دانشگاه صنعتی اصفهان
دانشکده مهندسی حمل و نقل

پژوهش عملیاتی

مسئله واگذاری

Assignment Problem

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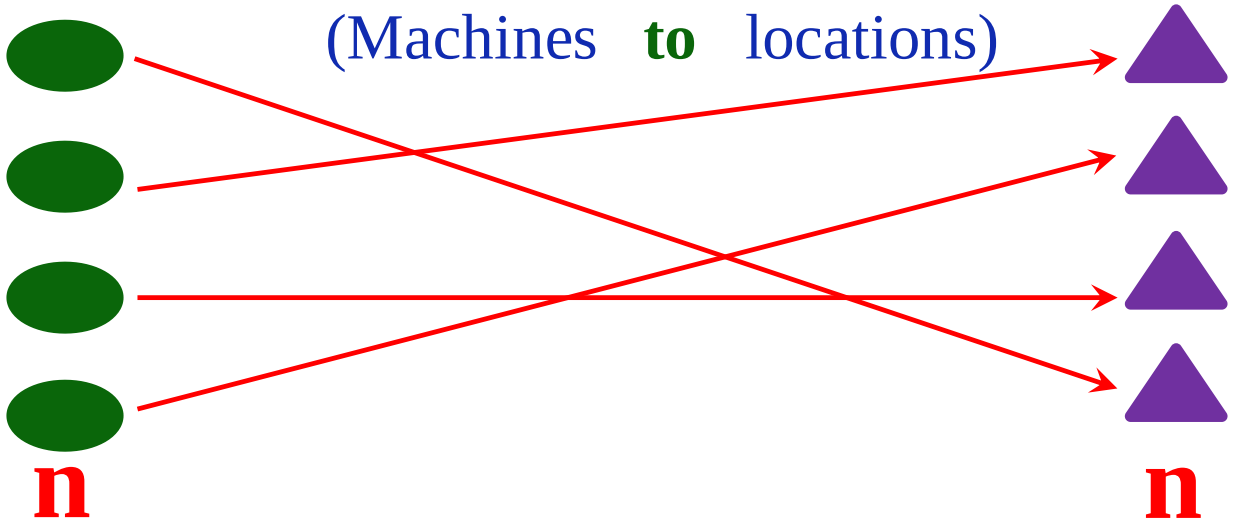
پاییز ۱۳۹۴

Assignment Problem:

Assignees are being assigned to perform tasks.

(People **to** Jobs)

(Machines **to** locations)

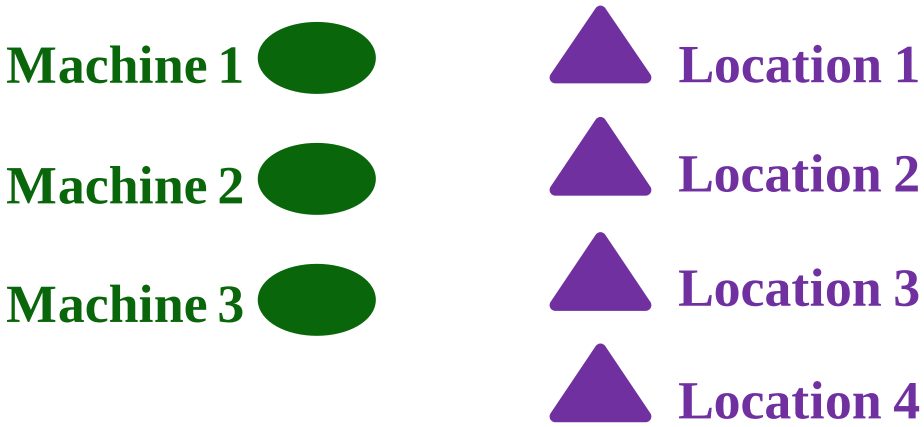


Assumptions:

1. Number of assignees = Number of tasks
2. Each assignee is to be assigned to exactly 1 task.
3. Each task is to be performed by exactly 1 assignee.
4. Cost C_{ij} = assignee i performing task j.
5. Objective: how to assign all to minimize total cost.

Example: JOB SHOP COMPANY

Assigning 3 new machines to 4 available locations in the shop



Location 2 is not considered suitable for Machine 2

How to formulate it as an assignment problem?

A dummy machine for extra location

How to prevent assignment of Machine 2 to location 2?

