

Dear Editor-in-Chief and Referees:

Thank you for your kind letter of “Decision on Manuscript ID 2036-6344-1-RV” on the 17th February, 2010. The title of the manuscript is: “Analysis of the inexact Uzawa algorithms for nonlinear saddle point problems”(by Jian-Lei Li and Liang Li). We are very grateful to your and the reviewers’ comments and thoughtful suggestions. We are sorry for the linguistic quality of the original manuscript. Based on these comments and suggestions, we have made careful modifications to the original manuscript, and carefully proof-read the manuscript to minimize typographical and grammatical errors. We believe that the manuscript has been greatly improved and hope it has reached your magazine’s standard.

Once again, we acknowledge your comments very much, which are valuable in improving the quality of our manuscript.

Sincerely yours

Jian-Lei Li and Liang Li.

Here are our responses to the reviewers’ comments and list of corrections.

**The reviewer’s comment:** The paper introduces new inexact Uzawa algorithms for solving nonlinear saddle point problems. The proposed idea is interesting, but it seems that further assumptions are needed in order to make the algorithm and the analysis mathematically correct. Here are the main concerns about the paper which need to be considered.

1. The Definition of the generalized Jacobian (bottom of page 3) is confusing. First, I think a gradient symbol is missing when defining  $\partial_B F$ . (In passing, why are the authors using the notation  $\partial_B$  since  $\partial F$  does not have anything to do with the matrix  $B$ ?) Second, the general Jacobian is a set of matrices which might not be symmetric in the general case. Nevertheless, the authors completely ignore this facts, and without no assumptions, in Algorithm1, they consider that general Jacobian is a singled valued function, talking about “ $A_i$  is the generalized Jacobian of  $F$  at  $x_i$ ” and “ $A_i$  is a positive definite operator” (top of page 4) and further ignore that  $A_i$  might not be a symmetric matrix.

**The authors’ answer:** We are very grateful to your comments on the manuscript. The purpose of this paper is to try to give the inexact Uzawa

algorithms for solving nonlinear saddle point problems. For our mistakes, the gradient symbol is missing when defining  $\partial_B F$ . Now, we have corrected it, i.e., “ $\partial_B F(x) = \left\{ \lim_{\xi \rightarrow x, \xi \in D_F} F(x) \right\}$ ” has been replaced by “ $\partial_B F(x) = \left\{ \lim_{\xi \rightarrow x, \xi \in D_F} \nabla F(x) \right\}$ ” (see the top of page 4). Moreover, in fact, the definition  $\partial_B$  has nothing to do with the matrix  $B$ , it is just a symbol. Please check the related reference [4]. In addition, we appreciate the reviewer for pointing out that the general Jacobian is a set of matrices which might not be symmetric in the general case. Now, we have redefined the norm under the positive definite operator and have given the new assumptions on the positive definite operator. Such as the Eqs. (2.1), (2.6), (2.8), (2.11) and (2.13), some more detailed discussion has been made, please check Section 2 in the revised manuscript.

**The reviewer’s comment:** 2. At the beginning of Section 2, the authors define “ $G$ -induced norm” without the assumption that  $G$  is a symmetric matrix. It might seem that this is an omission, but in the “proof” of Lemma 1, the authors are using such “norms” for  $G = A_i$  a possible non-symmetric matrix. If this is just a notation and the authors are not really using all the property of norms (specially the triangle inequality property), then the authors should clearly specify that.

**The authors’ answer:** We thank you very much for your comments for pointing out this omission. Now, in the revised manuscript, we have corrected the definition of the “ $G$ -induced norm”. For the positive definite operator  $G$ , we define the symmetric part  $G_s$  of the matrix  $G$  as  $G_s = \frac{1}{2}(G + G^T)$ .  $\|x\|_{G_s}$  represent the  $G$ -induced norm, namely  $\|x\|_{G_s} = (G_s x, x)^{\frac{1}{2}}$  for all  $x \in R^n$ .

**The reviewer’s comment:** 3. In the proof of Theorem 1, the authors are using that the square root of  $A_i$  exists and is a symmetric matrix, without assuming that  $A_i$  is a symmetric matrix. This does not have any justification and needs to be clarified.

**The authors’ answer:** We are very grateful to your comments and thoughtful suggestions. In the revised manuscript, we have modified the assumptions and the definition of the norms, and corrected the proof of Lemma. We use the square root of  $A_s$  which is the symmetric part of  $A$ . In addition, we also

give two other Lemmas to support the proofs of the theorems; see pages 6-7. At the same time, we also slightly modified the Theorem 1 and Theorem 2 which are the main results on the convergence of the inexact Uzawa algorithm for the nonlinear saddle point problems.

**The reviewer's comment:** 4. The numerical results are presented for a simple function  $F$  which is differentiable everywhere and the generalized Jacobian of  $F$  coincides with the standard Jacobian matrix at any point. In addition, in this case the Jacobian matrix is symmetric at any point. Thus, the theory presented would make sense and work only for this very particular case for  $F$ .

**The authors' answer:** We thank you very much for your comments on our manuscript. The numerical example comes from the reference [5]. Here, we know that the Jacobian matrix is symmetric, however, the numerical experiments don't effect the theory on the convergence analysis of the inexact Uzawa algorithm. In the future research, we will focus on the example of the Jacobian matrix which is nonsymmetric but positive definite.

In conclusion, according to the comments, we give a detailed revision on the original manuscript. At the same time, we have thoroughly revised the manuscript in order to express the idea more clearly. The major revision are listed in the following. We believe the manuscript has been greatly improved. Once again, thank you for the kind advice.

### Summary of the Revision Made

Since we have made thorough revision, here we only list those important ones.

1. In Section 2 (Algorithms):

(1) "For any  $n \times n$  positive definite matrix  $G$ ,  $\|x\|_G$  represent the  $G$ -induced norm, namely  $\|x\|_G = (Gx, x)^{\frac{1}{2}}$  for all  $x \in R^n$ " has been replaced by "For any  $n \times n$  positive definite matrix  $G$ , the symmetric part  $G_s$  of the matrix  $G$  is defined by

$$G_s = \frac{1}{2}(G + G^T),$$

$\|x\|_{G_s}$  represent the  $G$ -induced norm, namely  $\|x\|_{G_s} = (G_s x, x)^{\frac{1}{2}}$  for all  $x \in R^n$ ".

(2) “ $\partial_B F(x) = \left\{ \lim_{\xi \rightarrow x, \xi \in D_F} F(x) \right\}$ ” has been replaced by “ $\partial_B F(x) = \left\{ \lim_{\xi \rightarrow x, \xi \in D_F} \nabla F(x) \right\}$ ”

(see the top of page 4).

(3) For the convenient of the describe, we define the generalized Jacobian matrix of  $F$  at  $x_i$  as  $A$ . From lines 19 to 23 on page 4, we add the sentence “By (2.4),  $A$  is a positive definite operator. Hence,  $A_s$  is a symmetric positive definite operator, assume that  $A_s$  satisfies

$$(Av, w) \leq \alpha(A_s v, v)^{1/2}(A_s w, w)^{1/2}, \text{ for all } v, w \in R^n,$$

for some number  $\alpha$ . Clearly,  $\alpha \geq 1$ . Moreover, since  $A_s$  is positive definite, such an  $\alpha$  always exists.” Moreover, Eqs. (2.8), (2.11), (2.12) and (2.13) are changed.

2. In Section 3 (Analysis of the Uzawa algorithms for the nonlinear saddle point problems):

Since the assumption condition changed greatly in Section 2, in Section 3, there are lots of change, for example, we added to lemmas, and give the new proofs of Theorem 1 and Theorem 2. For details, you can check pages 6-11 in the revised manuscript.

3. In the reference, we added two references [30] and [31].

[30] J.H. Bramble, J.E. Pasciak, A.T. Vassilev, Uzawa type algorithms for nonsymmetric saddle point problems, Math. Comp., 69 (1999), pp. 667-689.

[31] Y. Saad, M.H. Schultz, GMRES: a generalized minimal residual algorithm for solving nonsymmetric linear systems, SIAM J. Sci. Comput., 7 (1986), pp. 856-869.

We have also corrected some minor spelling errors which are not listed here. We would like to thank the referee for helpful comments and suggestions on the manuscript.

**Thank you very much. Have a good time everyday!**

Sincerely yours,

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