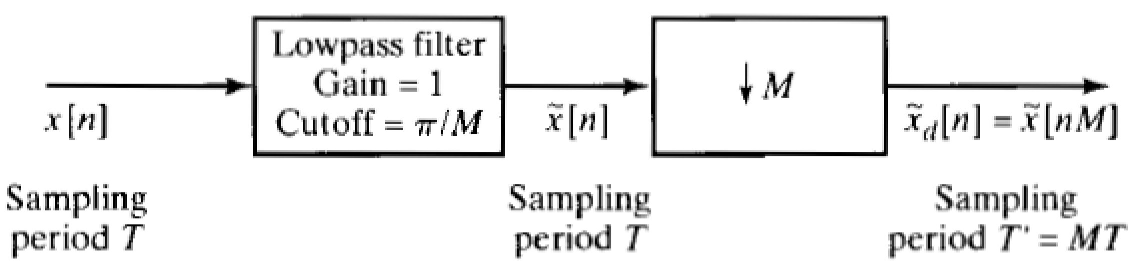
Sampling theorem

- Avoiding aliasing in downsampling via DT anti-aliasing filter
- The cut-off frequency: $\omega_N < \pi / M$



General system for sampling rate reduction by M.

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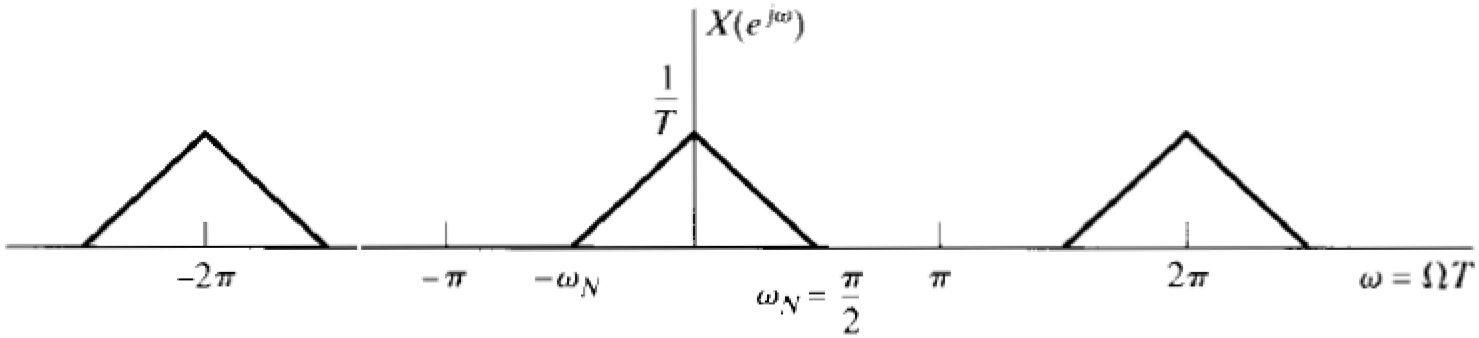
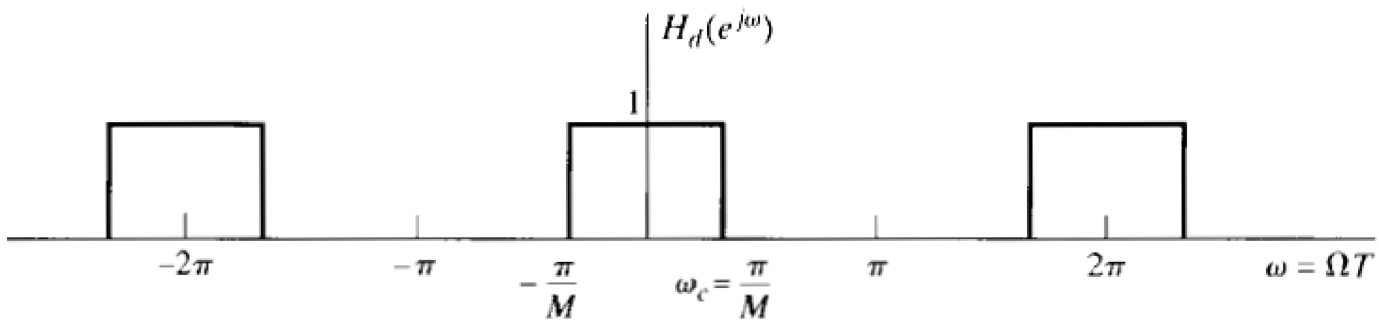
Sampling theorem

