Digital Signal Processing (DSP)

Fall 2014

Mohammad Mahdi Naghsh
mm_naghsh@cc.iut.ac.ir

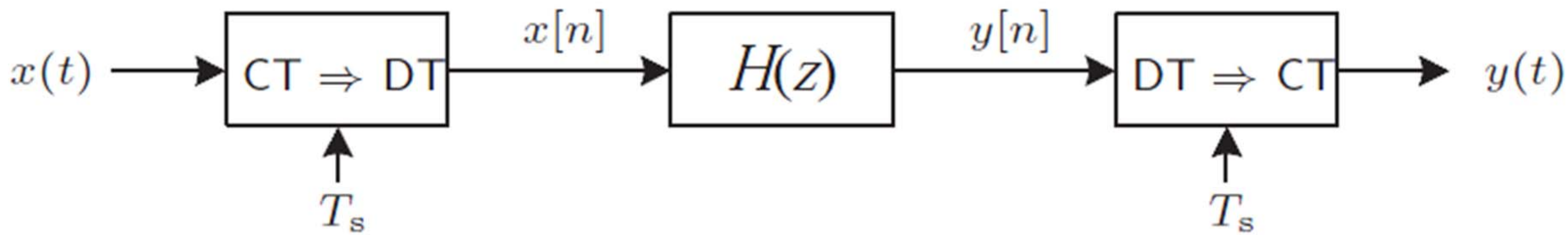
DIGITAL SIGNAL PROCESSING (DSP)

Lecture 7
Sampling Theorem

Mohammad Mahdi Naghsh ECE. Dept.,
Isfahan University of Technology

Sampling theorem

- DT processing of CT signals



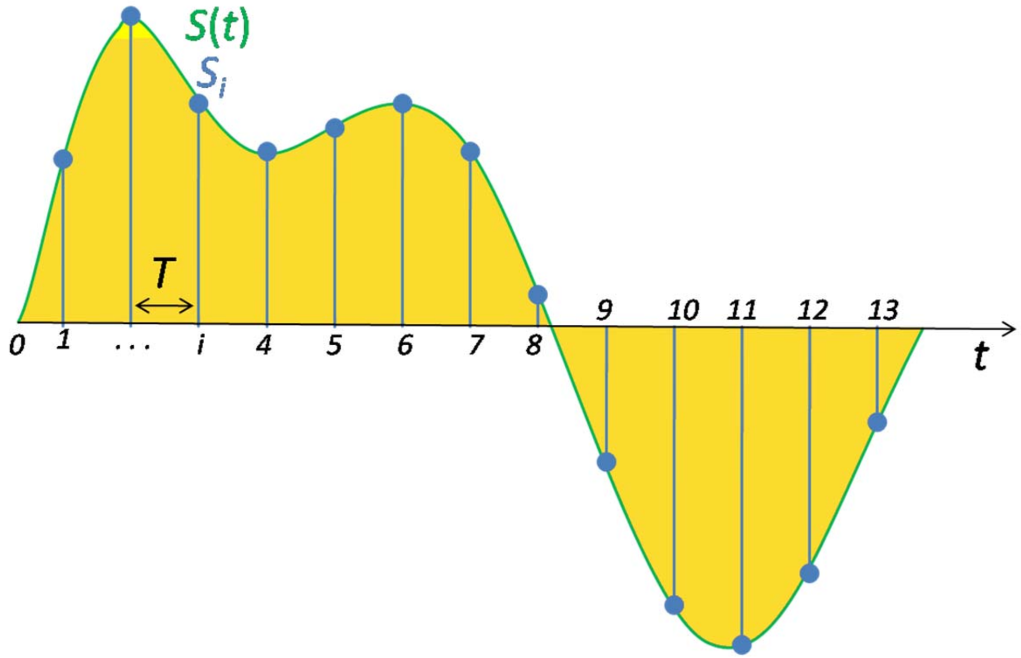
- We work on discrete-time signals not digital signals (more on this issue later!)

http://web.cecs.pdx.edu/~ece2xx/ECE223/Slides/Sampling.pdf

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- The main step: sampling $x[n] = x(t)|_{t=nT_s}$