A large cost M

Materials-handling cost data ($) for Job Shop Co.

		Location			
		1	2	3	4
Machine	1	13	16	12	11
	2	15	—	13	20
	3	5	7	10	6

Cost table for the Job Shop Co. assignment problem

		Task (Location)			
		1	2	3	4
Assignee (Machine)	1	13	16	12	11
	2	15	*M*	13	20
	3	5	7	10	6
	4(*D*)	0	0	0	0

No cost for assigning a Dummy machine

Variables

$$x_{ij} \begin{cases} 1 & \text{if assignee } i \text{ performs task } j \\ 0 & \text{if not} \end{cases}$$

i = 1, 2, ..., n
j = 1, 2, ..., n

Objective Function

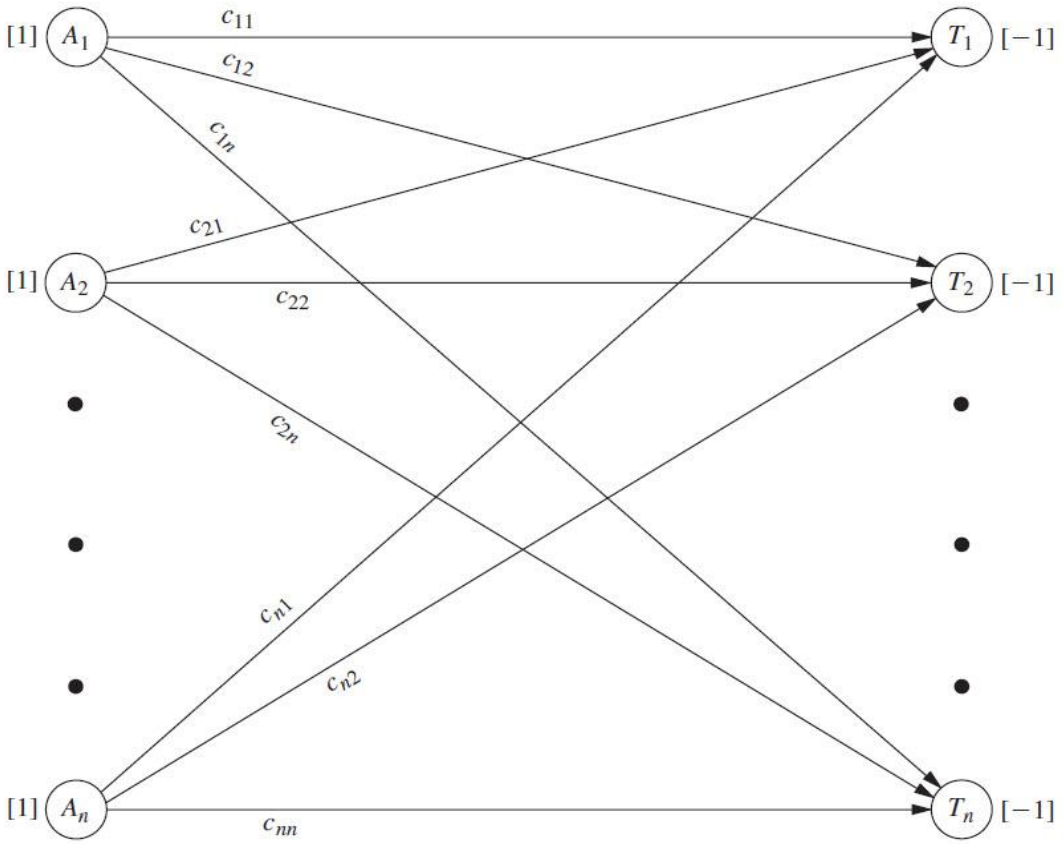
Minimize total cost $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$

Constraints

$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i = 1,2,...,n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \forall j = 1,2,...,n$$

$$x_{ij} \geq 0 \quad \forall i, j \quad (x_{ij} \text{ Binary})$$



$x_{ij} \geq 0 \quad \forall i, j$

$(x_{ij} \ \ Binary)$ LP problem ?