- Modified version of the aliased example





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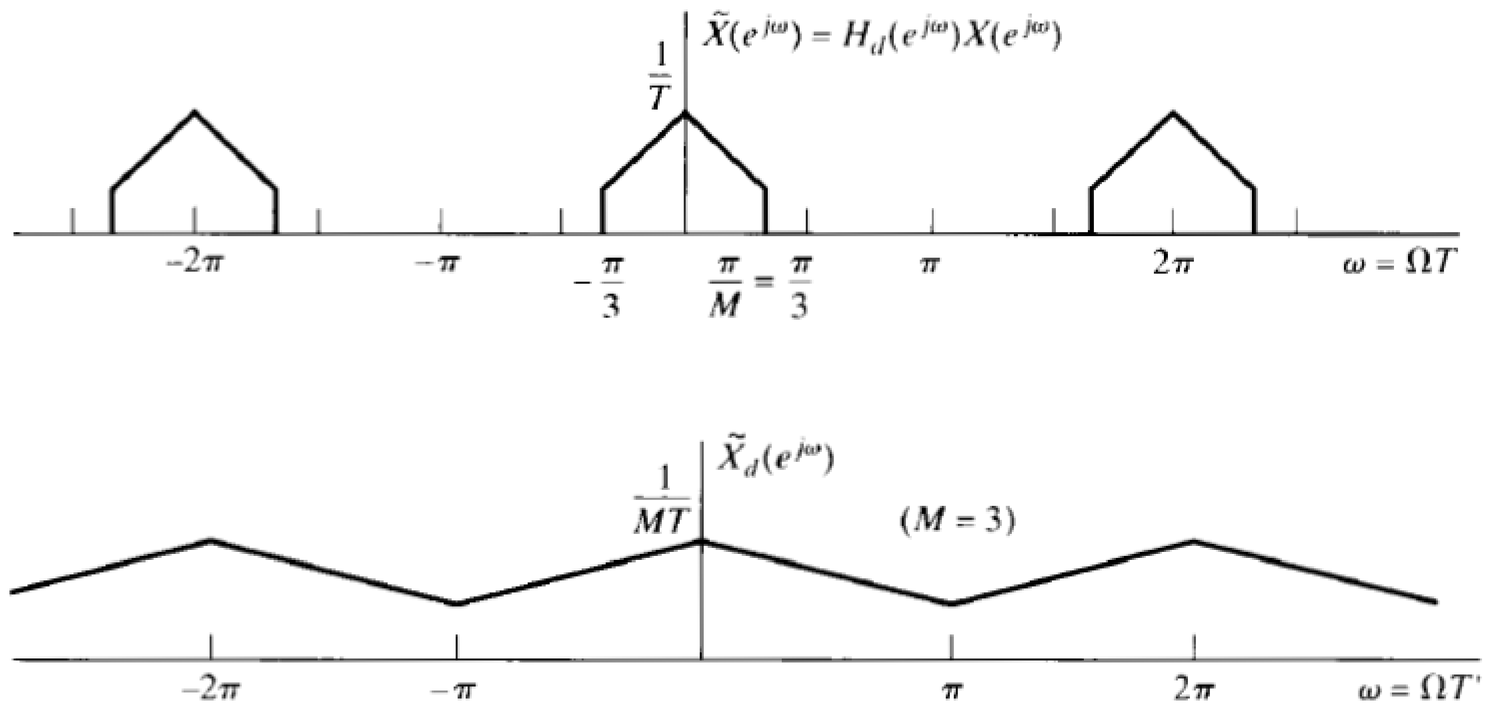
Sampling theorem

- Cont.



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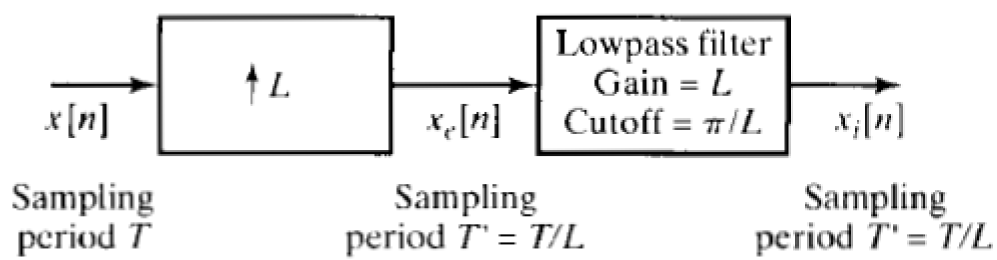
Sampling theorem

- Increasing the sampling rate by an integer factor $x_i[n] = x[n/L] = x_c(nT/L), \qquad n = 0, \pm L, \pm 2L, \ldots.$
  - Expander/upsampling

$$x_e[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots, \\ 0, & \text{otherwise}, \end{cases}$$



General system for sampling rate increase by L.