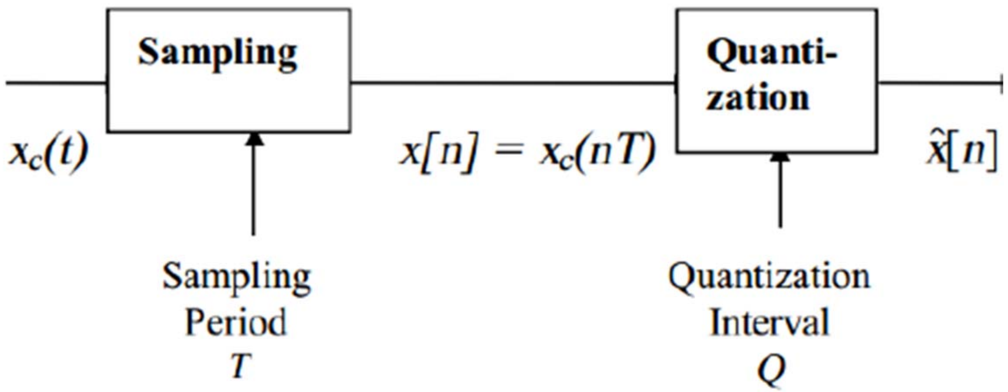


http://en.wikipedia.org/wiki/Sampling_%28signal_processing%29

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- Sampling: conversion of CT signals to discrete-time signals
- Quantization: conversion of the discrete-time signals to discrete-time discrete-values signals

