```
> restart;
> with(plots):
> with(linalg):
> f:=(x,y)->sqrt(6-x^2-y^2);
```

$$f := (x, y) \rightarrow \sqrt{6 - x^2 - y^2}$$

```
> fx:=subs(x=2,y=1,diff(f(x,y),x));
```

$$fx := -2$$

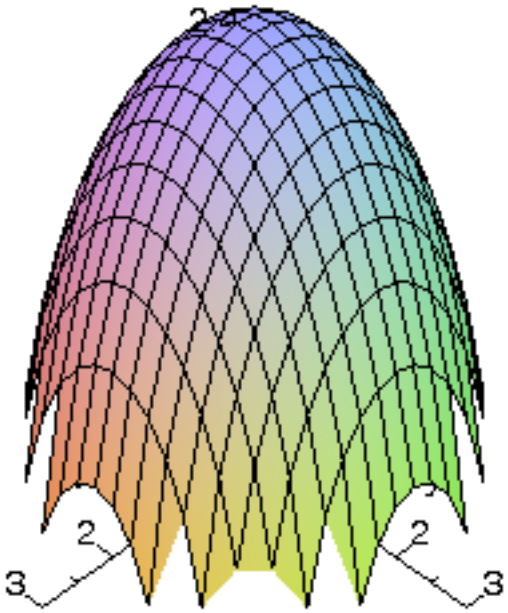
```
> fy:=subs(x=2,y=1,diff(f(x,y),y));
```

$$fy := -1$$

```
> fx*(x-2)+fy*(y-1)-(z-f(2,1))=0;
```

$$-2x + 6 - y - z = 0$$

```
> plot3d(f(x,y),x=-3..3,y=-3..3,axes=normal);
```



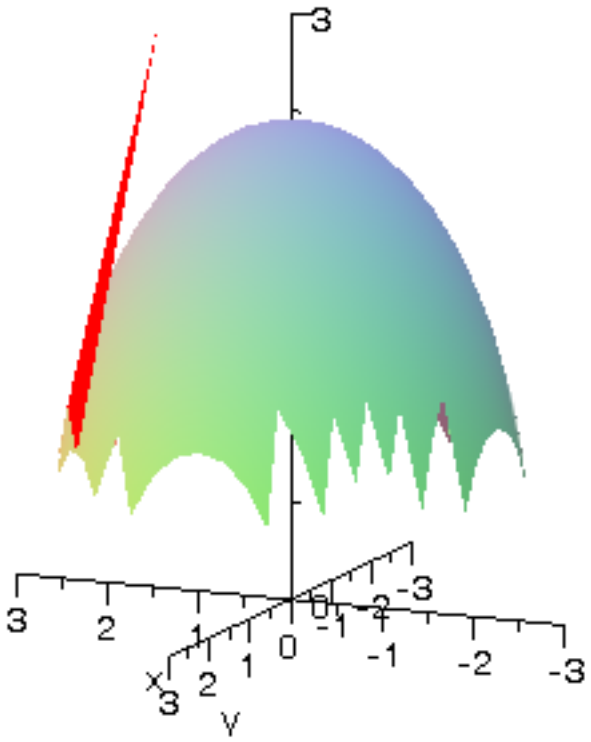

```
> F:=(x,y,z)->z-sqrt(6-x^2-y^2);
```

$$F := (x, y, z) \rightarrow z - \sqrt{6 - x^2 - y^2}$$

```
> grad(F(x,y,z),[x,y,z]);
```

$$\left[\frac{x}{\sqrt{6 - x^2 - y^2}} \quad \frac{y}{\sqrt{6 - x^2 - y^2}} \quad 1 \right]$$

```
> tanplain:=(a,b,c)->implicitplot3d(D[1](F)(a,b,c)*(x-a)+D[2](F)
  (a,b,c)*(y-b)+D[3](F)(a,b,c)*(z-c)=0,x=a-1..a+1,y=a-1..a+1,z=
  a-1..a+1,color=red):
> p1:=plot3d(f(x,y),x=-3..3,y=-3..3,axes=normal):
> display3d({p1,tanplain(2,1,1)},style=patchnogrid);
```




```
> f:=(x,y)->x*y^2-x^2*y+2*x*y+5;
```