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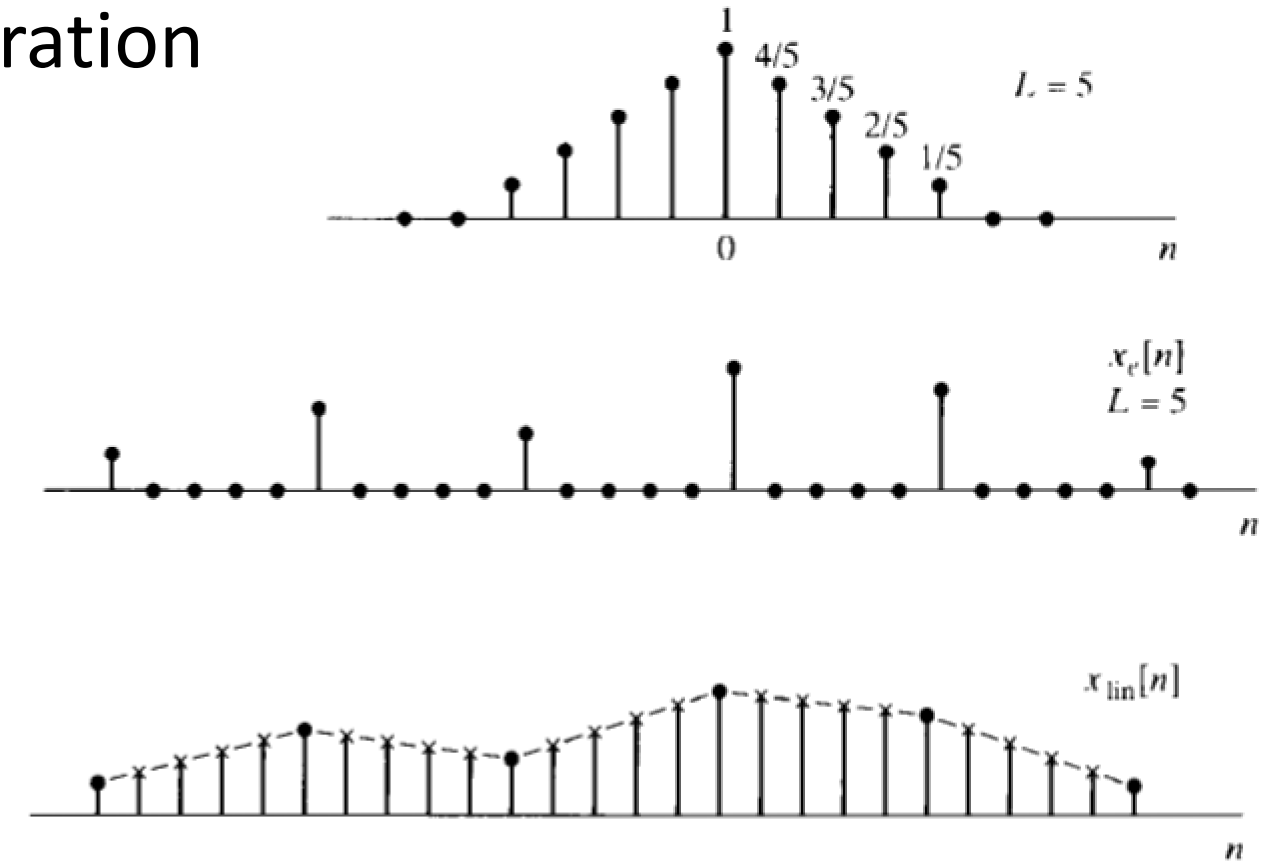
Sampling theorem

- Illustration



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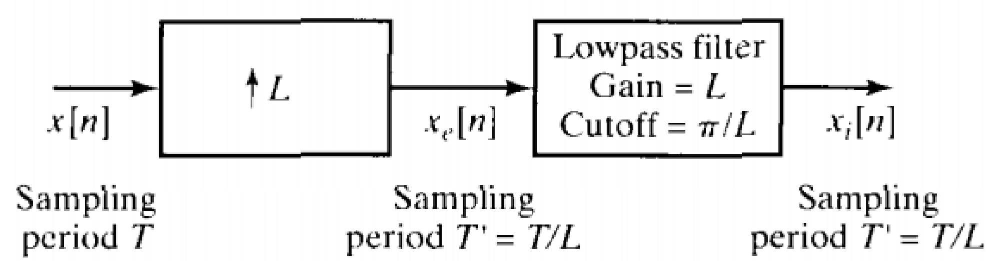
Sampling theorem

- We saw that

$$x_e[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots, \\ 0, & \text{otherwise}, \end{cases}$$

  – Meaning that $$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - kL].$$



General system for sampling rate increase by L.