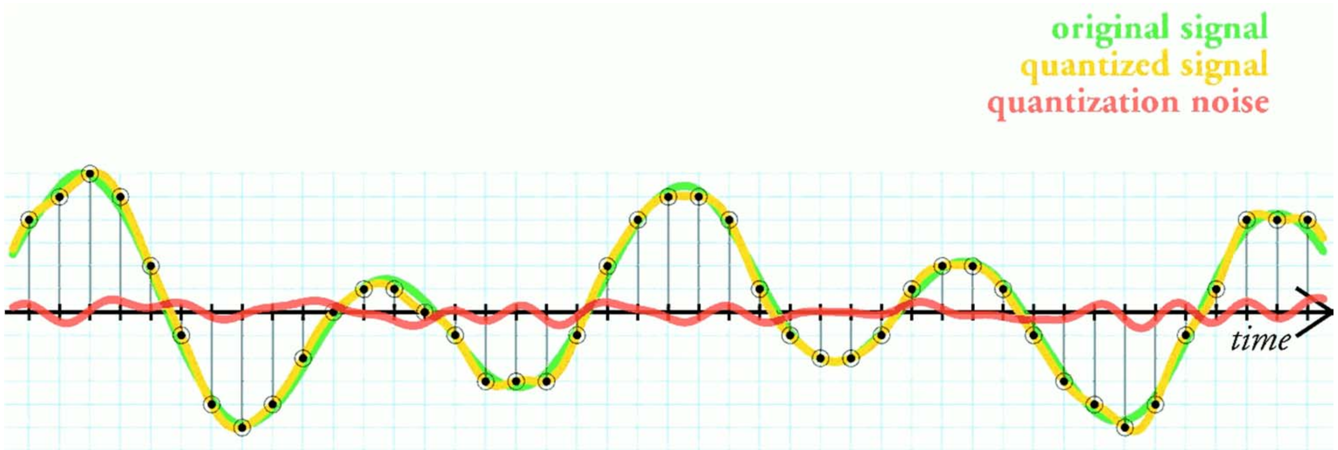


Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

http://www.psut.edu.jo/sites/alzoubi/project2/Puplications/Exp.%20PCM.pdf

Sampling theorem

- Quantization: quantization level
  - Non-linear
  - Associated with noise

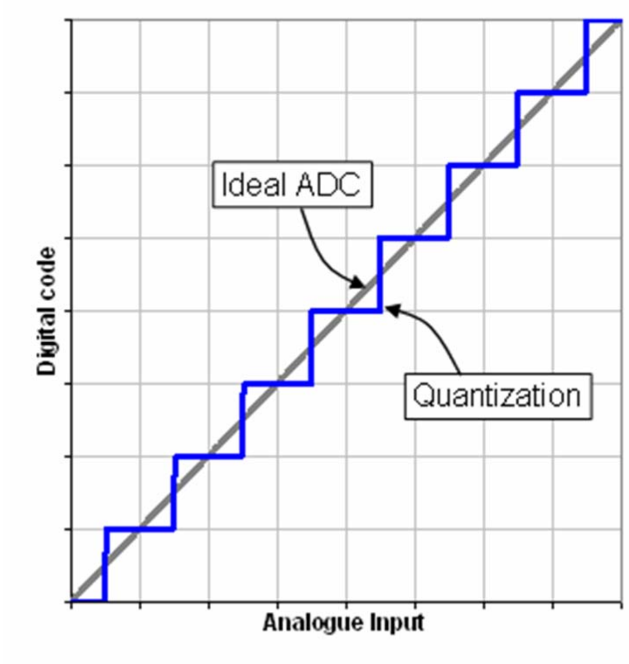


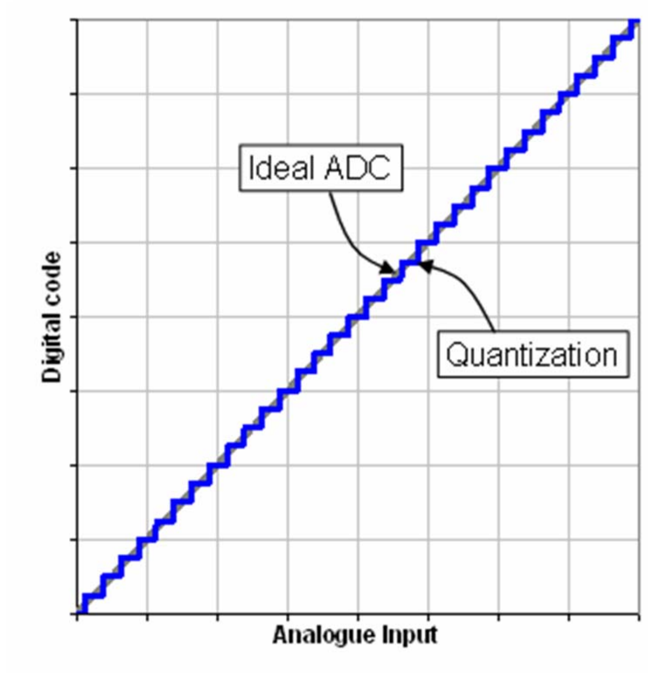
http://en.wikipedia.org/wiki/Quantization_%28signal_processing%29

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- More on the effects of the quantization level