Integer Solutions Property (in ransportation problem) model:

s_i and d_j are integers (= 1)
every BF solution (including optimal one) is “integer solution”

Deleting “Binary restriction”:

BF solutions automatically will satisfy the binary restriction

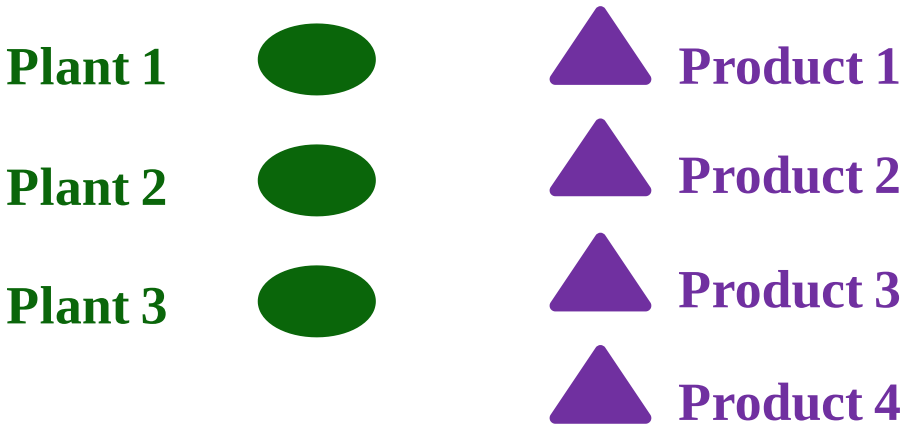
Example: Assigning Products to Plants

Production of 4 products, using 3plants that currently have excess production capacity.

		Unit Cost ($) for Product				Capacity Available
		1	2	3	4	
Plant	1	41	27	28	24	75
	2	40	29	—	23	75
	3	37	30	27	21	45
Production rate		20	30	30	40	

Required Production per day

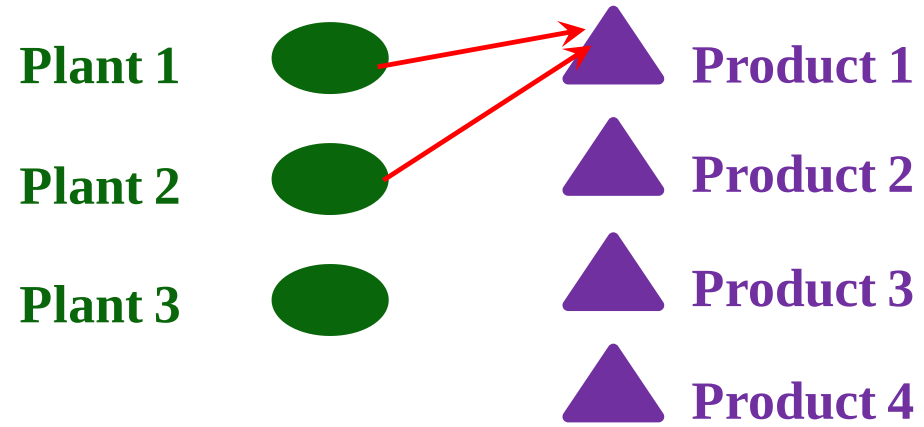
Plant 2 cannot produce product 3



Two kinds of options are available:

Option 1:

Permit product splitting
(the same product is produced in more than 1 plant)

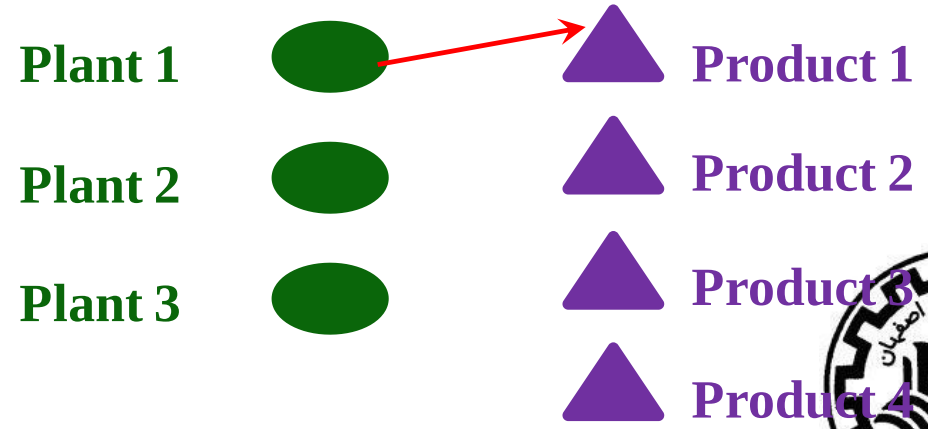


It leads to a transportation problem

Option 2:

Prohibit product splitting.

