```
> restart;
```

```
> Int(Int(Int(1/(x^2+y^2+z^2),z=-sqrt(4-x^2-y^2)..sqrt(4-x^2-y^2)
  ),y=-sqrt(4-x^2)..sqrt(4-x^2)),x=-2..2)=int(int(int(1/(x^2+y^2+
  z^2),z=-sqrt(4-x^2-y^2)..sqrt(4-x^2-y^2)),y=-sqrt(4-x^2)..sqrt(4
  -x^2)),x=-2..2);
```

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}\frac{1}{x^2+y^2+z^2}\,dz\,dy\,dx=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\frac{2\arctan\left(\frac{\sqrt{4-x^2-y^2}}{\sqrt{x^2+y^2}}\right)}{\sqrt{x^2+y^2}}\,dy\,dx$$

```
> with(linalg):
> x:=rho*cos(theta)*sin(phi);
```

$$x := \rho\cos(\theta)\sin(\phi)$$

```
> y:=rho*sin(theta)*sin(phi);
```

$$y := \rho\sin(\theta)\sin(\phi)$$

```
> z:=rho*cos(phi);
```

$$z := \rho\cos(\phi)$$

```
> jacobian([x,y,z],[rho,theta,phi]);
```

$$\begin{bmatrix} \cos(\theta)\sin(\phi) & -\rho\sin(\theta)\sin(\phi) & \rho\cos(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) & \rho\cos(\theta)\sin(\phi) & \rho\sin(\theta)\cos(\phi) \\ \cos(\phi) & 0 & -\rho\sin(\phi) \end{bmatrix}$$

```
> det(%);
```

$$-\cos(\theta)^2\sin(\phi)^3\rho^2-\sin(\theta)^2\sin(\phi)^3\rho^2-\rho^2\sin(\theta)^2\sin(\phi)\cos(\phi)^2$$
$$-\rho^2\cos(\theta)^2\cos(\phi)^2\sin(\phi)$$

```
> simplify(%);
```

$$-\sin(\phi)\rho^2$$

```
> 1/(x^2+y^2+z^2);
```

$$\frac{1}{\rho^2 \cos(\theta)^2 \sin(\phi)^2 + \rho^2 \sin(\theta)^2 \sin(\phi)^2 + \rho^2 \cos(\phi)^2}$$

```
> simplify(%);
```

$$\frac{1}{\rho^2}$$

```
> Int(Int(Int(abs(-sin(phi)*rho^2)*1/rho^2,rho=0..2),phi=0..Pi),
  theta=0..2*Pi)=int(int(int(abs(-sin(phi)*rho^2)*1/rho^2,rho=0.
  .2),phi=0..Pi),theta=0..2*Pi);
```

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{2} \frac{\left|\sin(\phi)\ \rho^2\right|}{\rho^2}\ d\rho\ d\phi\ d\theta = 8\ \pi$$

```
> 2/3*Pi+Int(Int(Int(abs(-sin(phi)*rho^2),rho=cos(phi)+sqrt(4*cos
  (phi)^2-3)..2/cos(phi)),phi=0..arctan(1/2)),theta=0..2*Pi)=2/3*
  Pi+int(int(int(abs(-sin(phi)*rho^2),rho=cos(phi)+sqrt(4*cos(phi)
  ^2-3)..2/cos(phi)),phi=0..arctan(1/2)),theta=0..2*Pi);
```

$$\frac{2}{3}\ \pi + \int_0^{2\pi}\int_0^{\arctan\left(\frac{1}{2}\right)}\int_{\cos(\phi)+\sqrt{4\cos(\phi)^2-3}}^{\frac{2}{\cos(\phi)}} \left|\sin(\phi)\ \rho^2\right|\ d\rho\ d\phi\ d\theta = \frac{489}{400}\ \pi - \frac{27}{32}\ \ln(3)\ \pi + \frac{27}{64}\ \ln(5)\ \pi$$

```
> evalf(%);
```

$$3.061568438 = 3.061568436$$

```
> Int(Int(Int(x^2*y^2,z=sqrt(x^2+y^2)..1),x=-sqrt(1-y^2)..sqrt(1-
  y^2)),y=-1..1)=int(int(int(x^2*y^2,z=sqrt(x^2+y^2)..1),x=-sqrt(1
  -y^2)..sqrt(1-y^2)),y=-1..1);
```

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{\sqrt{x^2+y^2}}^{1} x^2 y^2\ dz\ dx\ dy = \int_{-1}^{1}\left(-\frac{1}{2}\ y^2\sqrt{1-y^2} + \frac{1}{4}\ y^4\sqrt{1-y^2} - \frac{1}{8}\ y^6 \ln\left(-\sqrt{1-y^2}+1\right) + \frac{2}{3}\ y^2\left(1-y^2\right)^{3/2} + \frac{1}{8}\ y^6 \ln\left(\sqrt{1-y^2}+1\right)\right) dy$$

```
> evalf(%);
```

$$0.01869995627 = 0.01869995627$$

```
> x:=r*cos(theta);
```

$$x := r\cos(\theta)$$

```
> y:=r*sin(theta);
```

$$y := r\sin(\theta)$$

```
> z:=z;
```

$$z := z$$

```
> with(linalg):
> jacobian([x,y,z],[r,theta,z]);
```

$$\begin{bmatrix} \cos(\theta) & -r\sin(\theta) & 0 \\ \sin(\theta) & r\cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> det(%);
```

$$\cos(\theta)^2 r + \sin(\theta)^2 r$$

```
> simplify(%);
```

$$r$$

```
> x^2*y^2;
```

$$r^4 \cos(\theta)^2 \sin(\theta)^2$$

```
> Int(Int(Int(r*r^4*cos(theta)^2*sin(theta)^2,z=r..1),r=0..1),
  theta=0..2*Pi)=int(int(int(r*r^4*cos(theta)^2*sin(theta)^2,z=r.
  .1),r=0..1),theta=0..2*Pi);
```

$$\int_0^{2\pi}\int_0^1\int_r^1 \cos(\theta)^2 \sin(\theta)^2 r^5 \, dz \, dr \, d\theta = \frac{1}{168}\,\pi$$

```
> evalf(%);
```

$$0.01869995627 = 0.01869995627$$