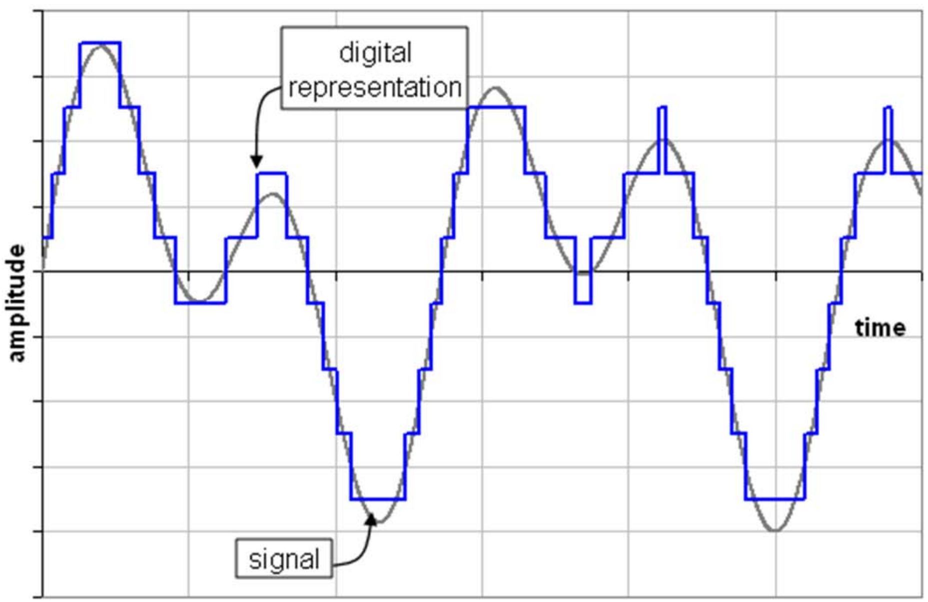


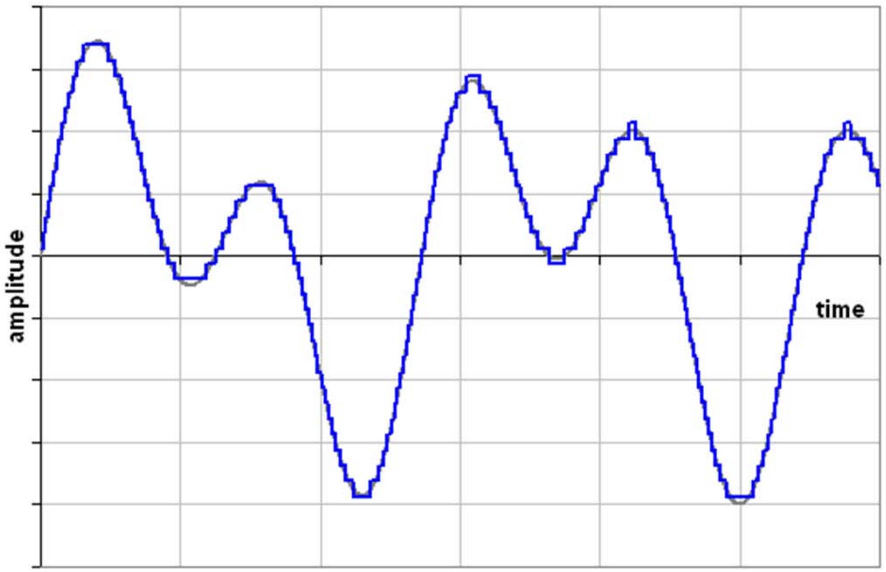
http://www.diracdelta.co.uk/science/source/q/u/quantization%20error/source.html#.VHROe8nzxtA

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- Example





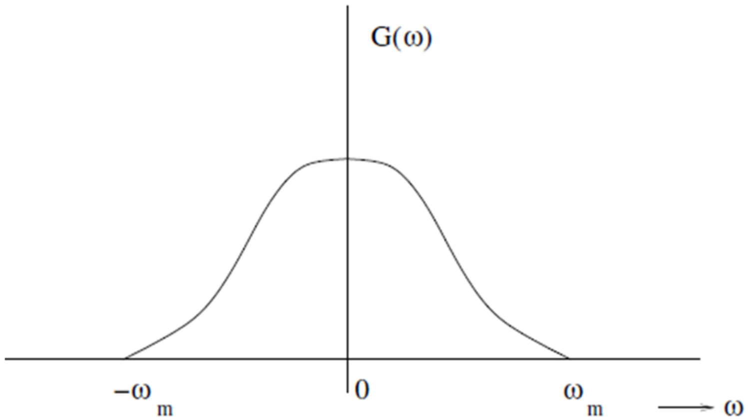
http://www.diracdelta.co.uk/science/source/q/u/quantization%20error/source.html#.VHROe8nzxtA

Sampling theorem

- Nyquest theorem (sampling theorem)

  "A bandlimited signal can be reconstructed exactly if it is sampled at a rate atleast twice the maximum frequency component in it."

- Illustration: band-limited signal



Sampling theorem

- Note that:

The maximum frequency component of $g(t)$ is f_m. To recover the signal $g(t)$ exactly from its samples it has to be sampled at a rate $f_s \geq 2f_m$.

- Nyquest rate: the minimum required sampling frequency (Hz)
- Sampling frequency (Hz)

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

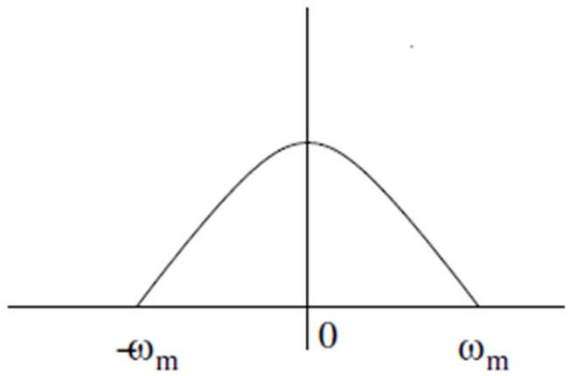
- Example: speech signal with 4 KHz bandwidth
  - Nyquest raet: 8000Hz
  - Sampling frequency: more than 8000Hz
  - Usual quantization level: 256

- ADC: sampling & quantization
  - Sampling frequency
  - Quantization level
  - Challenge!

