Linearization Techniques Used in PEVFCS Sizing and Placement

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I. Preface

Here, we address the linearization techniques employed in the paper titled, "PEV Fast-Charging Station Sizing and Placement in Coupled Transportation-Distribution Networks Considering Power Line Conditioning Capability". At first, the MINLP model is represented. Then, the employed linearization techniques are described. Finally, the resulted MILP model is expressed.

The notation used here are similar to that of the original paper. As needed, other symbols are defined throughout the text.

II. MINLP model

$$\min \sum_z (C_z^{fix}u_z + C_z^{sp}d_z) + \sum_t\sum_a\sum_z C_{z,a}^{wait}n_{z,a,t} + \beta\sum_t\sum_r\sum_z W_{r,t}x_{z,r,t}T_{z,r,t} + \sum_{(b,z)\in\Theta_b^z} C_b^S s_b^{st}u_z + \sum_t\sum_s\sum_{b\in\Phi^{mainB}} C_{s,t}^P p_{b,s,t} + \sum_t\sum_s\sum_{b\in\Phi^{mainB}} C_{s,t}^Q q_{b,s,t} \tag{1}$$

Subject to:

$$\sum_a n_{z,a,t} = \sum_r W_{r,t}x_{z,r,t}, \quad \forall z,t \tag{2}$$

$$n_{z,a,t} \le \Delta n_{z,a} d_z, \quad \forall z,a,t \tag{3}$$

$$\sum_z x_{z,r,t} = 1, \quad \forall t,r \tag{4}$$

$$\sum_r W_{r,t}x_{z,r,t} \le \frac{P_b^{sp}\times\Delta t}{Cu\times Rng} d_z, \quad \forall (z,b)\in\Theta_b^z, t \tag{5}$$

$$u_z \le d_z \le D^{max}u_z, \quad \forall z \tag{6}$$

$$u_z \in \{0,1\}, \quad \forall t,z \tag{7}$$

$$x_{z,r,t} \ge 0, \quad \forall z,r,t \tag{8}$$

$$n_{z,a,t} \ge 0, \quad \forall z,a,t \tag{9}$$

$$i_{h,b,s,t}^r = \sum_{k\in\Phi^B}(G_{h,b,k}^B v_{h,k,s,t}^r - B_{h,b,k}^B v_{h,k,s,t}^{im}), \quad \forall h,b,s,t \tag{10}$$

$$i_{h,b,s,t}^{im} = \sum_{k\in\Phi^B}(G_{h,b,k}^B v_{h,k,s,t}^{im} + B_{h,b,k}^B v_{h,k,s,t}^r), \quad \forall h,b,s,t \tag{11}$$

$$p_{b,s,t} = v_{h_1,b,s,t}^r i_{h_1,b,s,t}^r + v_{h_1,b,s,t}^{im} i_{h_1,b,s,t}^{im}, \quad \forall b,s,t \tag{12}$$

$$q_{b,s,t} = -v_{h_1,b,s,t}^r i_{h_1,b,s,t}^{im} + v_{h_1,b,s,t}^{im} i_{h,b,s,t}^r, \quad \forall b,s,t \tag{13}$$

$$p_{b,s,t} = P_{b,s,t}^G - p_{b,s,t}^D, \quad \forall b\notin\Phi^{mainB}, s,t \tag{14}$$

$$q_{b,s,t} = Q_{b,s,t}^G + Q_{b,s,t}^C - q_{b,s,t}^D, \quad \forall b\notin\Phi^{mainB}, s,t \tag{15}$$

$$p_{b,s,t}^D = P_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} P_{nl,s,t}^{NL} + p_{b,s,t}^{st}, \quad \forall b,s,t \tag{16}$$

$$q_{b,s,t}^D = Q_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} Q_{nl,s,t}^{NL} + q_{b,s,t}^{st}, \quad \forall b,s,t \tag{17}$$

$$i_{h,b,s,t}^r = -i_{h,b,s,t}^{st,r} - i_{h,b,s,t}^{NL,r}, \quad \forall h\ne h_1, b,s,t \tag{18}$$

$$i_{h,b,s,t}^{im} = -i_{h,b,s,t}^{st,im} - i_{h,b,s,t}^{NL,im}, \quad \forall h\ne h_1, b,s,t \tag{19}$$

$$P_{nl,s,t}^{NL} = i_{nl,b,s,t}^{NL,r1} v_{h_1,b,s,t}^r + i_{nl,b,s,t}^{NL,im1} v_{h_1,b,s,t}^{im}, \quad \forall (nl,b)\in\Theta_b^{nl}, s,t \tag{20}$$

$$Q_{nl,s,t}^{NL} = i_{nl,b,s,t}^{NL,r1} v_{h_1,b,s,t}^{im} - i_{nl,b,s,t}^{NL,im1} v_{h_1,b,s,t}^r, \quad \forall (nl,b)\in\Theta_b^{nl}, s,t \tag{21}$$

$$i_{h,b,s,t}^{NL,r} = \sum_{nl\in\Theta_b^{nl}} \left(C_{nl,h}^r i_{nl,b,s,t}^{NL,r1} - C_{nl,h}^{im} i_{nl,b,s,t}^{NL,im1}\right), \quad \forall h,b,s,t \tag{22}$$

$$i_{h,b,s,t}^{NL,im} = \sum_{nl\in\Theta_b^{nl}} \left(C_{nl,h}^r i_{nl,b,s,t}^{NL,im1} + C_{nl,h}^{im} i_{nl,b,s,t}^{NL,r1}\right), \quad \forall h,b,s,t \tag{23}$$

$$\underline{V}^{rms} \le v_{b,s,t}^{rms} \le \overline{V}^{rms}, \quad \forall b,s,t \tag{24}$$

$$v_{b,s,t}^{rms} = \sqrt{\sum_h \left(v_{h,b,s,t}^{mg}\right)^2}, \quad \forall b,s,t \tag{25}$$

$$v_{h,b,s,t}^{mg} = \sqrt{(v_{h,b,s,t}^r)^2 + \left(v_{h,b,s,t}^{im}\right)^2}, \quad \forall h,b,s,t \tag{26}$$