Also, Every plant should Produce At least 1 product



It leads to an assignment problem

Option 1 (Permit product splitting)

		Unit Cost ($) for Product				Capacity Available
		1	2	3	4	
Plant	1	41	27	28	24	75
	2	40	29	—	23	75
	3	37	30	27	21	45
Production rate		20	30	30	40	

Total capacity (75 + 75 + 45 = 195)

Total required production (20 + 30 + 30 + 40 = 120)

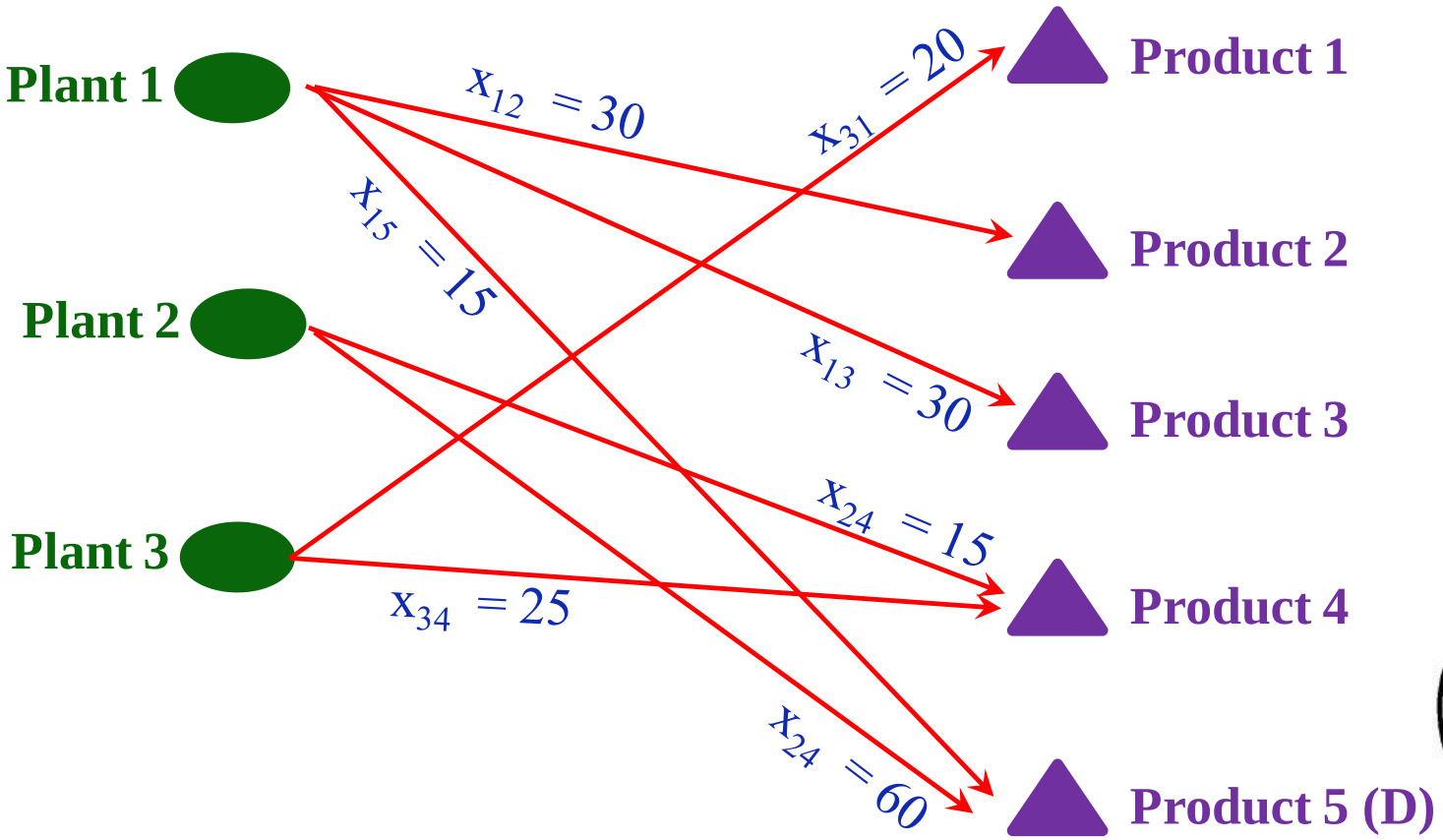
A dummy destination with demand of 75 is needed