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

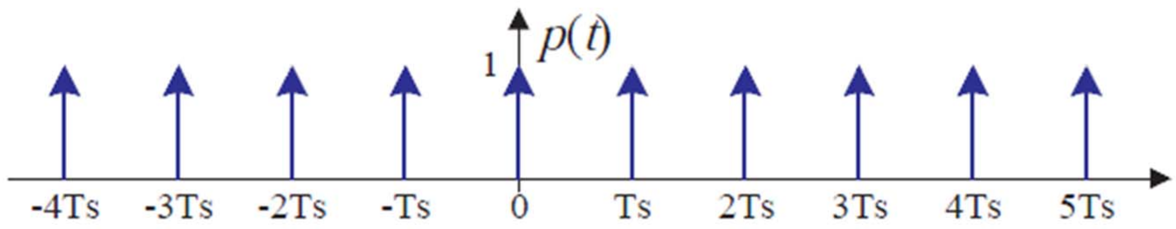
Sampling theorem

- Important proof:
  - Consider the band-limited signal *x(t)*
  - We use a modeling for the sampled signal
  - We calculate the FT to observe the proof



Sampling theorem

- Impulse sampling



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_\mathrm{s})$$

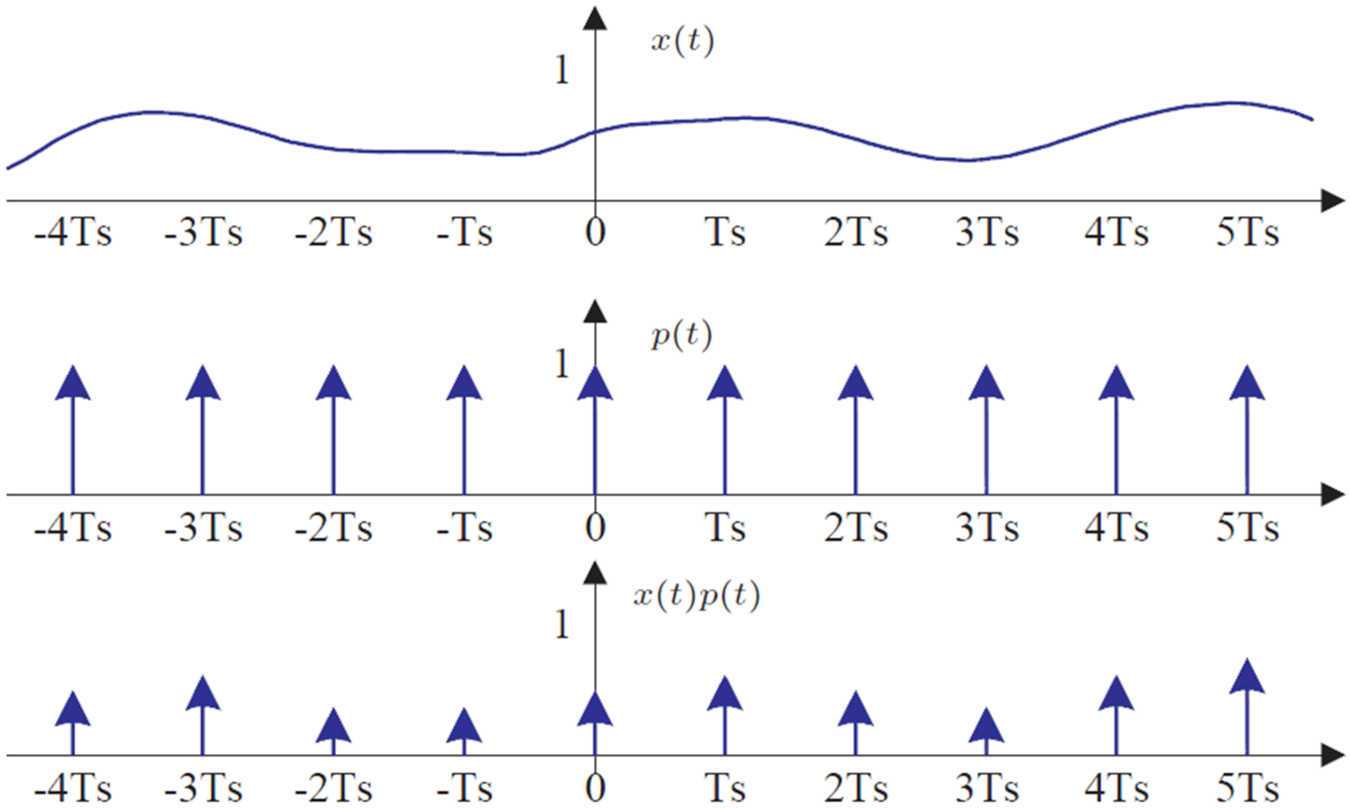
- – We model the sampled signal by weighted impulses

http://web.cecs.pdx.edu/~ece2xx/ECE223/Slides/Sampling.pdf

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- Example



Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

http://web.cecs.pdx.edu/~ece2xx/ECE223/Slides/Sampling.pdf

Sampling theorem

– Define

$$x_\delta(t) = x(t)\,p(t) = \sum_{n=-\infty}^{\infty} x(nT_\mathrm{s})\delta(t - nT_\mathrm{s})$$

