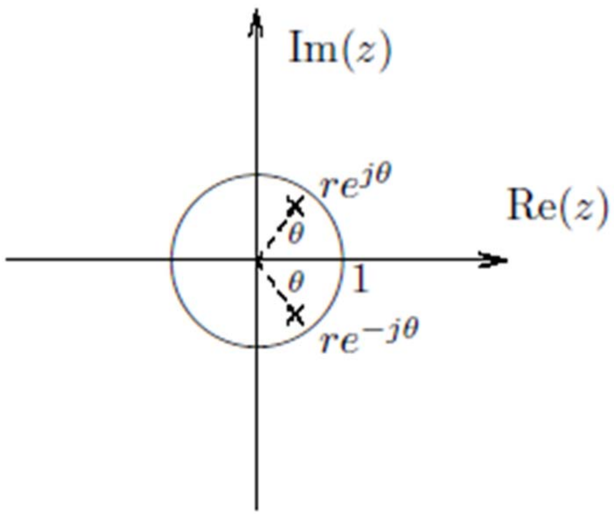
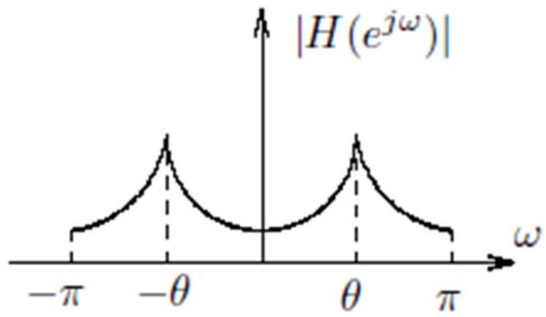
ZT & LTI systems

- Illustration



(c) Poles near the unit circle.



(d) A bandpass filter corresponding to (c).

https://engineering.purdue.edu/.../notes/Section1.6.pdf

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Example: phase distortion and delay

$$h_{\text{id}}[n] = \delta[n - n_d].$$

  - Which leads to

$$H_{\text{id}}(e^{j\omega}) = e^{-j\omega n_d},$$

$$|H_{\text{id}}(e^{j\omega})| = 1,$$
$$\sphericalangle H_{\text{id}}(e^{j\omega}) = -\omega n_d, \qquad |\omega| < \pi,$$

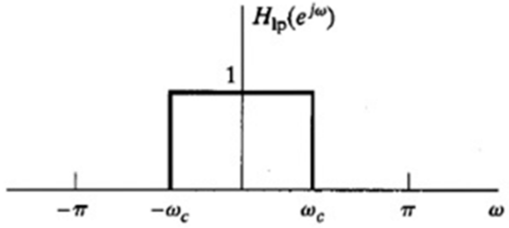
  - Linear phase: delay

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

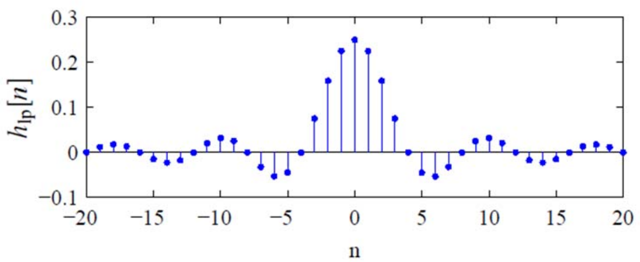
ZT & LTI systems

- Example: ideal LPF (with linear phase)

$$H_{\mathrm{lp}}(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi, \end{cases}$$



$$H_{\mathrm{lp}}(e^{j\omega}) = \begin{cases} e^{-j\omega n_d}, & |\omega| < \omega_c, \\ 0, & \omega_c < |\omega| \le \pi. \end{cases}$$



