

```
> restart;
```

```
> with(linalg):
```

```
> assume(t,real);
```

```
> r:=t->[exp(t)*sin(2*t),exp(t)*cos(2*t),2*exp(t)];
```

$$r := t \rightarrow [e^t \sin(2t), e^t \cos(2t), 2e^t]$$

```
> map(diff,r(t),t);
```

$$[e^t \sin(2t) + 2e^t \cos(2t), e^t \cos(2t) - 2e^t \sin(2t), 2e^t]$$

```
> rp:=t->[exp(t)*sin(2*t)+2*exp(t)*cos(2*t), exp(t)*cos(2*t)-2*exp(t)*sin(2*t), 2*exp(t)];
```

$$rp := t \rightarrow [e^t \sin(2t) + 2e^t \cos(2t), e^t \cos(2t) - 2e^t \sin(2t), 2e^t]$$

```
> normalize(rp(0));
```

$$\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

```
> normalize(rp(t));
```

$$\left[(e^t \sin(2t) + 2e^t \cos(2t)) / \right.$$

$$\left. (|2e^t \sin(t) \cos(t) + 4e^t \cos(t)^2 - 2e^{2t}|^2 + |-2e^t \cos(t)^2 + e^{2t} \right.$$

$$\left. + 4e^t \sin(t) \cos(t)|^2 + 4(e^{2t})^{1/2}, (e^t \cos(2t) - 2e^t \sin(2t)) / \right.$$

$$\left. (|2e^t \sin(t) \cos(t) + 4e^t \cos(t)^2 - 2e^{2t}|^2 + |-2e^t \cos(t)^2 + e^{2t} \right.$$

$$\left. + 4e^t \sin(t) \cos(t)|^2 + 4(e^{2t})^{1/2}, (2e^t) / \right.$$

$$\left. (|2e^t \sin(t) \cos(t) + 4e^t \cos(t)^2 - 2e^{2t}|^2 + |-2e^t \cos(t)^2 + e^{2t} \right.$$

$$\left. + 4e^t \sin(t) \cos(t)|^2 + 4(e^{2t})^{1/2} \right]$$

```
> simplify(%);
```

$$\left[\frac{1}{3} \sin(2t) + \frac{2}{3} \cos(2t) \quad \frac{1}{3} \cos(2t) - \frac{2}{3} \sin(2t) \quad \frac{2}{3} \right]$$

> **T:=t->vector([1/3*sin(2*t)+2/3*cos(2*t), 1/3*cos(2*t)-2/3*sin(2*t), 2/3]);**

$$T:=t \rightarrow \left[\frac{1}{3} \sin(2t) + \frac{2}{3} \cos(2t) \quad \frac{1}{3} \cos(2t) - \frac{2}{3} \sin(2t) \quad \frac{2}{3} \right]$$

> **map(diff,T(t),t);**

$$\left[\frac{2}{3} \cos(2t) - \frac{4}{3} \sin(2t) \quad -\frac{2}{3} \sin(2t) - \frac{4}{3} \cos(2t) \quad 0 \right]$$

> **Tp:=t->vector([2/3*cos(2*t)-4/3*sin(2*t), -2/3*sin(2*t)-4/3*cos(2*t), 0]);**

$$Tp:=t \rightarrow \left[\frac{2}{3} \cos(2t) - \frac{4}{3} \sin(2t) \quad -\frac{2}{3} \sin(2t) - \frac{4}{3} \cos(2t) \quad 0 \right]$$

> **normalize(Tp(t));**

$$\left[\frac{\frac{2}{3} \cos(2t) - \frac{4}{3} \sin(2t)}{\sqrt{\left| -\frac{4}{3} \cos(t)^2 + \frac{2}{3} + \frac{8}{3} \sin(t) \cos(t) \right|^2 + \left| \frac{4}{3} \sin(t) \cos(t) + \frac{8}{3} \cos(t)^2 - \frac{4}{3} \right|^2}}, \right. \\ \left. \left(-\frac{2}{3} \sin(2t) - \frac{4}{3} \cos(2t) \right) / \left(\left| -\frac{4}{3} \cos(t)^2 + \frac{2}{3} + \frac{8}{3} \sin(t) \cos(t) \right|^2 + \left| \frac{4}{3} \sin(t) \cos(t) + \frac{8}{3} \cos(t)^2 - \frac{4}{3} \right|^2 \right)^{1/2}, 0 \right]$$

> **simplify(%);**

$$\left[\frac{1}{5} (\cos(2t) - 2 \sin(2t)) \sqrt{5} \quad -\frac{1}{5} (\sin(2t) + 2 \cos(2t)) \sqrt{5} \quad 0 \right]$$

> **N:=t->vector([1/5*(cos(2*t)-2*sin(2*t))*5^(1/2), -1/5*(sin(2*t)+2*cos(2*t))*5^(1/2), 0]);**

$$N:=t \rightarrow \left[\frac{1}{5} (\cos(2t) - 2 \sin(2t)) \sqrt{5} \quad -\frac{1}{5} (\sin(2t) + 2 \cos(2t)) \sqrt{5} \quad 0 \right]$$

> **crossprod(T(t),N(t));**

$$\left[\frac{2}{15} (\sin(2t) + 2 \cos(2t)) \sqrt{5}, \frac{2}{15} (\cos(2t) - 2 \sin(2t)) \sqrt{5}, -\frac{1}{5} \left(\frac{1}{3} \sin(2t) \right) \right]$$

$$\left. \begin{aligned} & + \frac{2}{3} \cos(2t) \right) (\sin(2t) + 2 \cos(2t)) \sqrt{5} - \frac{1}{5} \left(\frac{1}{3} \cos(2t) \right. \\ & \left. - \frac{2}{3} \sin(2t) \right) (\cos(2t) - 2 \sin(2t)) \sqrt{5} \end{aligned} \right]$$

> **simplify(%);**

$$\left[\frac{2}{15} (\sin(2t) + 2 \cos(2t)) \sqrt{5} - \frac{2}{15} (-\cos(2t) + 2 \sin(2t)) \sqrt{5} - \frac{1}{3} \sqrt{5} \right]$$

> **B:=t->vector([2/15*(sin(2*t)+2*cos(2*t))*5^(1/2), 2/15*(cos(2*t)-2*sin(2*t))*5^(1/2), -1/3*5^(1/2)]);**

$$B := t \rightarrow \left[\frac{2}{15} (\sin(2t) + 2 \cos(2t)) \sqrt{5} \quad \frac{2}{15} (\cos(2t) - 2 \sin(2t)) \sqrt{5} \quad -\frac{1}{3} \sqrt{5} \right]$$

> **N1:=norm(Tp(t),2);**

$$N1 := \sqrt{\left| \frac{2}{3} \cos(2t) - \frac{4}{3} \sin(2t) \right|^2 + \left| \frac{2}{3} \sin(2t) + \frac{4}{3} \cos(2t) \right|^2}$$

> **N2:=norm(rp(t),2);**

$$N2 := \sqrt{|e^t \sin(2t) + 2 e^t \cos(2t)|^2 + |-e^t \cos(2t) + 2 e^t \sin(2t)|^2 + 4 (e^t)^2}$$

> **N1/N2;**

$$\frac{\sqrt{\left| \frac{2}{3} \cos(2t) - \frac{4}{3} \sin(2t) \right|^2 + \left| \frac{2}{3} \sin(2t) + \frac{4}{3} \cos(2t) \right|^2}}{\sqrt{|e^t \sin(2t) + 2 e^t \cos(2t)|^2 + |-e^t \cos(2t) + 2 e^t \sin(2t)|^2 + 4 (e^t)^2}}$$

> **simplify(%);**

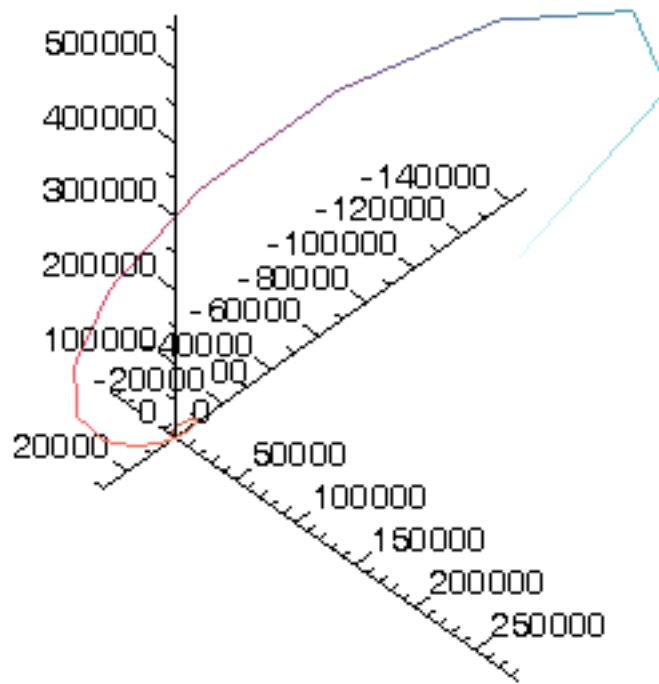
$$\frac{2}{9} \sqrt{5} e^{-t}$$

> **kappa:=t->2/9*5^(1/2)*exp(-t);**

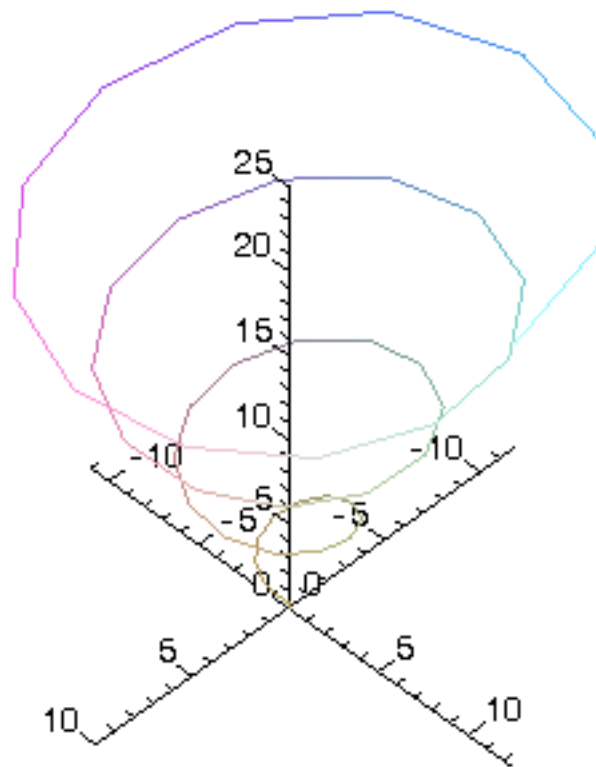
$$\kappa := t \rightarrow \frac{2}{9} \sqrt{5} e^{-t}$$

> **with(plots):**

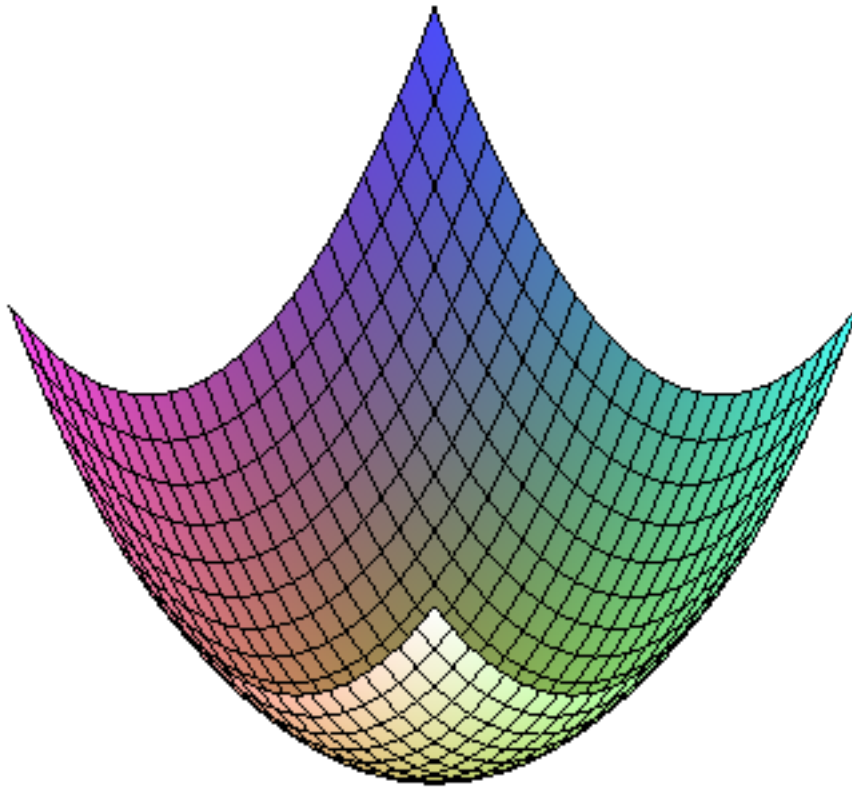
> **spacecurve({[exp(t)*sin(2*t), exp(t)*cos(2*t), 2*exp(t)]}, t=0..4*Pi);**



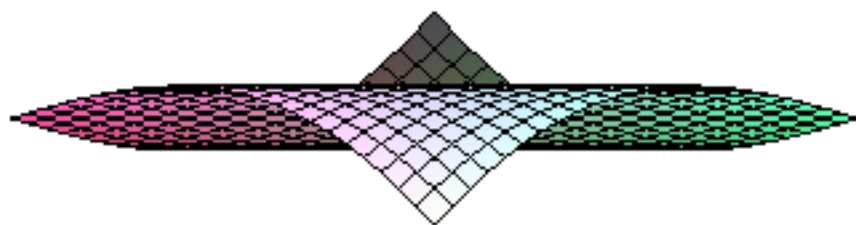
```
> spacecurve([t*sin(2*t),t*cos(2*t),2*t],t=0..4*Pi);
```

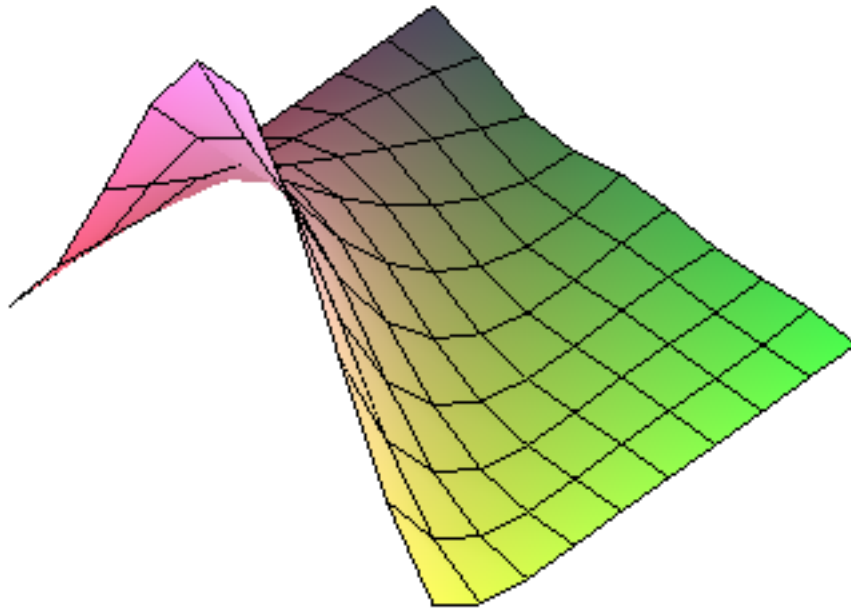


```
> plot3d(x^2+y^2,x=-2..2,y=-2..2);
```