$$THD_{b,s,t}^V \le THD^{max}, \quad \forall b,s,t \tag{27}$$

$$THD_{b,s,t}^V = \frac{\sqrt{\sum_{h\ne 1}\left(v_{h,b,s,t}^{mg}\right)^2}}{v_{h_1,b,s,t}^{mg}}, \quad \forall b,s,t \tag{28}$$

$$\frac{v_{h,b,s,t}^{mg}}{v_{h_1,b,s,t}^{mg}} \le IHD^{max}, \quad \forall h\ne h_1, b,s,t \tag{29}$$

$$i_{l,s,t}^{L,rms} \le I_l^{max}, \quad \forall l,s,t \tag{30}$$

$$i_{l,s,t}^{L,rms} = \sqrt{\sum_h \left(i_{h,l,s,t}^{L,mg}\right)^2}, \quad \forall l,s,t \tag{31}$$

$$i_{h,l,s,t}^{L,mg} = \sqrt{(i_{h,l,s,t}^{L,r})^2 + (i_{h,l,s,t}^{L,im})^2}, \quad \forall h,l,s,t \tag{32}$$

$$p_{b,t}^{st} = \frac{\left(\sum_{a\in\Psi^A} n_{z,a,t}\right)\times Cu\times Rng}{\Delta t\times\eta}, \quad \forall (b,z)\in\Theta_b^z, s,t \tag{33}$$

$$p_{b,s,t}^{st} = v_{h_1,b,s,t}^r i_{h_1,b,s,t}^{st,r} + v_{h_1,b,s,t}^{im} i_{h_1,b,s,t}^{st,im}, \quad \forall b,s,t \tag{34}$$

$$q_{b,s,t}^{st} = -v_{h_1,b,s,t}^r i_{h_1,b,s,t}^{st,im} + v_{h_1,b,s,t}^{im} i_{h_1,b,s,t}^{st,r}, \quad \forall b,s,t \tag{35}$$

$$i_{b,s,t}^{st,rms} \le \frac{s_b^{st}u_z}{\underline{V}^{rms}}, \quad \forall (b,z)\in\Theta_b^z, s,t \tag{36}$$

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$$i_{b,s,t}^{st,rms} = \sqrt{\sum_{h\in\Phi^H} \left(i_{h,b,s,t}^{st,mg} \right)^2}, \quad \forall b,\, s,\, t \tag{37}$$

$$i_{h,b,s,t}^{st,mg} = \sqrt{\left(i_{h,b,s,t}^{st,r} \right)^2 + \left(i_{h,b,s,t}^{st,im} \right)^2}, \quad \forall h,\, b,\, s,\, t \tag{38}$$

III. Linearization Techniques

In model (1)-(38), equations (12), (13), (20), (21), (25), (26), (28), (29), (31), (32) and (34)-(38) are nonlinear. In following subsections, we express linearization techniques.

A. Linearization of Harmonic Power Flow Constraints ((10)-(23))

Within the harmonic power flow constraints, (12), (13), (20) and (21) are nonlinear. To linearize the harmonic power flow constraints the method proposed in [1] is used. In [1], by using Taylor series for inverse of voltage on the complex plane, a linear approximation is presented. By using this method, (12)-(17) are replaced by (39) and (40).

$$\begin{aligned} i_{h_1,b,s,t}^{r} &= (P_{b,s,t}^{G} - (P_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} P_{nl,s,t}^{NL}))\left(2 - v_{h_1,b,s,t}^{r}\right) \\ &\quad + (Q_{b,s,t}^{G} + Q_{b,s,t}^{C} - (Q_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} Q_{nl,s,t}^{NL}))(v_{h_1,b,s,t}^{im}) \\ &+ i_{h_1,b,s,t}^{str}, \qquad \forall b \notin \Phi^{mainB},\, s,\, t \end{aligned} \tag{39}$$

$$\begin{aligned} i_{h_1,b,s,t}^{im} &= (P_{b,s,t}^{G} - (P_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} P_{nl,s,t}^{NL}))v_{h_1,b,s,t}^{im} \\ &\quad + (Q_{b,s,t}^{G} + Q_{b,s,t}^{C} - (Q_{b,s,t}^{D0} + \sum_{nl\in\Theta_b^{nl}} Q_{nl,s,t}^{NL}))\left(-2 + v_{h_1,b,s,t}^{im}\right) \\ &\quad + i_{h_1,b,s,t}^{stim}, \qquad \forall b \notin \Phi^{mainB},\, s,\, t \end{aligned} \tag{40}$$

The bus connected to the upstream network is considered as the reference bus. We assume that the magnitude and phase of the first (fundamental frequency) harmonic of reference bus voltage are 1 and 0, respectively. Thus, (12) and (13) for the reference bus can be formulated as (41) and (42), respectively.

$$p_{b,s,t} = \quad i_{h_1,b,s,t}^{r}, \quad \forall b \in \Phi^{mainB},\, s,\, t \tag{41}$$

$$q_{b,s,t} = -i_{h_1,b,s,t}^{im}, \quad \forall b \in \Phi^{mainB},\, s,\, t \tag{42}$$

Also, (20) and (21) are replaced by (43) and (44).

$$\bar{i}_{nl,b,s,t}^{NLr1} = P_{nl,s,t}^{NL}\left(2 - v_{h_1,b,s,t}^{r}\right) + Q_{nl,s,t}^{NL} v_{h_1,b,s,t}^{im}, \quad \forall\left(nl,\, b\right) \in \Theta_b^{nl},\, s,\, t \tag{43}$$

$$\bar{i}_{nl,b,s,t}^{NLim1} = Q_{nl,s,t}^{NL}\left(-2 + v_{h_1,b,s,t}^{r}\right) + P_{nl,s,t}^{NL} v_{h_1,b,s,t}^{im}, \quad \forall\left(nl,\, b\right) \in \Theta_b^{nl},\, s,\, t \tag{44}$$

Therefore, (10), (11), (39)-(42), (18), (19), (43), (44), (22) and (23) are the linear representation of harmonic power flow constraints (10)-(23).