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Sampling theorem

- The role of the LPF is interpolation!!
  - Note that

$$X_e(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x[k]\delta[n-kL] \right) e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega Lk} = X(e^{j\omega L}).$$

  - Frequency scaling

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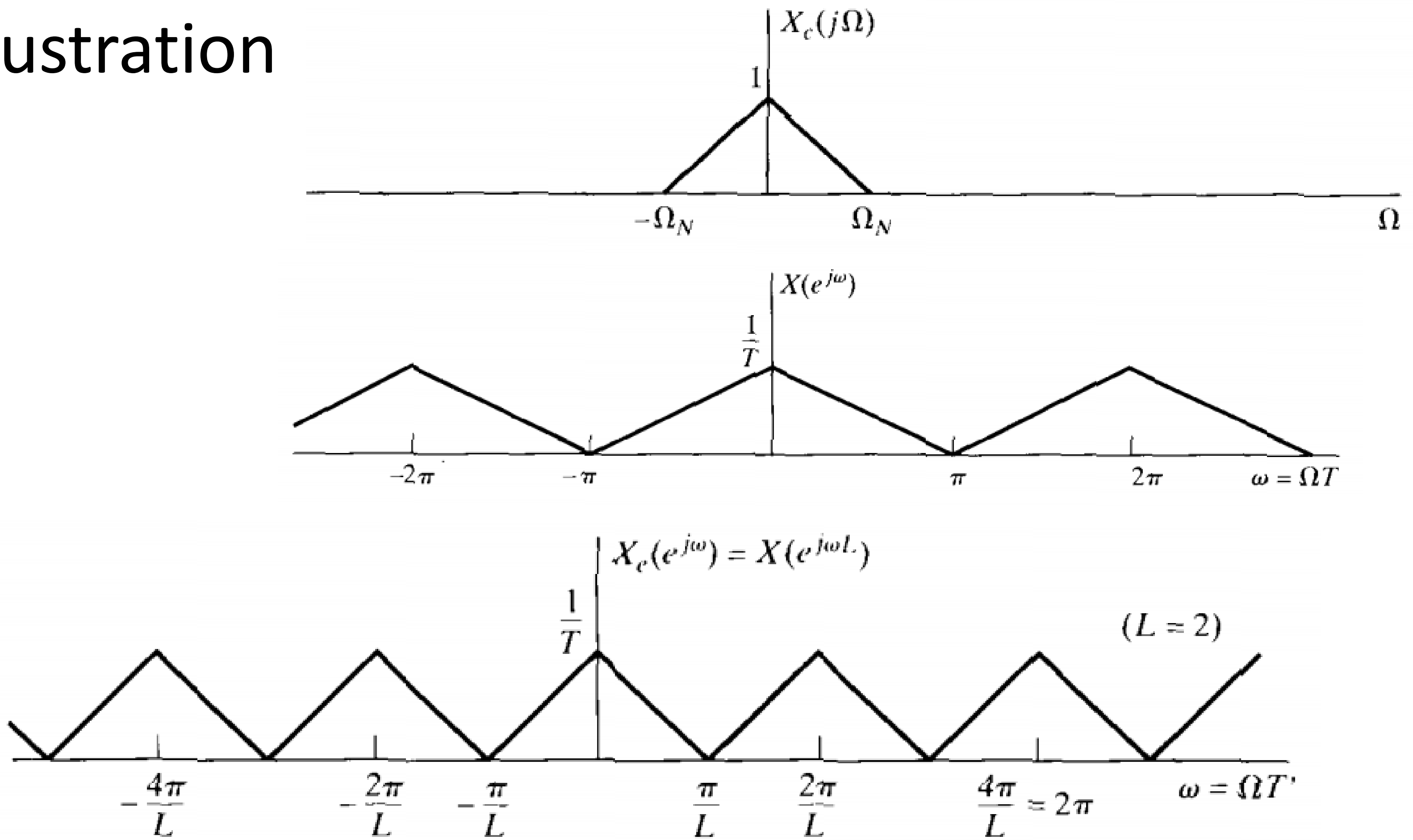
Sampling theorem

