional torque of the pulley}}{\text{Load torque on the pulley from the cable}} = e$$

Also, the following parameters are known:
J_p = moment of inertia of the pulley about the axis of rotation
r = radius of the pulley
m = equivalent mass of the platform and its payload.

(ii) Show that the maximum operating torque that is required from the stepper motor is given by:

$$T = \left[J_m + J_p + (1+e)mr^2\right]\frac{a}{r} + (1+e)rmg$$

in which J_m = moment of inertia of the motor rotor.

(iii) Suppose that $V = 8.0$ m/s, $L = 1.0$ m, $\Delta t = 1.0$ s, $m = 1.0$ kg, $J_p = 3.0 \times 10^{-4}$ kg.m², $r = 0.1$ m, and $e = 0.1$.

Four models of stepper motor are available, and their specifications given in the table and figure. Select the most appropriate motor (with the corresponding drive system) for this application. Clearly indicate all your computations and justify your choice.

(iv) What is the position resolution of the platform, as determined by the chosen motor?

توجه: موتور مناسب را از بین موتورهای اسلایدهای ۱۹ و ۲۰ انتخاب نمایید.