B. Linearization of PEVFCS Constraints ((33)-(38))

Among the PEVFCS constraints, (34)-(38) are nonlinear. Equation (34) has two terms that are the product of two variables. In order to linearize this equation, the method presented in [2], piecewise linear approximation of a function of two nonseparable variables, is used. This method is explained in appendix A. By this method, piecewise linear approximation of (34) is as (45)-(62).

$$p_{b,s,t}^{st} = \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{r} dV_i^{r} dI_j^{str} + \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{im} dV_i^{im} dI_j^{stim}, \quad \forall b,\, s,\, t \tag{45}$$

$$v_{h_1,b,s,t}^{r} = \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{r} dV_i^{r}, \quad \forall b,\, s,\, t \tag{46}$$

$$i_{h_1,b,s,t}^{st,r} = \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{r} dI_j^{str}, \quad \forall b,\, s,\, t \tag{47}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{r} = 1, \quad \forall b,\, s,\, t \tag{48}$$

$$\sum_{i=0}^{n}\sum_{j\in\Gamma^{+}(j,k)} \gamma_{b,s,t,i,j}^{r} \le a_{b,s,t,k}^{Vr}, \quad \forall b,\, s,\, t,\, k \in \Lambda_1 \tag{49}$$

$$\sum_{i=0}^{n}\sum_{j\in\Gamma^{0}(j,k)} \gamma_{b,s,t,i,j}^{r} \le 1 - a_{b,s,t,k}^{Vr}, \quad \forall b,\, s,\, t,\, k \in \Lambda_1 \tag{50}$$

$$\sum_{j=0}^{m}\sum_{i\in\Gamma^{+}(i,k)} \gamma_{b,s,t,i,j}^{r} \le a_{b,s,t,k}^{Ir}, \quad \forall b,\, s,\, t,\, k \in \Lambda_2 \tag{51}$$

$$\sum_{j=0}^{m}\sum_{i\in\Gamma^{0}(i,k)} \gamma_{b,s,t,i,j}^{r} \le 1 - a_{b,s,t,k}^{Ir}, \quad \forall b,\, s,\, t,\, k \in \Lambda_2 \tag{52}$$

$$\sum_{(i,j)\in\rho} \gamma_{b,s,t,i,j}^{r} \le a_{b,s,t}^{r0}, \quad \forall b,\, s,\, t \tag{53}$$

$$\sum_{(i,j)\in\sigma} \gamma_{b,s,t,i,j}^{r} \le 1 - a_{b,s,t}^{r0}, \quad \forall b,\, s,\, t \tag{54}$$

$$v_{h_1,b,s,t}^{im} = \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{im} dV_i^{im}, \quad \forall b,\, s,\, t \tag{55}$$

$$i_{h_1,b,s,t}^{im} = \sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{im} dI_j^{stim}, \quad \forall b,\, s,\, t \tag{56}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{m} \gamma_{b,s,t,i,j}^{im} = 1, \quad \forall b,\, s,\, t \tag{57}$$

$$\sum_{i=0}^{n}\sum_{j\in\Gamma^{+}(j,k)} \gamma_{b,s,t,i,j}^{im} \le a_{b,s,t,k}^{Vi}, \quad \forall b,\, s,\, t,\, k \in \Lambda_1 \tag{58}$$

$$\sum_{i=0}^{n}\sum_{j\in\Gamma^{0}(j,k)} \gamma_{b,s,t,i,j}^{im} \le 1 - a_{b,s,t,k}^{Vi}, \quad \forall b,\, s,\, t,\, k \in \Lambda_1 \tag{59}$$

$$\sum_{j=0}^{m}\sum_{i\in\Gamma^{+}(i,k)} \gamma_{b,s,t,i,j}^{im} \le a_{b,s,t,k}^{Ii}, \quad \forall b,\, s,\, t,\, k \in \Lambda_2 \tag{60}$$

$$\sum_{j=0}^{w}\sum_{i\in\Gamma^{0}(i,k)} \gamma_{b,s,t,i,j}^{im} \le 1 - a_{b,s,t,k}^{Ii}, \quad \forall b,\, s,\, t,\, k \in \Lambda_2 \tag{61}$$

$$\sum_{(i,j)\in\rho} \gamma_{b,s,t,i,j}^{im} \le a_{b,s,t}^{i0}, \quad \sum_{(i,j)\in\sigma} \gamma_{b,s,t,i,j}^{im} \le 1 - a_{b,s,t}^{i0}, \quad \forall b,\, s,\, t \tag{62}$$

where, $\Lambda_1 = \{1, ..., \log_2^n\}$, $\Lambda_2 = \{1, ..., \log_2^m\}$, n and m are total number of breakpoints on each axis, and $\gamma_{b,s,t,i,j}^{r} \in [0, 1]$ and $\gamma_{b,s,t,i,j}^{im} \in [0, 1]$ are auxiliary variables. Also, $a_{b,s,t,k}^{Vr}$ and $a_{b,s,t,k}^{Vi}$ are binary variables used for implementation of SOS2 constraints for the first coordinate, $a_{b,s,t,k}^{Ir}$ and $a_{b,s,t,k}^{Ii}$ are binary variables used for implementation of SOS2 constraints for the second coordinate, $a^{r0}_{b,s,t}$ and $a^{i0}_{b,s,t}$ are binary variables used to select one of the two triangle inside the square selected in the grid induced by triangulation. dV^{r}_{i}, dV^{im}_{i}, dI^{str}_{j} and dI^{stim}_{j} are the breakpoints on each axis.

Equation (35) also has non-linear terms like (34). However, since current-based equations are used for DN power flow implementation in MILP model (according to (39) and (40)), (35) is not used in our formulation. Therefore, we do not need to linearize it.