- Illustration



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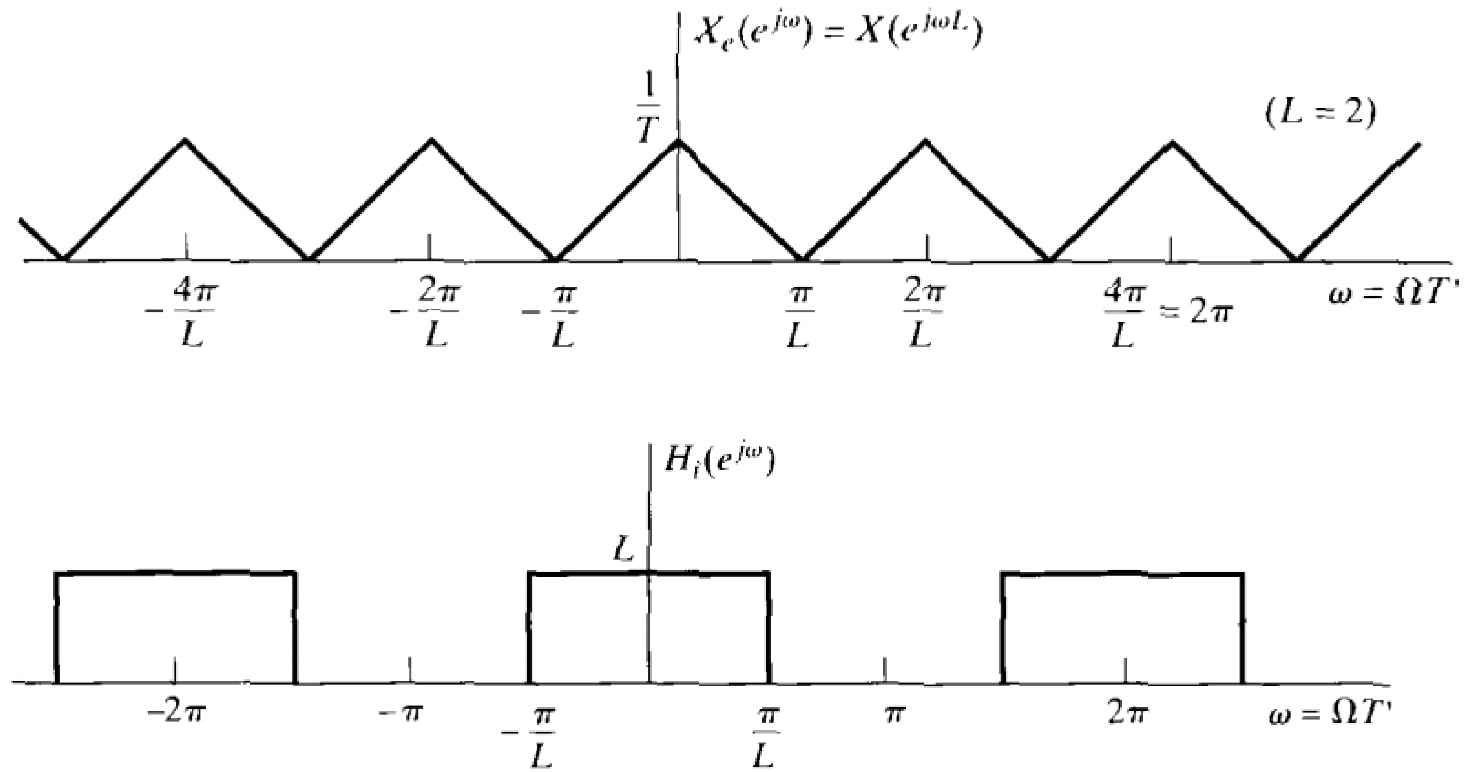
Sampling theorem

– And, the filtering:



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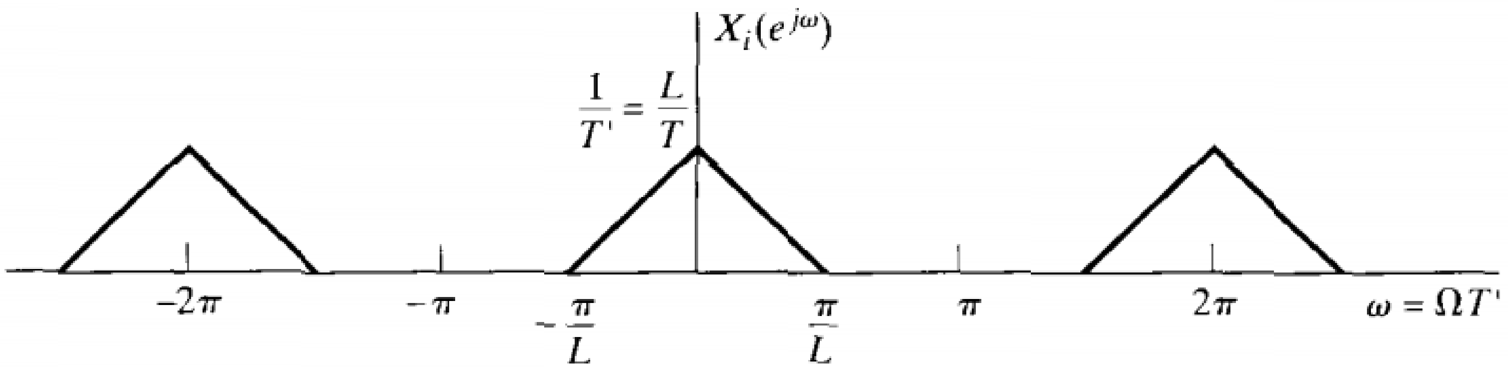
Sampling theorem

– Output



– Note that $T' = T/L$

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Sampling theorem

- Time-domain analysis
  - The impulse response of the LPF

$$h_i[n] = \frac{\sin(\pi n/L)}{\pi n/L}.$$

  - We also had

$$x_e[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-kL].$$

  - Thus:

$$x_i[n] = \sum_{k=-\infty}^{\infty} x[k]\frac{\sin[\pi(n-kL)/L]}{\pi(n-kL)/L}$$

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Sampling theorem

- This is the sinc interpolation
  - Note that

$$h_i[0] = 1,$$
$$h_i[n] = 0, \qquad n = \pm L, \pm 2L, \ldots.$$

  - Which means that

$$x_i[n] = x[n/L] = x_c(nT/L) = x_c(nT'), \qquad n = 0, \pm L, \pm 2L, \ldots,$$

- Linear/quadratic/qubic interpolators

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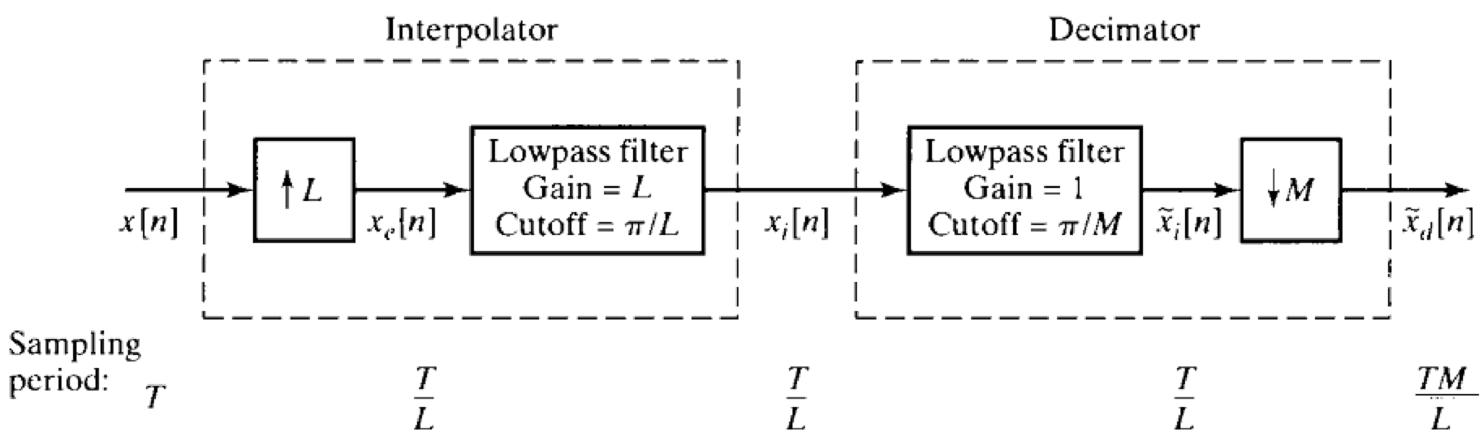
Sampling theorem