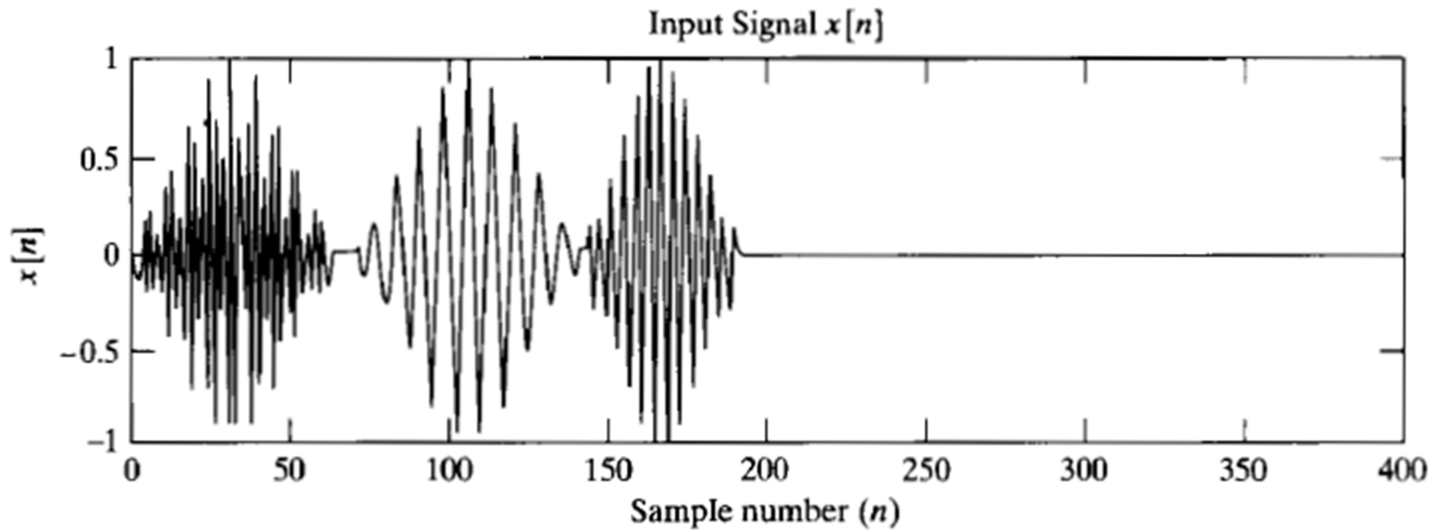
$$\frac{\sin \omega_c n}{\pi n}, \qquad -\infty < n < \infty.$$

$$h_{\mathrm{lp}}[n] = \frac{\sin \omega_c (n - n_d)}{\pi (n - n_d)}, \qquad -\infty < n < \infty.$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

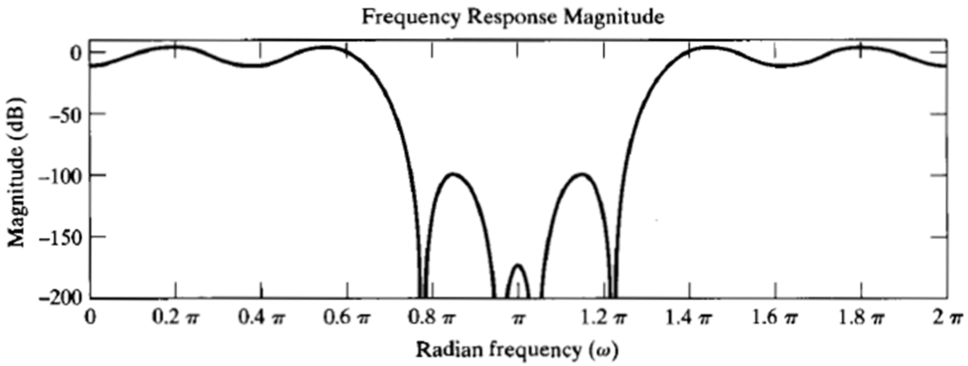
- Illustration
- Filtering: selection of frequency components

