

```
> restart;
```

```
> f:=(x,y)->piecewise(x=0 and y=0,0,x*(x^2-y^2)/(x^2+y^2));
```

$$f := (x, y) \rightarrow \text{piecewise}\left(x = 0 \text{ and } y = 0, 0, \frac{x(x^2 - y^2)}{x^2 + y^2}\right)$$

```
> f(0,0);
```

0

```
> limit(f(x,y),{x=0,y=0});
```

$$\text{limit}\left(\frac{x(x^2 - y^2)}{x^2 + y^2}, \{x = 0, y = 0\}\right)$$

```
> limit(f(r*cos(theta),r*sin(theta)),r=0);
```

0

```
> fx0:=limit((f(0+x,0)-f(0,0))/x,x=0);
```

$fx0 := 1$

```
> Diff(f(x,y),x)=diff(f(x,y),x);
```

$$\frac{\partial}{\partial x} \left( \frac{x(x^2 - y^2)}{x^2 + y^2} \right) = \frac{x^2 - y^2}{x^2 + y^2} + \frac{2x^2}{x^2 + y^2} - \frac{2x^2(x^2 - y^2)}{(x^2 + y^2)^2}$$

```
> simplify(%);
```

$$\frac{\partial}{\partial x} \left( \frac{x(x^2 - y^2)}{x^2 + y^2} \right) = \frac{x^4 - y^4 + 4x^2y^2}{(x^2 + y^2)^2}$$

```
> fx:=(x,y)->piecewise(x=0and y=0,0,(x^4-y^4+4*x^2*y^2)/(x^2+y^2)^2);
```

$$fx := (x, y) \rightarrow \text{piecewise}\left(x = 0 \text{ and } y = 0, 0, \frac{x^4 - y^4 + 4x^2y^2}{(x^2 + y^2)^2}\right)$$

```
> limit(fx(x,0),x=0);
```

1

```
> is(limit(fx(x,0),x=0)=fx(0,0));
```

false

```
> Diff(f(x,y),x$2,y$3)=diff(f(x,y),x$2,y$3);
```

$$\frac{\partial^5}{\partial y^3 \partial x^2} \left( \frac{x(x^2 - y^2)}{x^2 + y^2} \right) = -\frac{144yx}{(x^2 + y^2)^3} + \frac{576y^3x}{(x^2 + y^2)^4} + \frac{1152(x^2 - y^2)xy^3}{(x^2 + y^2)^5}$$

$$-\frac{432(x^2-y^2)xy}{(x^2+y^2)^4} - \frac{768x^3y^3}{(x^2+y^2)^5} - \frac{3840x^3(x^2-y^2)y^3}{(x^2+y^2)^6} + \frac{1152x^3(x^2-y^2)y}{(x^2+y^2)^5}$$

> **f:=(x,y)->x^2-x\*y;**

$$f := (x, y) \rightarrow x^2 - xy$$

> **Delta:=f(-2+h,1+k)-f(-2,1);**

$$\Delta := (-2+h)^2 - (-2+h)(1+k) - 6$$

> **simplify(%);**

$$-5h + h^2 + 2k - hk$$

> **Diff(f(x,y),x)=diff(f(x,y),x);**

$$\frac{\partial}{\partial x} (x^2 - xy) = 2x - y$$

> **Diff(f(x,y),y)=diff(f(x,y),y);**

$$\frac{\partial}{\partial y} (x^2 - xy) = -x$$

> **fx:=(x,y)->2\*x-y;**

$$fx := (x, y) \rightarrow 2x - y$$

> **fy:=(x,y)->-x;**

$$fy := (x, y) \rightarrow -x$$

> **fx(-2,1);**

$$-5$$

> **fy(-2,1);**

$$2$$

> **f(-2+h,1+k)-f(-2,1)-(fx(-2,1)\*h +fy(-2,1)\*k);**

$$(-2+h)^2 - (-2+h)(1+k) - 6 + 5h - 2k$$

> **simplify(%);**

$$h^2 - hk$$

> **eta[1]:=(h,k)->h;**

$$\eta_1 := (h, k) \rightarrow h$$

> **eta[2]:=(h,k)->-h;**

$$\eta_2 := (h, k) \rightarrow -h$$

> **Limit(eta[1](h,k),{h=0,k=0})=limit(eta[1](h,k),{h=0,k=0});**

$$\text{Limit}(h, \{h=0, k=0\}) = 0$$

> **Limit(eta[2](h,k),{h=0,k=0})=limit(eta[2](h,k),{h=0,k=0});**

$$\text{Limit}(-h, \{h=0, k=0\}) = 0$$

> **f:=(x,y)->sin(x^2+y^2);**

$$f := (x, y) \rightarrow \sin(x^2 + y^2)$$

> Delta:=f(0+h,0+k)-f(0,0);

$$\Delta := \sin(h^2 + k^2)$$

> Diff(f(x,y),x)=diff(f(x,y),x);

$$\frac{\partial}{\partial x} \sin(x^2 + y^2) = 2 \cos(x^2 + y^2) x$$

> Diff(f(x,y),y)=diff(f(x,y),y);

$$\frac{\partial}{\partial y} \sin(x^2 + y^2) = 2 \cos(x^2 + y^2) y$$

> fx0:=limit((f(0+x,0)-f(0,0))/x,x=0);

$$fx0 := 0$$

> fy0:=limit((f(0,y+0)-f(0,0))/y,y=0);

$$fy0 := 0$$

> fx:=(x,y)->2\*cos(x^2+y^2)\*x;

$$fx := (x, y) \rightarrow 2 \cos(x^2 + y^2) x$$

> fy:=(x,y)->2\*cos(x^2+y^2)\*y;

$$fy := (x, y) \rightarrow 2 \cos(x^2 + y^2) y$$

> fx(0,0);

$$0$$

> fy(0,0);

$$0$$

> (f(0+h,0+k)-f(0,0))-(fx(0,0)\*h +fy(0,0)\*k);

$$\sin(h^2 + k^2)$$

> eta[1]:=(h,k)->1/2 \*sin(h^2+k^2)/h;

$$\eta_1 := (h, k) \rightarrow \frac{1}{2} \frac{\sin(h^2 + k^2)}{h}$$

> eta[2]:=(h,k)->1/2 \*sin(h^2+k^2)/k;

$$\eta_2 := (h, k) \rightarrow \frac{1}{2} \frac{\sin(h^2 + k^2)}{k}$$

> Limit(eta[1](r\*cos(theta),r\*sin(theta)),r=0)=limit(eta[1](r\*cos(theta),r\*sin(theta)),r=0);

$$\lim_{r \rightarrow 0} \frac{1}{2} \frac{\sin(r^2 \cos(\theta)^2 + r^2 \sin(\theta)^2)}{r \cos(\theta)} = 0$$

> Limit(eta[2](r\*cos(theta),r\*sin(theta)),r=0)=limit(eta[2](r\*cos(theta),r\*sin(theta)),r=0);

$$\lim_{r \rightarrow 0} \frac{1}{2} \frac{\sin(r^2 \cos(\theta)^2 + r^2 \sin(\theta)^2)}{r \sin(\theta)} = 0$$

```
> with(plots):
```

```
> plot3d(f(x,y), x=-0.1..0.1, y=-0.1..0.1);
```