The next non-linear equations are (36)-(38). In order to linearize them, at first, by some manipulations, (63) is obtained.

$$\sum_{h}\left((i^{st,r}_{h,b,s,t})^2+(i^{st,im}_{h,b,s,t})^2\right)\le\frac{(s^{st}_{b})^2 u_z}{(\bar{V}^{rms})^2},\quad \forall\,(b,z)\in\Theta^{z}_{b},\,s,\,t \qquad (63)$$

The right hand side of (63) include the product of a binary variable and a continuous variable. In order to get rid of this nonlinear term, (63) is substituted by (64) and (65).

$$s^{st}_{b}\le Mu_z,\quad \forall\,(b,z)\in\Theta^{z}_{b} \qquad (64)$$

$$\sum_{h}\left((i^{st,r}_{h,b,s,t})^2+(i^{st,im}_{h,b,s,t})^2\right)\le\frac{(s^{st}_{b})^2}{(\bar{V}^{rms})^2},\quad \forall\,(b,z)\in\Theta^{z}_{b},\,s,\,t \qquad (65)$$

where, M is a large positive value.

The method utilized in [3], linear approximation of a circle, is generalized to linearize (65). This method is described in appendix B. Thus, (65) is substituted by (66), as a linear approximation.

$$\begin{aligned}
&i^{st,r}_{h,b,s,t}\cos(2\pi\frac{k}{n})+i^{st,im}_{h,b,s,t}\sin(2\pi\frac{k}{n})\le\alpha^{st}_{h,b,s,t},\\
&\qquad\qquad \forall h,\,b,\,s,\,t,\,k\in\{1,\dots,n\}\\
&\alpha^{st}_{h_1,b,s,t}\cos(2\pi\frac{k}{n})+\alpha^{st}_{h_2,b,s,t}\sin(2\pi\frac{k}{n})\le\beta^{st}_{1,b,s,t}\\
&\qquad\vdots\\
&\gamma^{st}_{1,b,s,t}\cos(2\pi\frac{k}{n})+\gamma^{st}_{2,b,s,t}\sin(2\pi\frac{k}{n})\le\frac{s^{st}_{b}}{\bar{V}^{rms}},\\
&\qquad\qquad \forall b,\,s,\,t,\,k\in\{0,\dots,\frac{n}{4}\}
\end{aligned} \qquad (66)$$

where $\alpha^{st}_{h,b,s,t}$, $\beta^{st}_{1,b,s,t}$, $\gamma^{st}_{1,b,s,t}$ and $\gamma^{st}_{2,b,s,t}$ are auxiliary variables used for expressing (65) as a circle equation.

Therefore (33), (45)-(62), (64) and (66) are the linear representation of PEVFCS constraints (33)-(38).

C. Linearization of Operation and Security Constraints ((24)-(32))

Among the operation and security constraints, (25), (26), (28), (29), (31) and (32) are nonlinear.

In order to linearize (25) and (26), (24)-(26) are merged together and (67)-(68) are obtained.

$$\sum_{h}\left((v^{r}_{h,b,s,t})^2+(v^{im}_{h,b,s,t})^2\right)\le\left(\bar{V}^{rms}\right)^2,\quad \forall b,\,s,\,t \qquad (67)$$

$$\sum_{h}\left((v^{r}_{h,b,s,t})^2+(v^{im}_{h,b,s,t})^2\right)\ge\left(\underline{V}^{rms}\right)^2,\quad \forall b,\,s,\,t \qquad (68)$$

To linearize (67), the method proposed for linearizing (65) is used. Therefore, (69) is obtained as the linear approximation of (67).

$$\begin{aligned}
&v^{r}_{h,b,s,t}\cos(2\pi\frac{k}{n})+v^{im}_{h,b,s,t}\sin(2\pi\frac{k}{n})\le\alpha^{v}_{h,b,s,t},\\
&\qquad\qquad \forall h,\,b,\,s,\,t,\,k\in\{1,\dots,n\}\\
&\alpha^{v}_{h_1,b,s,t}\cos(2\pi\frac{k}{n})+\alpha^{v}_{h_2,b,s,t}\sin(2\pi\frac{k}{n})\le\beta^{v}_{1,b,s,t}\\
&\qquad\vdots\\
&\gamma^{v}_{1,b,s,t}\cos(2\pi\frac{k}{n})+\gamma^{v}_{2,b,s,t}\sin(2\pi\frac{k}{n})\le\bar{V}^{rms},\\
&\qquad\qquad \forall b,\,s,\,t,\,k\in\{0,\dots,\frac{n}{4}\}
\end{aligned} \qquad (69)$$

where $\alpha^{v}_{h,b,s,t}$, $\beta^{v}_{1,b,s,t}$, $\gamma^{v}_{1,b,s,t}$ and $\gamma^{v}_{2,b,s,t}$ are auxiliary variables used for piecewise linear expression.

For linearizing (68) we can use another method. According to (68) and (27), we can see two extreme conditions of (68) as follow:

$$(27)\quad and\quad (68)\Rightarrow(v^{r}_{h_1,b,s,t})^2+(v^{im}_{h_1,b,s,t})^2\ge\frac{(\underline{V}^{rms})^2}{1+THD^2}$$

$$Extreme\ condition\ 1: THD=THD^{max}$$

$$\Rightarrow(v^{r}_{h_1,b,s,t})^2+(v^{im}_{h_1,b,s,t})^2\ge\frac{(\underline{V}^{rms})^2}{1+(THD^{max})^2}$$

$$(e.g.\ THD^{max}=0.05\Rightarrow(v^{r}_{h_1,b,s,t})^2+(v^{im}_{h_1,b,s,t})^2\ge0.998(\underline{V}^{rms})^2)$$

$$Extreme\ condition\ 2: THD=0\ \Rightarrow\ (v^{r}_{h_1,b,s,t})^2+(v^{im}_{h_1,b,s,t})^2\ge(\underline{V}^{rms})^2$$

Therefore, (68) can be simplified and rewritten as (70).

$$(v^{r}_{h_1,b,s,t})^2+(v^{im}_{h_1,b,s,t})^2\ge\left(\underline{V}^{rms}\right)^2,\quad \forall b,\,s,\,t \qquad (70)$$

We should linearize (70) instead of (68). In order to linearize the left hand side of (70), the variables used in (45) can be employed. Therefore, (70) can be replaced by (71).

$$\sum_{i=0}^{n}\sum_{j=0}^{m}\left(\gamma^{r}_{b,s,t,i,j}(dV^{r}_{i})^2+\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{im}_{b,s,t,i,j}(dV^{im}_{i})^2\right)\ge\left(\underline{V}^{rms}\right)^2,\quad \forall b,\,s,\,t \qquad (71)$$

where actually the left hand side of (71) is $(v^{mg}_{h_1,b,s,t})^2$.