– Next we compute the CTFT

$$x(t)\,p(t) \quad \overset{\mathcal{FT}}{\Longleftrightarrow} \quad \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

– Remember that (lecture 6):

$$P(j\omega) \quad \overset{\mathcal{FT}}{\Longleftrightarrow} \quad \frac{2\pi}{T_{\mathrm{s}}} \sum_{k=-\infty}^{\infty} \delta\left(\omega - k\frac{2\pi}{T_{\mathrm{s}}}\right)$$

– Therefore,

$$x(t)\,p(t) \quad \overset{\mathcal{FT}}{\Longleftrightarrow} \quad \frac{1}{2\pi} X(j\omega) * \frac{2\pi}{T_{\mathrm{s}}} \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

– with $\omega_s \quad \triangleq \quad \dfrac{2\pi}{T_{\mathrm{s}}}$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

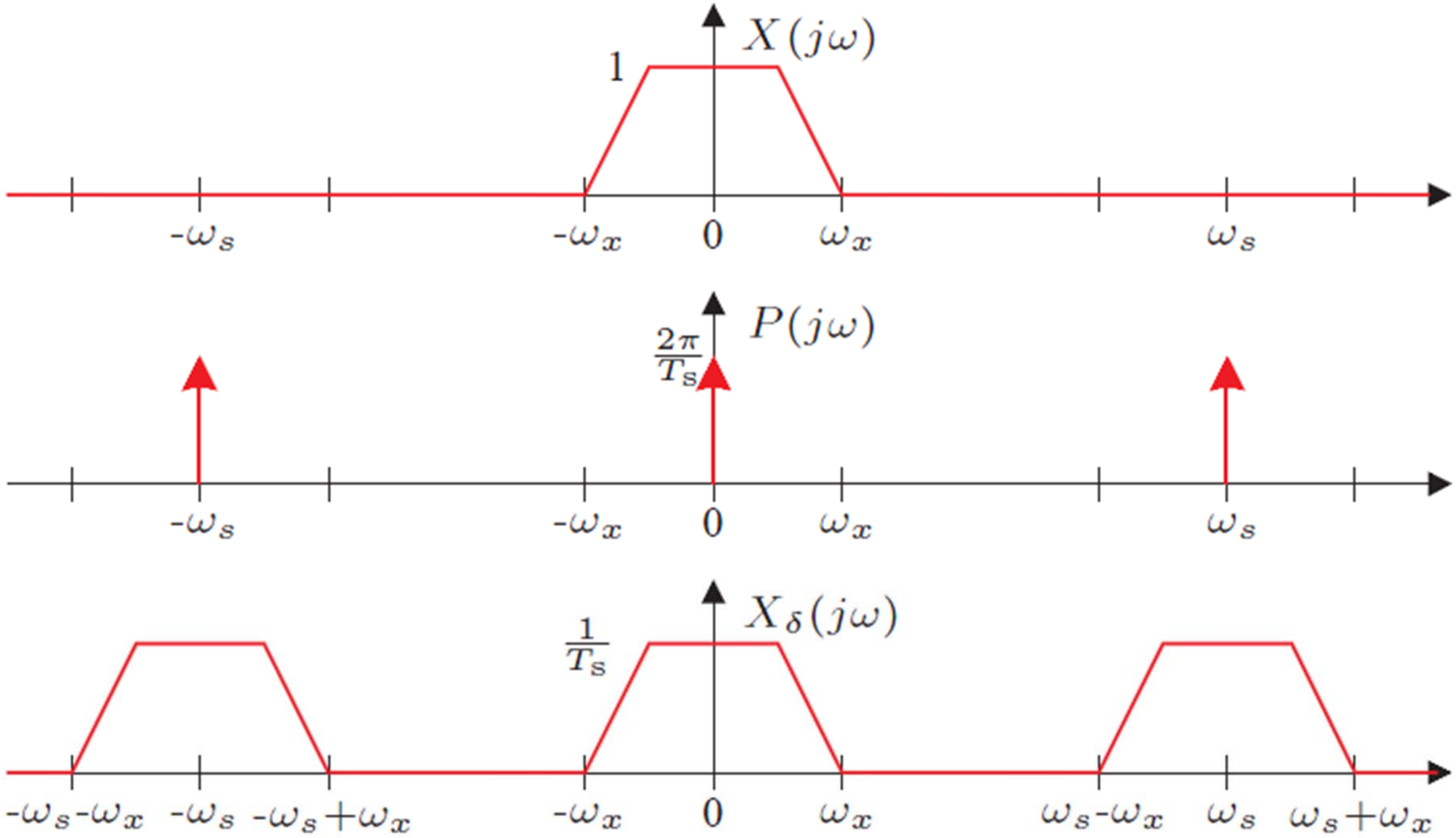
Sampling theorem

– Finally:

$$x(t)\,p(t) \quad \overset{\mathcal{FT}}{\Longleftrightarrow} \quad \frac{1}{T_{\mathrm{s}}} \sum_{k=-\infty}^{\infty} X\left(j(\omega - k\omega_s)\right)$$

- By sampling, we’ll have copies of the spectrum at harmonics of the sampling frequency

Sampling theorem



Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology
http://web.cecs.pdx.edu/~ece2xx/ECE223/Slides/Sampling.pdf

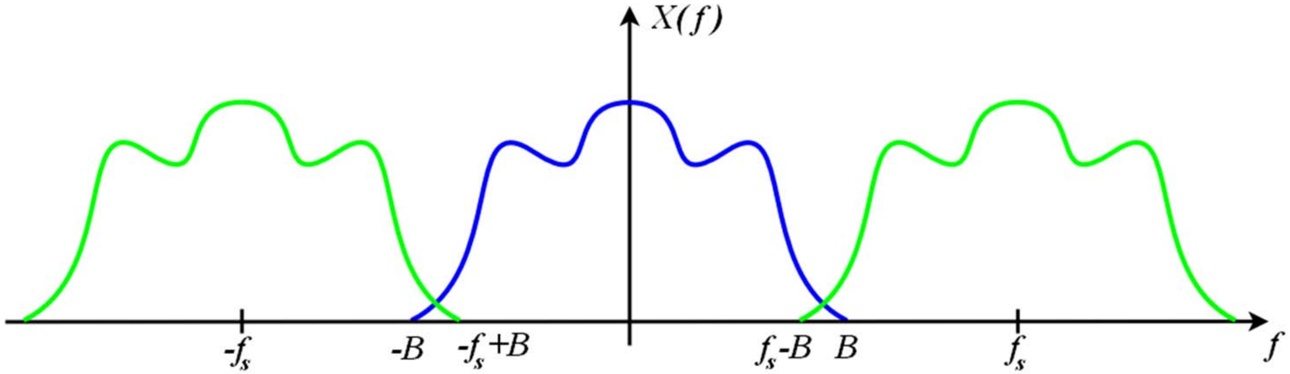
Sampling theorem

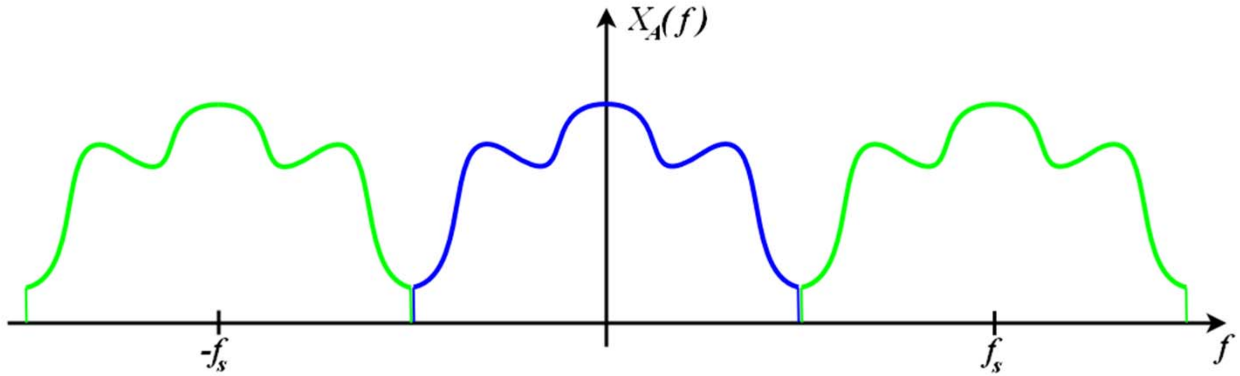
- Observations from the diagram
  - Choosing a large sampling frequency
    - Gaps between the copies of the spectrum
  - Choosing a small sampling frequency
    - Overlapping of the copies of the spectrum
    - Spectra overlapping: **aliasing**
    - **Important concept**