- Changing the rate by a non-integer rational factor



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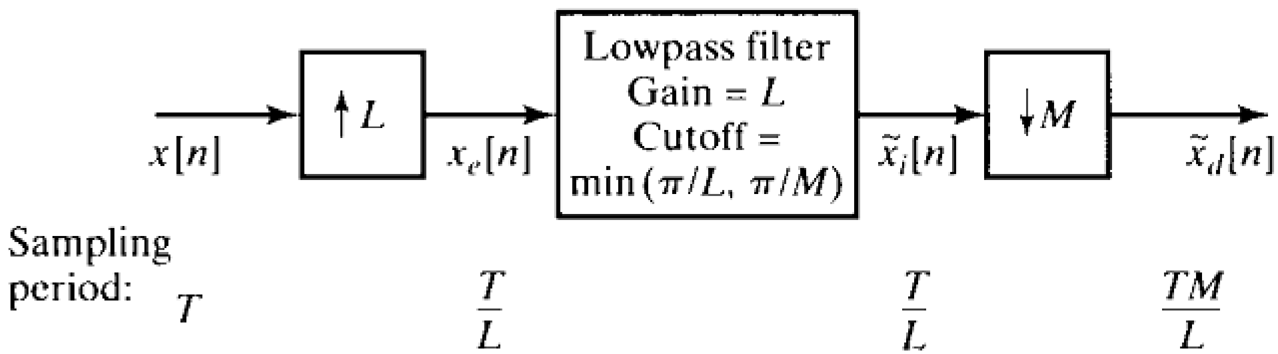
Sampling theorem

- The equivalent system



- Example $T' = (3/2)T$

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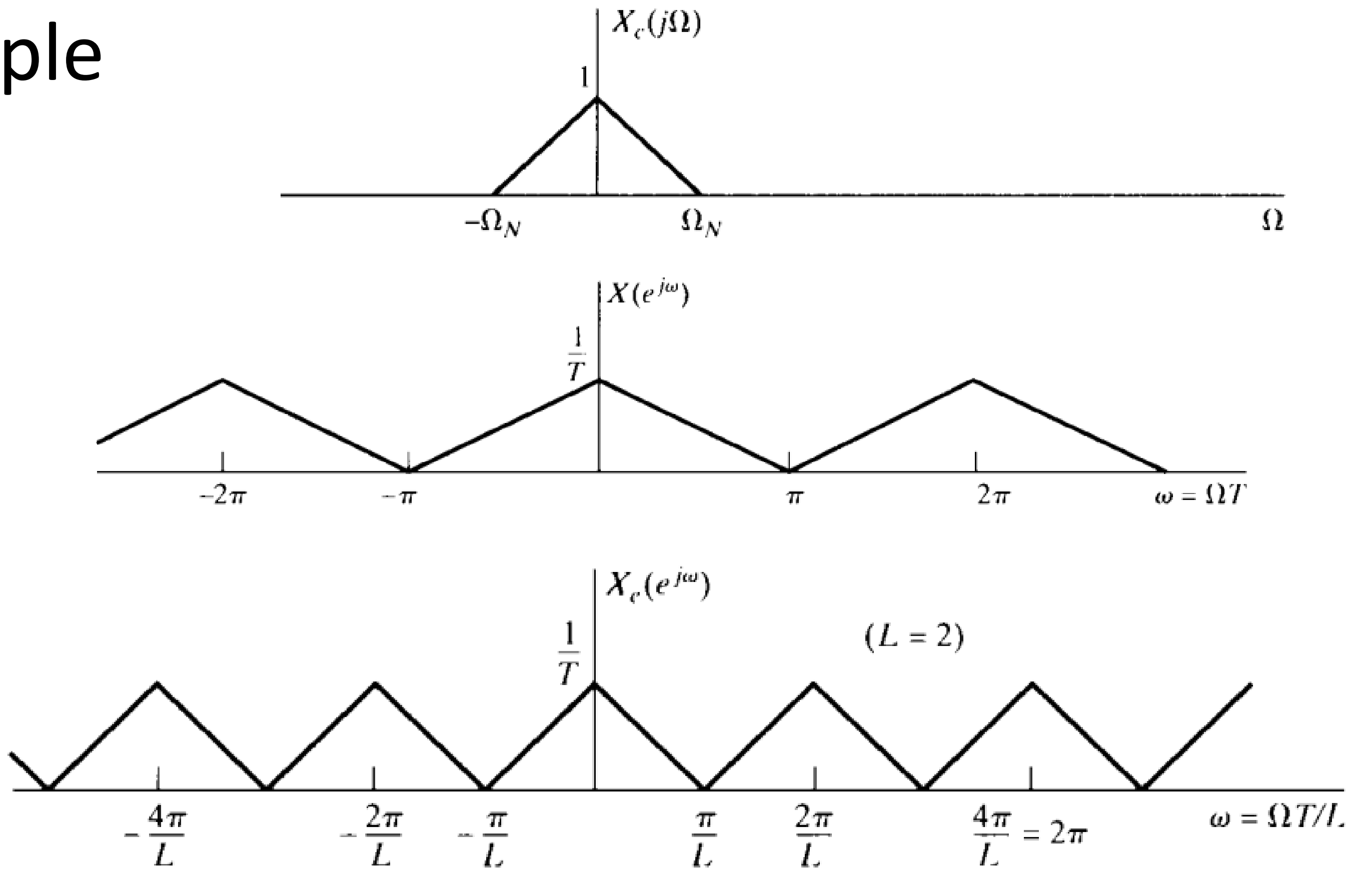
Sampling theorem