$$f := (x, y) \to x\,y^2 - x^2\,y + 2\,y\,x + 5$$

```
> solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0},{x,y});
```

$$\{x = 0, y = 0\},\ \{y = 0, x = 2\},\ \{x = 0, y = -2\},\ \left\{x = \frac{2}{3}, y = -\frac{2}{3}\right\}$$

```
> A:=diff(f(x,y),x$2);
```

$$A := -2\,y$$

```
> B:=diff(f(x,y),y$2);
```

$$B := 2\,x$$

```
> C:=diff(f(x,y),x,y);
```

$$C := 2\,y - 2\,x + 2$$

```
> Delta:=A*B-C^2;
```

$$\Delta := -4\,x\,y - (2\,y - 2\,x + 2)^2$$

```
> simplify(%);
```

$$4\,x\,y - 4\,y^2 - 8\,y - 4\,x^2 + 8\,x - 4$$

```
> subs(x=0,y=0,Delta), subs(x=0,y=0,A);
```

$$-4, 0$$

```
> subs(x=2,y=0,Delta), subs(x=2,y=0,A);
```

$$-4, 0$$

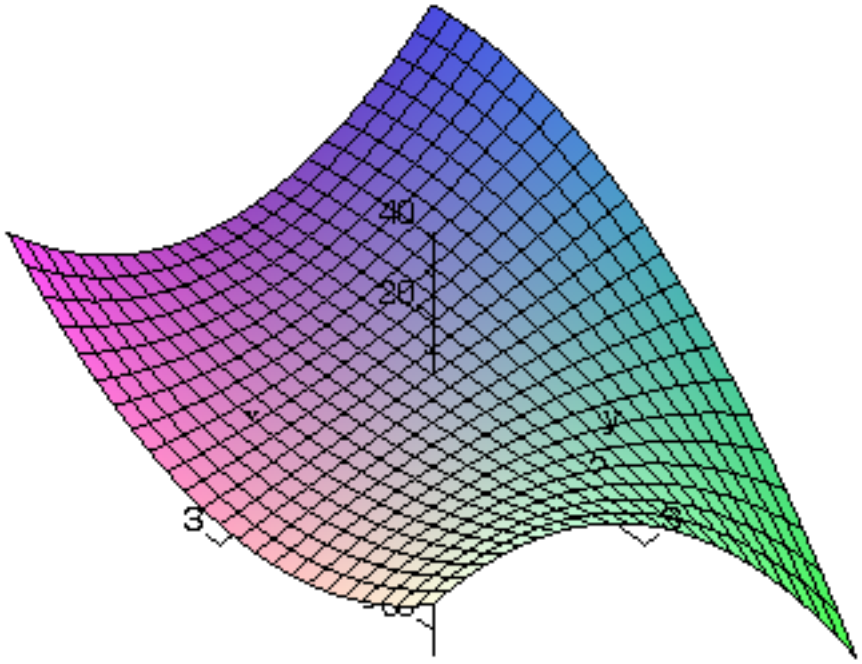
```
> subs(x=0,y=-2,Delta), subs(x=0,y=-2,A);
```

$$-4, 4$$

```
> subs(x=2/3,y=-2/3,Delta), subs(x=2/3,y=-2/3,A);
```

$$\frac{4}{3}, \frac{4}{3}$$

```
> plot3d(f(x,y),x=-3..3,y=-3..3,axes=normal);
```




```
> f:=(x,y)->sqrt(6-x^2-y^2);
```

$$f := (x, y) \rightarrow \sqrt{6 - x^2 - y^2}$$

```
> solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0},{x,y});
```

$$\{x = 0, y = 0\}$$

```
> A:=diff(f(x,y),x$2);
```

$$A := -\frac{x^2}{(6-x^2-y^2)^{3/2}} - \frac{1}{\sqrt{6-x^2-y^2}}$$

```
> B:=diff(f(x,y),y$2);
```

$$B := -\frac{y^2}{(6-x^2-y^2)^{3/2}} - \frac{1}{\sqrt{6-x^2-y^2}}$$

```
> C:=diff(f(x,y),x,y);
```