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- Aliasing





Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

http://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem

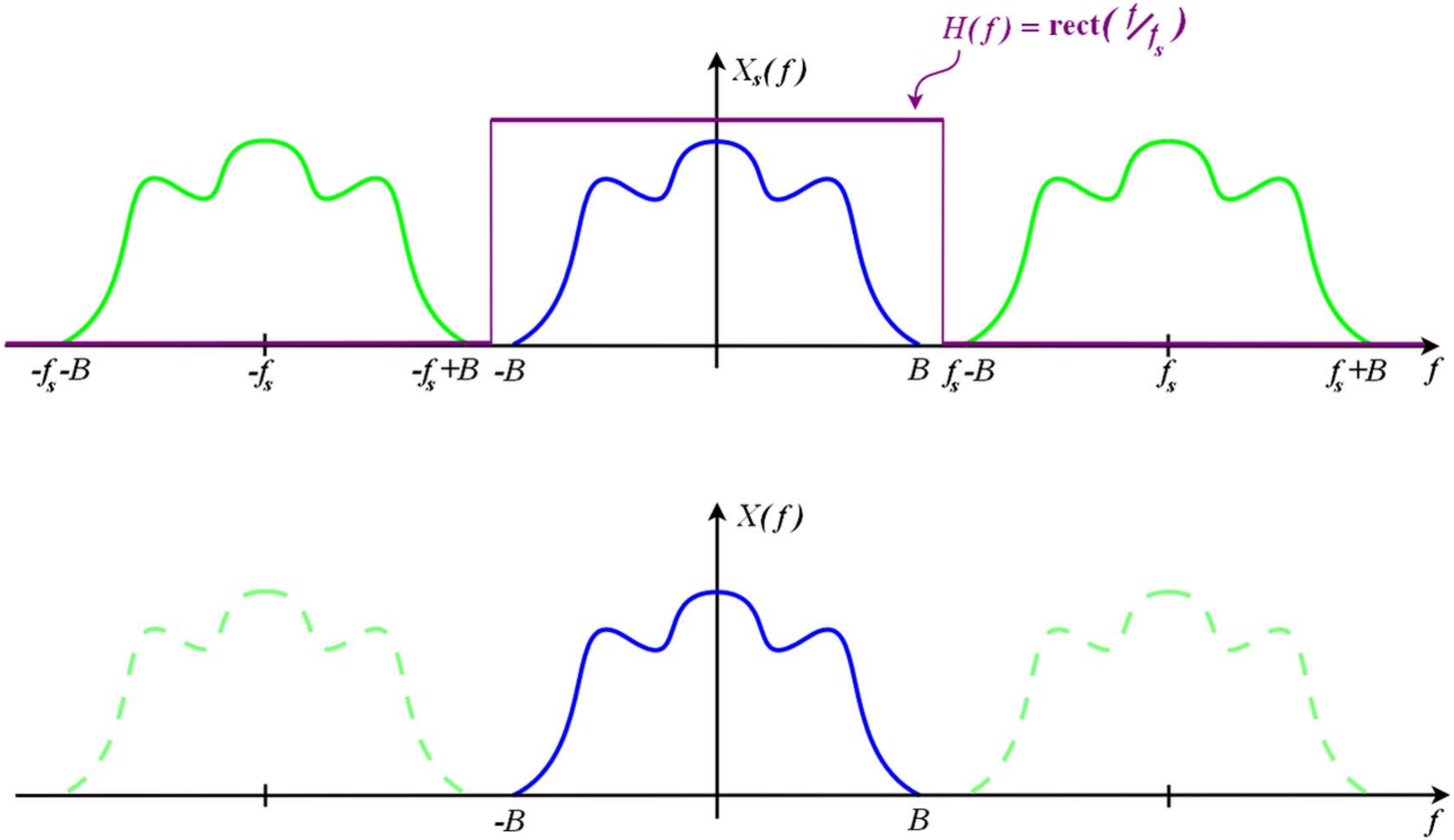
Sampling theorem

- Reconstruction (sampling)
  - Clearly assume no aliasing via proper selection of the sampling frequency: above Nyquest rate
  - We employ an ideal LPF to select the spectrum of the CT signal
- Note: quantization is not reversible!!
  - Quantization noise

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

Sampling theorem

- Reconstruction



Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

http://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem