PROBLEM OF THE WEEK
Solution of Problem No. 5 (Fall 2013 Series)

Problem:

A standard six sided die is rolled forever. Let T_k be the total of all the dots rolled in the first k rolls. Find the probability that one of the T_k is eight.

Solution 1: (by Hubert Desprez, Paris, France)

There are 20 disjoint possibilities for our event with $2 \leq k \leq 8$; they are permutations of:

26 35 44
116 125 134 224 233
1115 1124 1133 1223 2222
11114 11123 11222
111113 111122
11111112
11111111

$$p = \frac{5}{6^2} + \frac{21}{6^3} + \frac{35}{6^4} + \frac{35}{6^5} + \frac{21}{6^6} + \frac{7}{6^7} + \frac{1}{6^8}$$

$$p = \frac{450295}{6^8} \simeq 0.2681$$

Solution 2: (by David Stoner, Student at South Aiken High School, Aiken, S. Carolina)

Let $P(n)$ denote the probability that n occurs in the sequence $T_k, k \geq 0$. Clearly, $P(-1) = P(-2) = P(-3) = P(-4) = P(-5) = 0$ and $P(0) = 1$. Now note that for $n \geq 1$, we have $P(n) = \frac{1}{6}\Big(P(n-1)+P(n-2)+P(n-3)+P(n-4)+P(n-5)+P(n-6)\Big)$ (This follows

from considering the scenario after the first roll.) Now we can directly apply this to find:

$$P(1) = \frac{1}{6}$$
$$P(2) = \frac{7}{36}$$
$$P(3) = \frac{49}{216}$$
$$P(4) = \frac{343}{1296}$$
$$P(5) = \frac{2401}{7776}$$
$$P(6) = \frac{16807}{46656}$$
$$P(7) = \frac{70993}{279936}$$
$$P(8) = \frac{450295}{1679616}$$

This is about 0.268094.

The problem was also solved by:

Undergraduates: Bennett Marsh (Jr. Engr.), Elisha Rothenbush (Fr. Chem, Math, Phys), Jingbo Wu (So. Tech.)

Graduates: Sambit Palit (ECE), Tairan Yuwen (Chemistry), Samson Zhou (CS)

Others: Pierre Castelli (Antibes, France), Hongwei Chen (Professor, Christopher Newport Univ., Virginia), Tom Engelsman (Tampa, FL), Andrew Garmon (Christopher Newport University alumni), Elie Ghosn (Montreal, Quebec), Srikanth Gopalan (Professor, Boston Univ.), Mohammed Hamami (AT & T), Kipp Johnson (Valley Catholic HS teacher, Oregon), Peter Kornya (Retired Faculty, Ivy Tech), Steven Landy (Physics Faculty, IUPUI), Wei-Xiang Lien (Miaoli, Taiwan), Vladimir B. Lukianov (Lecturer, Tel-Aviv), Sorin Rubinstein (TAU faculty,Tel Aviv, Israel), Krishnaraj Sambath, Craig Schroeder (Postdoc. UCLA), Mark Senn (Systems Programmer, Purdue Univ.), Bruce Grayson & Rick Shilling (Orlando, FL), Aaron Tang (Student, National Univ. of Singapore), Motohiro Tsuchiya (Graduate student, Bethesda, MD)