PROBLEM OF THE WEEK
Solution of Problem No. 9 (Fall 2013 Series)

Problem:

Let E be the points of the $x-y$ plane which lie inside an elipse centered at the origin, and let D be those points inside the unit circle centered at the origin. Prove that the area of $D \cap E$ is at least as large as the area of D intersected with any translation of E. (This is, show $|D \cap E| \geq |D \cap \{(x,y)+(a,b)\colon (x,y) \in E\}|$ for every a, b.)

Solution: (by the Panel)

Equivalently fix E, centered at $(0,0)$, and let the center of D vary. If ℓ is any vertical line then $|D \cap E \cap \ell| \leq \min(|D \cap \ell|, |E \cap \ell|)$, with equality when the center of D lies on the x–axis. By Cavalieri's principle, $|D \cap E|$ is increased by moving the center of D to its projection on the x–axis. Slicing with horizontal lines and applying the same argument again moves the center to the origin.

The problem was also solved by:
Steven Landy (IUPUI Physics Faculty) and Craig Schroeder (UCLA Postdoc.)