Equation (28) is another non-linear constraint among the operation and security constraints. For linearizing it, (27) and (28) are merged together. By some manipulations, (72) is obtained that is the substitute for (27) and (28).

$$\sum_{h\ne1}\left((v^{r}_{h,b,s,t})^2+(v^{im}_{h,b,s,t})^2\right)\le(THD^{\max})^2(v^{mg}_{h_1,b,s,t})^2,\ \forall b,\,s,\,t \qquad (72)$$

To linearize (72), the same method explained for (65) and (67) can be utilized. To do this, first we need to define $v^{mg}_{h_1,b,s,t}$. As mentioned before, the left hand side of (71) is linear approximation of $(v^{mg}_{h_1,b,s,t})^2$. Therefore, in order to obtain $v^{mg}_{h_1,b,s,t}$ we use the first order Taylor series approximation of $\sqrt{x}$ about X=1, as follow.

$$v^{mg}_{h_1,b,s,t}=\sqrt{(v^{mg}_{h_1,b,s,t})^2}\simeq0.5(v^{mg}_{h_1,b,s,t})^2+0.5 \qquad (73)$$

Therefore, $v^{mg}_{h_1,b,s,t}$ can be obtained as (74).

$$v^{mg}_{h_1,b,s,t} = 0.5\left(\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{r}_{b,s,t,i,j}(dV_i^{r})^2 + \sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{im}_{b,s,t,i,j}(dV_i^{im})^2\right) + 0.5 \qquad \forall b,\, s,\, t \tag{74}$$

Considering that $0.95 \le v^{mg}_{h_1,b,s,t} \le 1.05$ (according to (67) and (70)), the relative percentage error of this Taylor approximation is less than 0.13% which is acceptable. So we substitute (72) by (75), as a linear approximation.

$$\begin{aligned} & v^{r}_{h,b,s,t}\ \cos(2\pi\frac{k}{n}) + v^{im}_{h,b,s,t}\ \sin(2\pi\frac{k}{n}) \le \alpha^{THD}_{h,b,s,t}, \\ & \qquad \forall h \ne h_1,\, b,\, s,\, t,\, k \in \{1, ..., n\} \\ & \alpha^{THD}_{2,b,s,t}\ \cos(2\pi\frac{k}{n}) + \alpha^{THD}_{3,b,s,t}\ \sin(2\pi\frac{k}{n}) \le \beta^{THD}_{1,b,s,t} \\ & \qquad \vdots \\ & \gamma^{THD}_{1,b,s,t}\ \cos(2\pi\frac{k}{n}) + \gamma^{THD}_{2,b,s,t}\ \sin(2\pi\frac{k}{n}) \le THD^{\max} v^{mg}_{h_1,b,s,t}, \\ & \qquad \forall b,\, s,\, t,\, k \in \{0, ..., \frac{n}{4}\} \end{aligned} \tag{75}$$

where $\alpha^{THD}_{h,b,s,t}$, $\beta^{THD}_{1,b,s,t}$, $\gamma^{THD}_{1,b,s,t}$ and $\gamma^{THD}_{2,b,s,t}$ are auxiliary variables used for piecewise linear approximation.

The next non-linear constraint among the operation and security constraints is (29). For linearizing (29), first it is rewritten as follow.

$$(v^{r}_{h,b,s,t})^2 + (v^{im}_{h,b,s,t})^2 \le (IHD^{\max})^2 (v^{mg}_{h_1,b,s,t})^2, \quad \forall h \ne h_1,\, b,\, s,\, t \tag{76}$$

Given the form of (76), it can be linearized using the piecewise linear approximation of a circle, that is described in the appendix B. Therefore, (76) can be stated as (77).

$$\begin{aligned} & v^{r}_{h,b,s,t}\ \cos(2\pi\frac{k}{n}) + v^{im}_{h,b,s,t}\ \sin(2\pi\frac{k}{n}) \le IHD^{\max} v^{mg}_{h_1,b,s,t}, \\ & \qquad \forall h \ne h_1,\, b,\, s,\, t,\, k \in \{1, ..., n\} \end{aligned} \tag{77}$$

The last non-linear constraints among the operation and security constraints is (30)-(32). For the piecewise linear expression of them, the method mentioned for (65) is used. Therefore, (78) are the piecewise linear approximation of (30)-(32).

$$\begin{aligned} & i^{L,r}_{h,l,s,t}\ \cos(2\pi\frac{k}{n}) + i^{L,im}_{h,l,s,t}\ \sin(2\pi\frac{k}{n}) \le \alpha^{L}_{h,l,s,t}, \\ & \qquad \forall h,\, l,\, s,\, t,\, k \in \{1, ..., n\} \\ & \alpha^{L}_{1,l,s,t}\ \cos(2\pi\frac{k}{n}) + \alpha^{L}_{2,l,s,t}\ \sin(2\pi\frac{k}{n}) \le \beta^{L}_{1,l,s,t} \\ & \qquad \vdots \\ & \gamma^{L}_{1,l,s,t}\ \cos(2\pi\frac{k}{n}) + \gamma^{L}_{2,l,s,t}\ \sin(2\pi\frac{k}{n}) \le I_l^{max}, \\ & \qquad \forall l,\, s,\, t,\, k \in \{0, ..., \frac{n}{4}\} \end{aligned} \tag{78}$$

where $\alpha^{L}_{h,l,s,t}$, $\beta^{L}_{1,l,s,t}$, $\gamma^{L}_{1,l,s,t}$ and $\gamma^{L}_{2,l,s,t}$ are auxiliary variables used for piecewise linear approximation.

Therefore, (69), (71), (74), (75), (77) and (78) are the linear approximation of operation and security constraints (24)-(32).

IV. MILP Model

Using the linearization techniques presented in section III, the MINLP model introduced in section II, can be recast as the MILP model (79)-(105).

$$\begin{aligned} \min \quad & \sum_z (C_z^{fix} u_z + C_z^{sp} d_z) + \sum_t\sum_a\sum_z C^{wait}_{z,a} n_{z,a,t} \\ & + \beta \sum_t\sum_r\sum_z W_{r,t} x_{z,r,t} T_{z,r,t} + \sum_{(b,z)\in\Theta_b^z} C_b^S s_b^{st} u_z \\ & + \sum_t\sum_s\sum_{b\in\Phi^{mainB}} C^P_{s,t} p_{b,s,t} + \sum_t\sum_s\sum_{b\in\Phi^{mainB}} C^Q_{s,t} q_{b,s,t} \end{aligned} \tag{79}$$

Subject to:

$$\sum_a n_{z,a,t} = \sum_r W_{r,t} x_{z,r,t}, \quad \forall z,\, t \tag{80}$$

$$n_{z,a,t} \le \Delta n_{z,a} d_z, \quad \forall z,\, a,\, t \tag{81}$$

$$\sum_z x_{z,r,t} = 1, \quad \forall t,\, r \tag{82}$$

$$\sum_r W_{r,t} x_{z,r,t} \le \frac{P_b^{sp} \times \Delta t}{Cu \times Rng} d_z, \quad \forall\, (z,\, b) \in \Theta_b^z,\, t \tag{83}$$

$$u_z \le d_z \le D^{max} u_z, \quad \forall z \tag{84}$$

$$u_z \in \{0, 1\}, \quad \forall t,\, z \tag{85}$$

$$x_{z,r,t} \ge 0, \quad \forall z,\, r,\, t \tag{86}$$

$$n_{z,a,t} \ge 0, \quad \forall z,\, a,\, t \tag{87}$$

$$i^{r}_{h,b,s,t} = \sum_{k\in\Phi^B} (G^B_{h,b,k} v^{r}_{h,k,s,t} - B^B_{h,b,k} v^{im}_{h,k,s,t}), \quad \forall h,\, b,\, s,\, t \tag{88}$$

$$i^{im}_{h,b,s,t} = \sum_{k\in\Phi^B} (G^B_{h,b,k} v^{im}_{h,k,s,t} + B^B_{h,b,k} v^{r}_{h,k,s,t}), \quad \forall h,\, b,\, s,\, t \tag{89}$$

$$\begin{aligned} i^{r}_{h_1,b,s,t} = {} & (P^G_{b,s,t} - (P^{D0}_{b,s,t} + \sum_{nl\in\Theta_b^{nl}} P^{NL}_{nl,s,t}))\left(2 - v^{r}_{h_1,b,s,t}\right) \\ & + (Q^G_{b,s,t} + Q^C_{b,s,t} - (Q^{D0}_{b,s,t} + \sum_{nl\in\Theta_b^{nl}} Q^{NL}_{nl,s,t}))(v^{im}_{h_1,b,s,t}) \\ & + i^{str}_{h_1,b,s,t}, \qquad \forall b \notin \Phi^{mainB},\, s,\, t \end{aligned} \tag{90}$$

$$\begin{aligned} i^{im}_{h_1,b,s,t} = {} & (P^G_{b,s,t} - (P^{D0}_{b,s,t} + \sum_{nl\in\Theta_b^{nl}} P^{NL}_{nl,s,t}))v^{im}_{h_1,b,s,t} \\ & + (Q^G_{b,s,t} + Q^C_{b,s,t} - (Q^{D0}_{b,s,t} + \sum_{nl\in\Theta_b^{nl}} Q^{NL}_{nl,s,t}))\left(-2 + v^{im}_{h_1,b,s,t}\right) \\ & + i^{stim}_{h_1,b,s,t}, \qquad \forall b \notin \Phi^{mainB},\, s,\, t \end{aligned} \tag{91}$$

$$p_{b,s,t} = i^{r}_{h_1,b,s,t}, \quad \forall b \in \Phi^{mainB},\, s,\, t \tag{92}$$

$$q_{b,s,t} = -i^{im}_{h_1,b,s,t}, \quad \forall b \in \Phi^{mainB},\, s,\, t \tag{93}$$

$$i^{r}_{h,b,s,t} = -i^{st,r}_{h,b,s,t} - i^{NL,r}_{h,b,s,t}, \quad \forall h \ne h_1,\, b,\, s,\, t \tag{94}$$

$$i^{im}_{h,b,s,t} = -i^{st,im}_{h,b,s,t} - i^{NL,im}_{h,b,s,t}, \quad \forall h \ne h_1,\, b,\, s,\, t \tag{95}$$

$$\begin{aligned} \bar{i}^{NLr1}_{nl,b,s,t} = P^{NL}_{nl,s,t}\left(2 - v^{r}_{h_1,b,s,t}\right) + Q^{NL}_{nl,s,t} v^{im}_{h_1,b,s,t}, \\ \forall\, (nl,\, b) \in \Theta_b^{nl},\, s,\, t \end{aligned} \tag{96}$$

$$\begin{aligned} \bar{i}^{NLim1}_{nl,b,s,t} = Q^{NL}_{nl,s,t}\left(-2 + v^{r}_{h_1,b,s,t}\right) + P^{NL}_{nl,s,t} v^{im}_{h_1,b,s,t}, \\ \forall\, (nl,\, b) \in \Theta_b^{nl},\, s,\, t \end{aligned} \tag{97}$$

$$i^{NL,r}_{h,b,s,t} = \sum_{nl\in\Theta_b^{nl}} \left(C^{r}_{nl,h} i^{NL,r1}_{nl,b,s,t} - C^{im}_{nl,h} i^{NL,im1}_{nl,b,s,t}\right), \quad \forall h,\, b,\, s,\, t \tag{98}$$

$$i^{NL,im}_{h,b,s,t} = \sum_{nl\in\Theta_b^{nl}} \left(C^{r}_{nl,h} i^{NL,im1}_{nl,b,s,t} + C^{im}_{nl,h} i^{NL,r1}_{nl,b,s,t}\right), \quad \forall h,\, b,\, s,\, t \tag{99}$$