		Cost per Unit Distributed					
		Destination (Product)					
		1	2	3	4	5(D)	Supply
Source (Plant)	1	41	27	28	24	0	75
	2	40	29	*M*	23	0	75
	3	37	30	27	21	0	45
Demand		20	30	30	40	75	

Option 1 (Permit product splitting)

Optimal Value: **3260 $**

Optimal Solution:



Option 2 (Prevent product splitting)

Also, Every plant should Produce At least 1 product

		Unit Cost ($) for Product				Capacity Available
		1	2	3	4	
Plant	1	41	27	28	24	75
	2	40	29	—	23	75
	3	37	30	27	21	45
Production rate		20	30	30	40	

Assignment Problem: Plants to Products

- One of the plants will need to be assigned 2 products.
- Plant 3 cannot produce more than 1 product.
- Either Plant 1 or Plant 2 will produce 2 product.

Assignees (n) = tasks (n)

How to make assignment of an extra product possible?

Plant 1 and Plant 2 each are split into two assignees

Option 2 (Prevent product splitting)

		Unit Cost ($) for Product				Capacity Available
		1	2	3	4	
Plant	1	41	27	28	24	75
	2	40	29	—	23	75
	3	37	30	27	21	45
Production rate		20	30	30	40	

960 = 24 * 40

Plant 3 Should Produce At least 1 real product

Assignment Problem: Plants to Products

Cost table

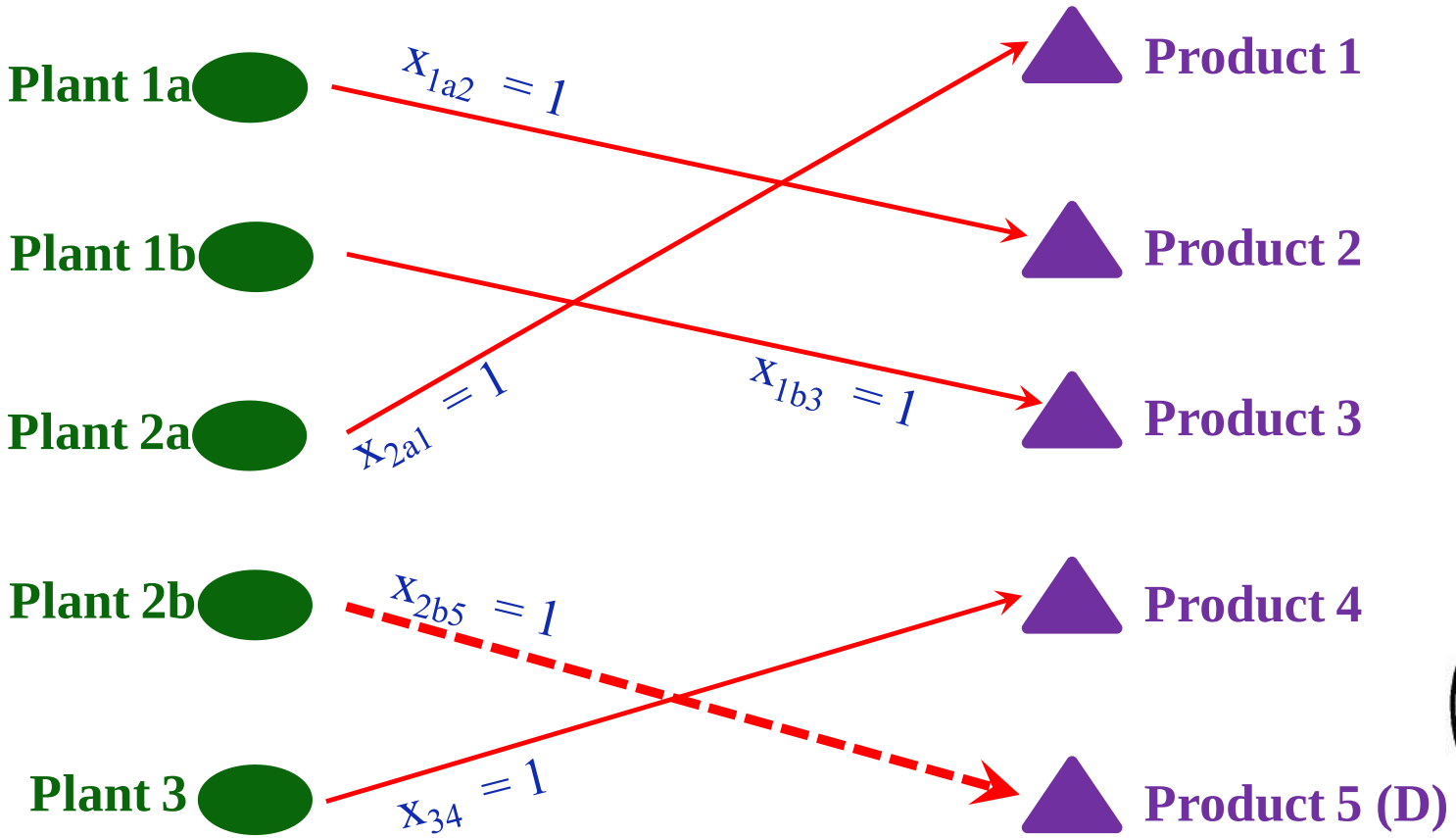
		Task (Product)				
		1	2	3	4	5(D)
Assignee (Plant)	1a	820	810	840	960	0
	1b	820	810	840	960	0
	2a	800	870	M	920	0
	2b	800	870	M	920	0
	3	740	900	810	840	M