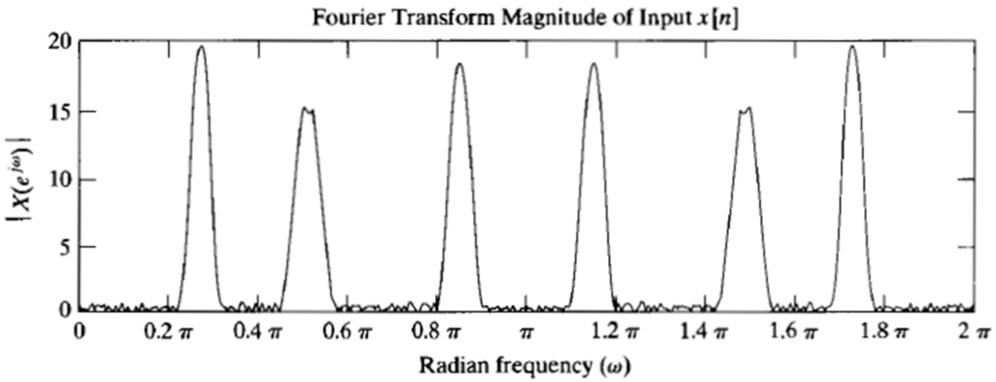


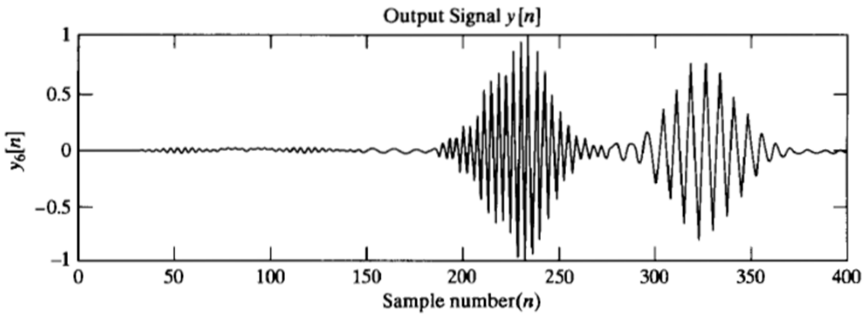
Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Illustration (cont.)







Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- All-pass systems: constant magnitude for all frequencies
  - Delay system
- An example of first order system

$$H_{\mathrm{ap}}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$

$$H_{\mathrm{ap}}(e^{j\omega}) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}} \quad = e^{-j\omega}\frac{1 - a^* e^{j\omega}}{1 - ae^{-j\omega}}.$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Note: numerator and denominator of the second factor are complex conjugate

$$e^{-j\omega}\frac{1-a^*e^{j\omega}}{1-ae^{-j\omega}}.$$

  – therefore,

$$|H_{ap}(e^{j\omega})| = 1$$

  – Applications: e.g., phase compensation purposes

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Inverse systems
  - Remember (LTI case)
  - convolution

$$x[n] \rightarrow \boxed{h_1[n]} \rightarrow \boxed{h_2[n]} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{h_1[n] * h_2[n]} \rightarrow y[n]$$

$$x_1[n] * x_2[n] \stackrel{z}{\longleftrightarrow} X_1(z)X_2(z),$$

$$x[n] \rightarrow \boxed{h_2[n] * h_1[n]} \rightarrow y[n]$$

$$x[n] \rightarrow \boxed{h_2[n]} \rightarrow \boxed{h_1[n]} \rightarrow y[n]$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Definition (inverse system):

The system $H_i(z)$ is the inverse system to $H(z)$ if

$$G(z) = H(z)H_i(z) = 1$$

- or

$$g[n] = h[n] * h_i[n] = \delta[n].$$

- and hence,

$$H(z) = \frac{1}{H_i(z)}$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- ROC?

The question of which ROC to associate with $H_i(z)$ is answered by the convolution theorem — for the previous equation to hold the regions of convergence of $H(z)$ and $H_i(z)$ must overlap.

- Example: an LTI system

$$H(z) = \frac{1 - 0.5z^{-1}}{1 - 0.9z^{-1}} \quad \text{ROC } |z| > 0.9.$$

  – Therefore,

$$H_i(z) = \frac{1 - 0.9z^{-1}}{1 - 0.5z^{-1}}$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

– We have

$$H_i(z) = \frac{1 - 0.9z^{-1}}{1 - 0.5z^{-1}}$$

– ROC: two possible choices

– To be overlapped

$$|z| > 0.5$$

– Finally:

$$h_i[n] = (0.5)^n u[n] - 0.9(0.5)^{n-1} u[n-1]$$

– Causal and stable

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Observation
  - the poles of H(z) are zeros of the inverse system
  - the zeros of H(z) are poles of the inverse system
- Remember
  - An LTI system is causal and stable iff all the poles are located within the unit circle
- *When both the H(z) and the inverse systems are causal and stable?*

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- The previous question leads us to the following definition:
  - Minimum phase systems
  - All the poles and zeros are located within the unit circle
    - The H(z) is causal and stable
    - The inverse system is causal and stable