$$
\begin{aligned}
&v^{r}_{h,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + v^{im}_{h,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \alpha^{v}_{h,b,s,t}, \\
&\qquad\qquad \forall h,\ b,\ s,\ t,\ k \in \{1,\ ...,\ n\} \\
&\alpha^{v}_{h_1,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + \alpha^{v}_{h_2,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \beta^{v}_{1,b,s,t} \\
&\qquad \vdots \\
&\gamma^{v}_{1,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + \gamma^{v}_{2,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \bar{V}^{rms}, \\
&\qquad\qquad \forall b,\ s,\ t,\ k \in \{0,\ ...,\ \frac{n}{4}\}
\end{aligned} \tag{100}
$$

$$
\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{r}_{b,s,t,i,j}\,(dV^{r}_{i})^2 + \sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{im}_{b,s,t,i,j}\,(dV^{im}_{i})^2 \ge \left(\underline{V}^{rms}\right)^2, \quad \forall b,\ s,\ t \tag{101}
$$

$$
\begin{aligned}
v^{mg}_{h_1,b,s,t} = 0.5\Bigg(&\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{r}_{b,s,t,i,j}\,(dV^{r}_{i})^2 \\
&+\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma^{im}_{b,s,t,i,j}\,(dV^{im}_{i})^2 \Bigg) + 0.5 \qquad \forall b,\ s,\ t
\end{aligned} \tag{102}
$$

$$
\begin{aligned}
&v^{r}_{h,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + v^{im}_{h,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \alpha^{THD}_{h,b,s,t}, \\
&\qquad\qquad \forall h \ne h_1,\ b,\ s,\ t,\ k \in \{1,\ ...,\ n\} \\
&\alpha^{THD}_{2,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + \alpha^{THD}_{3,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \beta^{THD}_{1,b,s,t} \\
&\qquad \vdots \\
&\gamma^{THD}_{1,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + \gamma^{THD}_{2,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le THD^{\max} v^{mg}_{h_1,b,s,t}, \\
&\qquad\qquad \forall b,\ s,\ t,\ k \in \{0,\ ...,\ \frac{n}{4}\}
\end{aligned} \tag{103}
$$

$$
\begin{aligned}
&v^{r}_{h,b,s,t}\ \cos(2\pi\ \frac{k}{n}) + v^{im}_{h,b,s,t}\ \sin(2\pi\ \frac{k}{n}) \le IHD^{\max} v^{mg}_{h_1,b,s,t}, \\
&\qquad\qquad \forall h \ne h_1,\ b,\ s,\ t,\ k \in \{1,\ ...,\ n\}
\end{aligned} \tag{104}
$$

$$
\begin{aligned}
&i^{L,r}_{h,l,s,t}\ \cos(2\pi\ \frac{k}{n}) + i^{L,im}_{h,l,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \alpha^{L}_{h,l,s,t}, \\
&\qquad\qquad \forall h,\ l,\ s,\ t,\ k \in \{1,\ ...,\ n\} \\
&\alpha^{L}_{1,l,s,t}\ \cos(2\pi\ \frac{k}{n}) + \alpha^{L}_{2,l,s,t}\ \sin(2\pi\ \frac{k}{n}) \le \beta^{L}_{1,l,s,t} \\
&\qquad \vdots \\
&\gamma^{L}_{1,l,s,t}\ \cos(2\pi\ \frac{k}{n}) + \gamma^{L}_{2,l,s,t}\ \sin(2\pi\ \frac{k}{n}) \le I^{max}_{l}, \\
&\qquad\qquad \forall l,\ s,\ t,\ k \in \{0,\ ...,\ \frac{n}{4}\}
\end{aligned} \tag{105}
$$

APPENDIX A

PIECEWISE LINEAR APPROXIMATION OF A FUNCTION OF TWO NONSEPARABLE VARIABLES

In [4], three methods for piecewise linear approximation of a function of two nonseparable variables are presented. In this paper we use the triangulation method. In this method each coordinates of the function are divided into several pieces, and the space of the two coordinates is divided into a number of triangles, as shown in Fig. 1. Then, by using convex combination of the function values, evaluated at the vertices of one of the triangles, the function is approximated. For this purpose, the following equations are created.

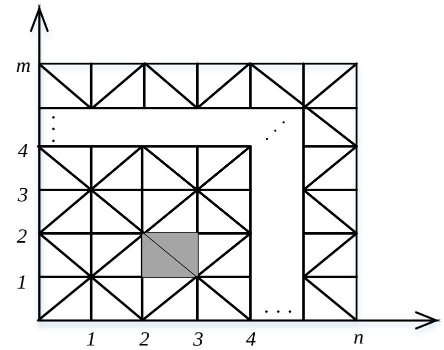

Fig. 1. Triangulations of the space of the two coordinates.

$$F\ (x,\ y) = \sum_{i=0}^{n}\sum_{j=0}^{m}\gamma_{i,\,j}\, f\ \left(x_i,\ y_j\right) \tag{a.1}$$

$$x = \sum_{i=0}^{n}\sum_{j=0}^{m}\gamma_{i,\,j}\, x_i \tag{a.2}$$

$$y = \sum_{i=0}^{n}\sum_{j=0}^{m}\gamma_{i,\,j}\, y_j \tag{a.3}$$

$$\sum_{i=0}^{n}\sum_{j=0}^{m}\gamma_{i,\,j} = 1 \tag{a.4}$$

where, x_i and y_j are the breakpoints on X axis and Y axis respectively, and $f\,(x_i,\ y_j)$ is the function value at each break points. n and m are the number of considered breakpoints on each coordinates. $F(x,\ y)$ is the approximated function and $\gamma_{i,\,j} \in [0,1]$ is the coefficient of the convex combination of the vertices of the selected triangle, in which only three value of that can be nonzero and these values should be corresponding to one of the triangles. In order to apply these conditions on $\gamma_{i,\,j}$, [4] uses $O(n \times m)$ binary and continuous variables and $O(n \times m)$ constraints. Therefore, it imposes a high computational burden on the problem, in cases with a large number of pieces. In order to reduce the use of binary variables and extra constraints, in [2] a formulation is proposed, that uses $O(n \times m)$ continuous variables and $O(\log_2\ n + \log_2\ m)$ binary variables and constraints. This formulation is shown in (a.5)-(a.10).