$$C := -\frac{x\,y}{(6-x^2-y^2)^{3/2}}$$

```
> Delta:=A*B-C^2;
```

$$\Delta := \left(-\frac{x^2}{(6-x^2-y^2)^{3/2}} - \frac{1}{\sqrt{6-x^2-y^2}}\right)\left(-\frac{y^2}{(6-x^2-y^2)^{3/2}} - \frac{1}{\sqrt{6-x^2-y^2}}\right) - \frac{x^2 y^2}{(6-x^2-y^2)^3}$$

```
> simplify(%);
```

$$\frac{6}{(-6+x^2+y^2)^2}$$

```
> subs(x=0,y=0,Delta);
```

$$\frac{1}{6}$$

```
> subs(x=0,y=0,A);
```

$$-\frac{1}{6}\sqrt{6}$$

```
> f:=(x,y,z)->(x-2)^2+(y-1)^2+(z+2)^2;
```

$$f := (x, y, z) \to (x-2)^2 + (y-1)^2 + (z+2)^2$$

```
> g:=(x,y,z)->x^2+y^2+z^2-1;
```

$$g := (x, y, z) \to x^2 + y^2 + z^2 - 1$$

```
> h:=(x,y,z)->f(x,y,z)-lambda *g(x,y,z);
```

$$h := (x, y, z) \to f(x, y, z) - \lambda\, g(x, y, z)$$

```
> points:=evalf(solve({diff(h(x,y,z),x)=0,diff(h(x,y,z),y)=0,diff
  (h(x,y,z),z)=0,g(x,y,z)=0},{x,y,z,lambda}));
```

$$points := \{x = -0.6666666667, z = 0.6666666667, y = -0.3333333333, \lambda = 4.\}, \{y = 0.3333333333, \lambda = -2., z = -0.6666666667, x = 0.6666666667\}$$

```
> point_list:=[points];
```

$$point\_list := [\{x = -0.6666666667, z = 0.6666666667, y = -0.3333333333, \lambda = 4.\}, \{y$$

$= 0.3333333333, \lambda = -2., z = -0.6666666667, x = 0.6666666667\}]$

```
> map(subs,point_list,f(x,y,z));
```

$[16.00000000, 3.999999999]$

```
> fvalues:=op(%);
```

$fvalues := 16.00000000, 3.999999999$

```
> max_distance:=sqrt(max(fvalues));
```

$max\_distance := 4.000000000$

```
> min_distance:=sqrt(min(fvalues));
```

$min\_distance := 2.000000000$

```
> f:=(x,y)->x^2-y^2;
```

$$f := (x, y) \rightarrow x^2 - y^2$$

```
> evalf(solve({diff(f(x,y),x)=0,diff(f(x,y),y)=0},{x,y}));
```

$\{x = 0., y = 0.\}$

```
> det:=diff(f(x,y),x$2)*diff(f(x,y),y$2)-diff(f(x,y),x,y)^2;
```

$det := -4$

```
> subs(x=0,y=0,det);
```

-4

```
> g:=(x,y)->x^2+y^2-1;
```

$$g := (x, y) \rightarrow x^2 + y^2 - 1$$

```
> h:=(x,y)->f(x,y)+lambda*g(x,y);
```

$$h := (x, y) \rightarrow f(x, y) + \lambda\, g(x, y)$$

```
> points:=evalf(solve({diff(h(x,y),x)=0,diff(h(x,y,z),y)=0,g(x,y)=
  0},{x,y,lambda}));
```

$points := \{y = 0., \lambda = -1., x = 1.\}, \{y = 0., \lambda = -1., x = -1.\}, \{x = 0., \lambda = 1., y = 1.\}, \{x = 0., \lambda = 1., y = -1.\}$

```
> point_list:=[points];
```

$point\_list := [\{y = 0., \lambda = -1., x = 1.\}, \{y = 0., \lambda = -1., x = -1.\}, \{x = 0., \lambda = 1., y = 1.\}, \{x = 0., \lambda = 1., y = -1.\}]$

```
> map(subs,point_list,f(x,y));
```

$[1., 1., -1., -1.]$