Plant 3 must be assigned a real product
(a choice of product 1, 2, 3, or 4)

Option 2 (Prevent product splitting)

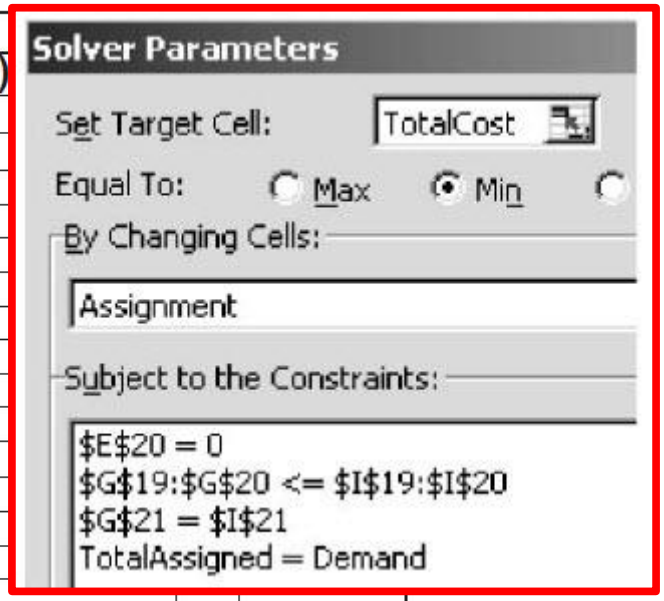
Optimal Value: **3290 $**

Optimal Solution:



Option 2 (Prevent product splitting)

	A	B	C	D	E	F			
1	Better Products Co. Production Planning Problem (Option 2)								
2									
3		Unit Cost	Product 1	Product 2	Product 3	Product 4			
4		Plant 1	$41	$27	$28	$24			
5		Plant 2	$40	$29	-	$23			
6		Plant 3	$37	$30	$27	$21			
7									
8		Required Production	20	30	30	40			
9									
10									
11		Cost ($/day)	Product 1	Product 2	Product 3	Product 4			
12		Plant 1	$820	$810	$840	$960			
13		Plant 2	$800	$870	-	$920			
14		Plant 3	$740	$900	$810	$840			
15									
16									
17							Total		
18		Assignment	Product 1	Product 2	Product 3	Product 4	Assignments		Supply
19		Plant 1	0	1	1	0	2	<=	2
20		Plant 2	1	0	0	0	1	<=	2
21		Plant 3	0	0	0	1	1	=	1
22		Total Assigned	1	1	1	1			
23			=	=	=	=			Total Cost
24		Demand	1	1	1	1			$3,290



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a), b), c),

d) Specify the optimal solution using excel

F. Hillier, G. J. Lieberman, “Introduction to Operations Research”, Ninth Edition, 2010.