$$\sum_{i=0}^{n}\ \sum_{j \in \Gamma^{+}(j,k)} \gamma_{i,\,j} \le a^{1}_{k}, \quad k = 1,\ 2,\ \dots,\ log^{m}_{2} \tag{a.5}$$

$$\sum_{i=0}^{n}\ \sum_{j \in \Gamma^{0}(j,k)} \gamma_{i,\,j} \le 1 - a^{1}_{k}, \quad k = 1,\ 2,\ \dots,\ log^{m}_{2} \tag{a.6}$$

$$\sum_{j=0}^{m}\ \sum_{i \in \Gamma^{+}(i,k)} \gamma_{i,\,j} \le a^{2}_{k}, \quad k = 1,\ 2,\ \dots,\ log^{n}_{2} \tag{a.7}$$

$$\sum_{j=0}^{m}\ \sum_{i \in \Gamma^{0}(i,k)} \gamma_{i,\,j} \le 1 - a^{2}_{k}, \quad k = 1,\ 2,\ \dots,\ log^{n}_{2} \tag{a.8}$$

$$\sum_{(i,\,j) \in \rho} \gamma_{i,\,j} \le a^{0} \tag{a.9}$$

$$\sum_{(i,j)\in\sigma}\gamma_{i,j} \le 1-a^0 \quad \text{(a.10)}$$

Consider $N=\{0,1,2,...,n\}$ and $i\in M$, in which M is the set of breakpoints and i is the index of breakpoints. A function $B:\{1,2,...,n\}\to\{0,1\}^{\log_2^n}$ is compatible with an SOS2 constraint on $(\gamma_i)_{i=0}^n$ if it is bijective and for all $i\in\{1,...,n-1\}$ the vectors $B(i)$ and $B(i+1)$ differ in at most one component. Also, let $B(i)=(a_1,a_2,...,a_{\log_2^n})$, for all $a_k\in\{0,1\}$ and $B(0)=B(1)$. Then according to this compatible function, $\Gamma^+(i,k)$ and $\Gamma^0(i,k)$ are defined as follow.

$$\Gamma^+(i,k)=\{i|\forall B(i)\,\&\,B(i+1),\ a_k=1,\ i=\{1,2,...,n-1\}\cup\{i|\forall B(i),\ a_k=1,\ i=\{0,n\}\}$$

$$\Gamma^0(i,k)=\{i|\forall B(i)\,\&\,B(i+1),\ a_k=0,\ i=\{1,2,...,n-1\}\cup\{i|\forall B(i),\ a_k=0,\ i=\{0,n\}\}$$

Equations (a.5), (a.6) and (a.7), (a.8) are created to implement SOS2 condition on each coordinate, and (a.5)-(a.8) are used to select a square in the grid induced by the triangulation. Then, one of the two triangles is selected inside this square, by using (a.9), (a.10) and defining ρ and σ as follow.

$$\rho=\left\{(i,\ j)\in\{0,...,n\}.\{0,...,m\}:\ (i\ \text{ is even})\,\&\,(j\ \text{ is odd})\right\}$$

$$\sigma=\left\{(i,\ j)\in\{0,...,n\}.\{0,...,m\}:\ (i\ \text{ is odd})\,\&\,(j\ \text{ is even})\right\}$$

APPENDIX B

GENERALIZATION OF PIECEWISE LINEAR MODELING OF A CIRCLE

In [3] a method for piecewise linear approximation of areas outside a circle is presented, as shown in Fig. 2. According to this model, a non-linear equation (b.1) is converted to (b.2).

$$A^2+B^2\le C^2 \quad \text{(b.1)}$$

$$A\cos(2\pi\frac{k}{n})+B\sin(2\pi\frac{k}{n})\le C \qquad \forall k\in\{1,...,n\} \quad \text{(b.2)}$$

In this paper, we generalize this method to piecewise linear approximation of (b.3).

$$A^2+B^2+C^2+D^2+E^2\le F^2 \quad \text{(b.3)}$$

According to (b.3), (b.4)-(b.7) can be extracted.

$$A^2+B^2\le F_1^2 \quad \text{(b.4)}$$

$$C^2+D^2\le F_2^2 \quad \text{(b.5)}$$

$$F_1^2+F_2^2\le F_3^2 \quad \text{(b.6)}$$

$$E^2+F_3^2\le F^2 \quad \text{(b.7)}$$

Thus, (b.8)-(b.11) can be considered as piecewise linear approximation of (b.3).

$$A\cos(2\pi\frac{k}{n})+B\sin(2\pi\frac{k}{n})\le F_1 \qquad \forall k\in\{1,...,n\} \quad \text{(b.8)}$$

$$C\cos(2\pi\frac{k}{n})+D\sin(2\pi\frac{k}{n})\le F_2 \qquad \forall k\in\{0,...,n\} \quad \text{(b.9)}$$

$$F_1\cos(2\pi\frac{k}{n})+F_2\sin(2\pi\frac{k}{n})\le F_3 \qquad \forall k\in\{0,...,\frac{n}{4}\} \quad \text{(b.10)}$$

$$E\cos(2\pi\frac{k}{n})+F_3\sin(2\pi\frac{k}{n})\le F \qquad \forall k\in\{0,...,\frac{n}{2}\} \quad \text{(b.11)}$$

Note that the number of k's in (b.10) are a quarter of number of that in (b.8), since in (b.10), F_1 and F_2 are positive and so (b.10) is sufficient to express only for a quarter of circle. Therefore, the number of extra equations are reduced.

Similarly, these formulations can be further expanded for equations with more square terms in the left hand side of (b.3).

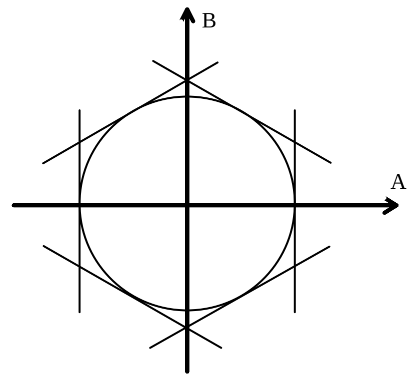

Fig. 2. Piecewise linearization of a circle.

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