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Frequency response

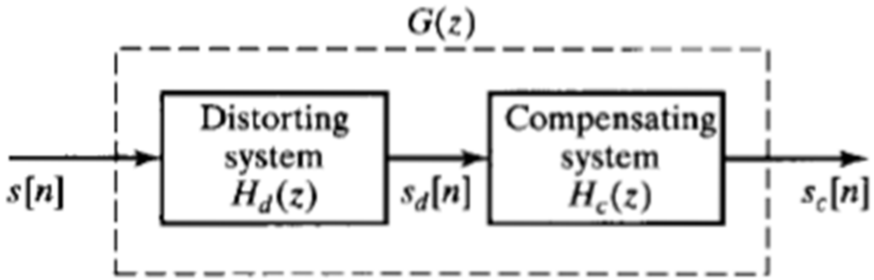
$$H(e^{j\omega}) = \frac{1}{H_i(e^{j\omega})}$$

- Note: some LTI systems have no inverse!!
  - Example: ideal LPF
  - Some frequencies are set to zero and can not be recovered

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Application: a wireless communication channel



$$H_c(z) = \frac{1}{H_{d\,\min}(z)}$$

- Then,

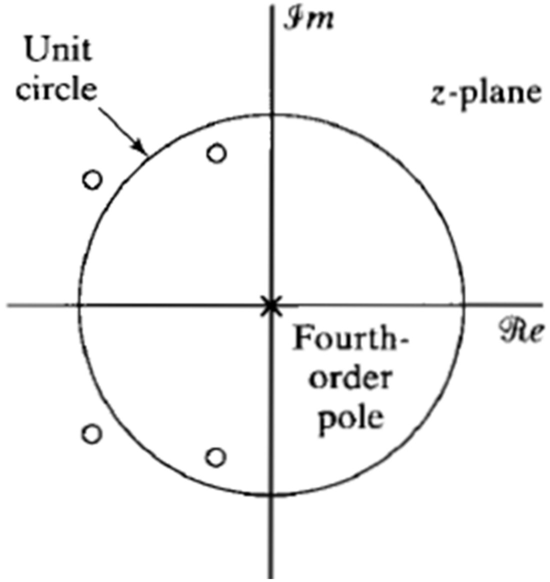
$$G(z) = H_d(z)H_c(z) = H_{\text{ap}}(z)$$

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

- Illustration: compensation for FIR systems

$$H_d(z) = (1 - 0.9e^{j0.6\pi} z^{-1})(1 - 0.9e^{-j0.6\pi} z^{-1}) \times (1 - 1.25e^{j0.8\pi} z^{-1})(1 - 1.25e^{-j0.8\pi} z^{-1}).$$

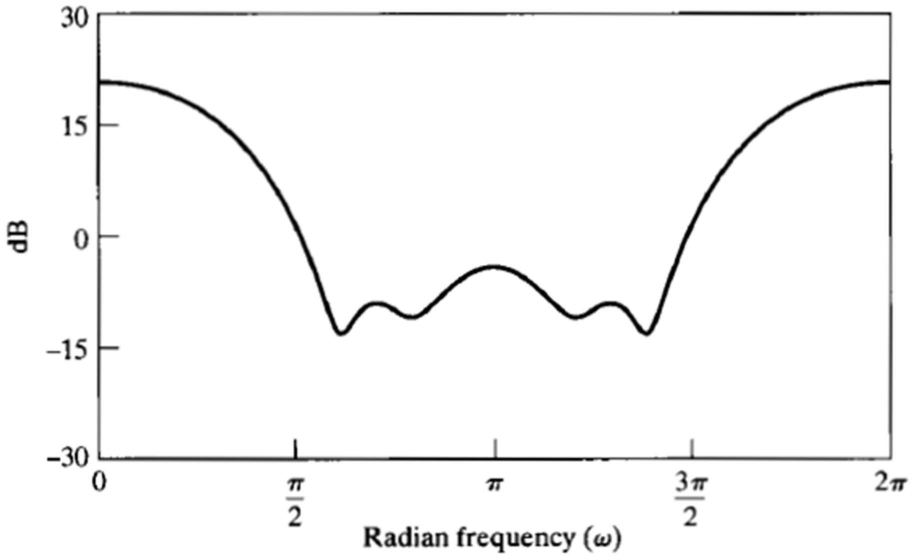


  - Causal
  - non-minimum phase

Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology

ZT & LTI systems

– Frequency response



Mohammad Mahdi Naghsh
ECE. Dept., Isfahan University of Technology