- Example



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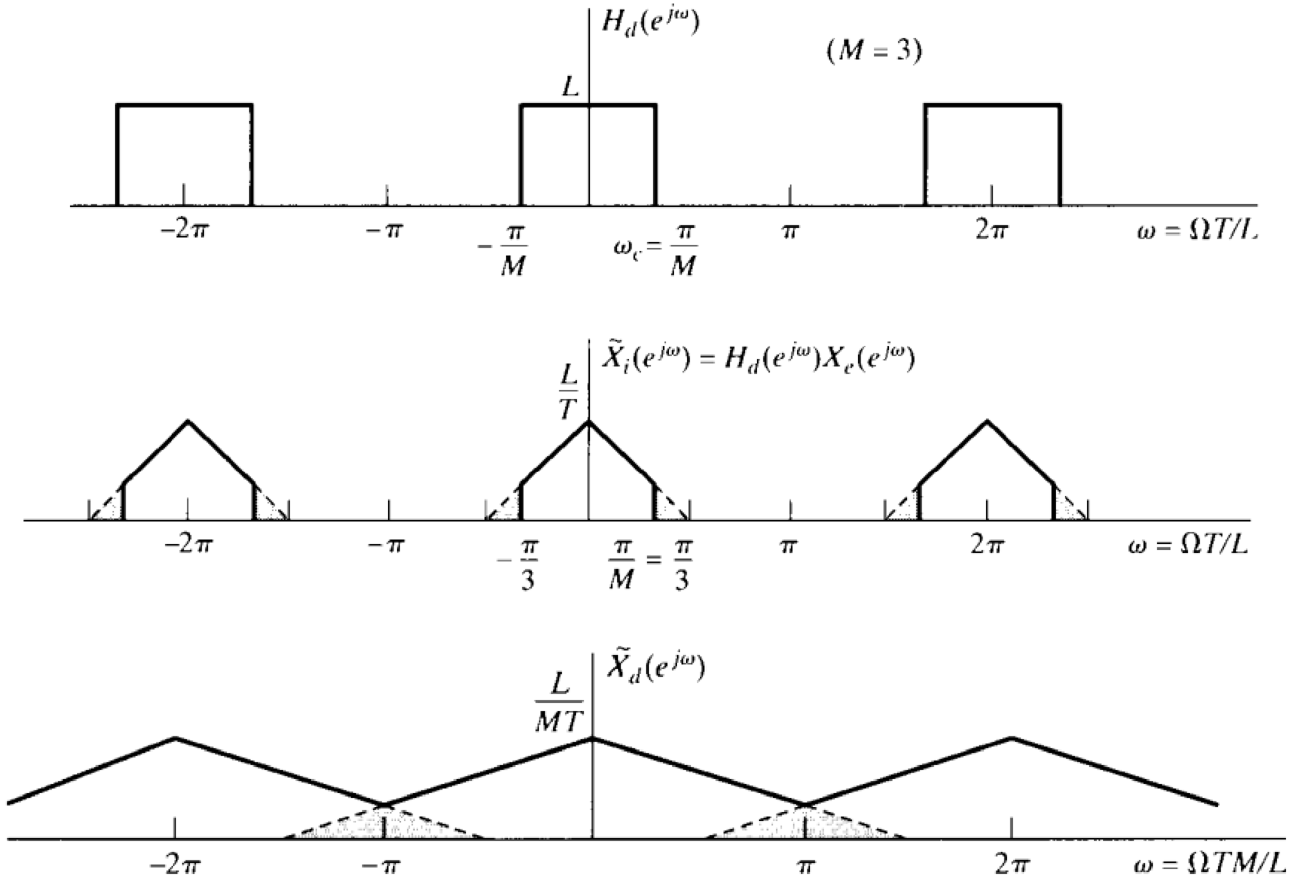
Sampling theorem

- Cont.



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Sampling theorem

- Applications
  - Multi-rate signal processing
    - More efficient signal processing algorithm via using upsampling/downsampling
  - A/D and D/A
  - Filterbanks

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