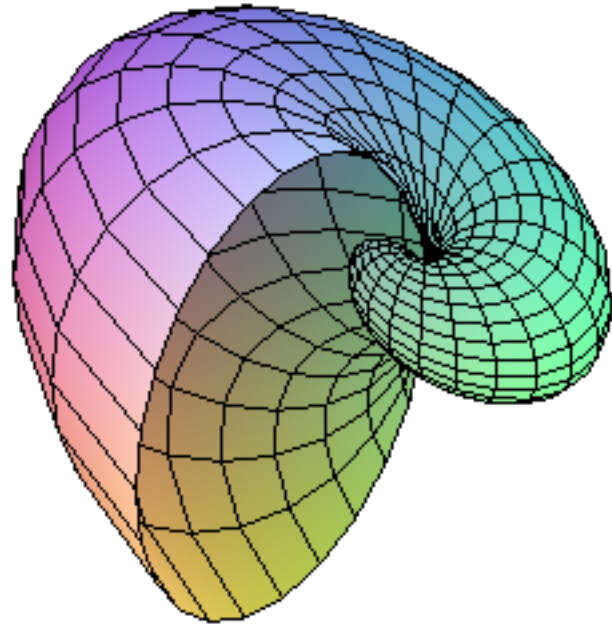


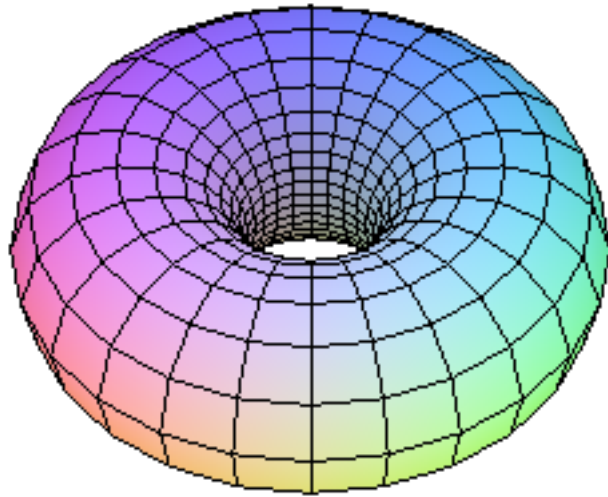
```
> plot3d(sin(x+y), x=-1..1, y=-1..1);  
plot3d(binomial, 0..5, 0..5, grid=[10,10]);
```



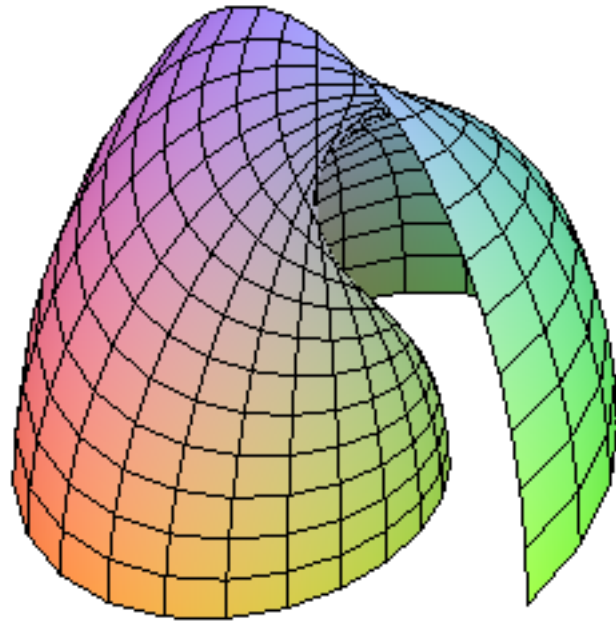


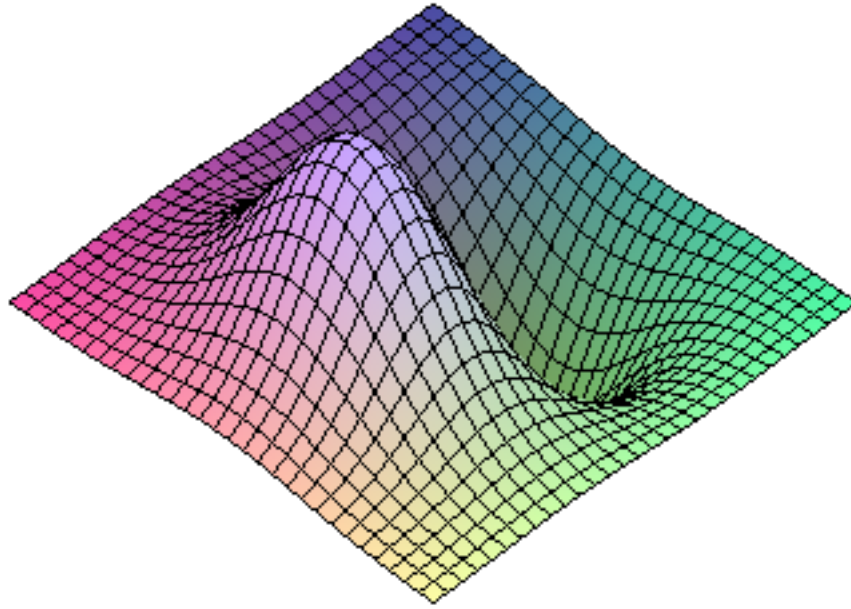
```
> plot3d((1.3)^x * sin(y), x=-1..2*Pi, y=0..Pi, coords=spherical,  
style=patch);  
plot3d([1,x,y], x=0..2*Pi, y=0..2*Pi, coords=toroidal(10),  
scaling=constrained);
```



```
> plot3d([x*sin(x)*cos(y), x*cos(x)*cos(y), x*sin(y)], x=0..2*Pi,  
y=0..Pi);  
plot3d(x*exp(-x^2-y^2), x=-2..2, y=-2..2, grid=[30,30]);
```





```
> plot3d(x^2-y^2,x=-10..10,y=-10..10,orientation=[80,70]);
```

