Digital Signal Processing (DSP)

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DIGITAL SIGNAL PROCESSING (DSP)

Lecture 6

A review on continuous-time Fourier transform

Most of the materials from www.site.uottawa.ca/~jpyao/courses/...files/ch 4.pdf

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CT Fourier transform

- Similar to the discrete-time case
- The result is not periodic generally
- We consider the CT signal x(t)

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

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CT Fourier transform

- Remember
  - DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] \rightarrow \boxed{\text{LTI } h[n]} \rightarrow y[n] = x[n] * h[n]$$

  - Eigen-function

$$x[n] = e^{j\omega n}$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)}$$

$$= e^{j\omega n}\left(\sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}\right)$$

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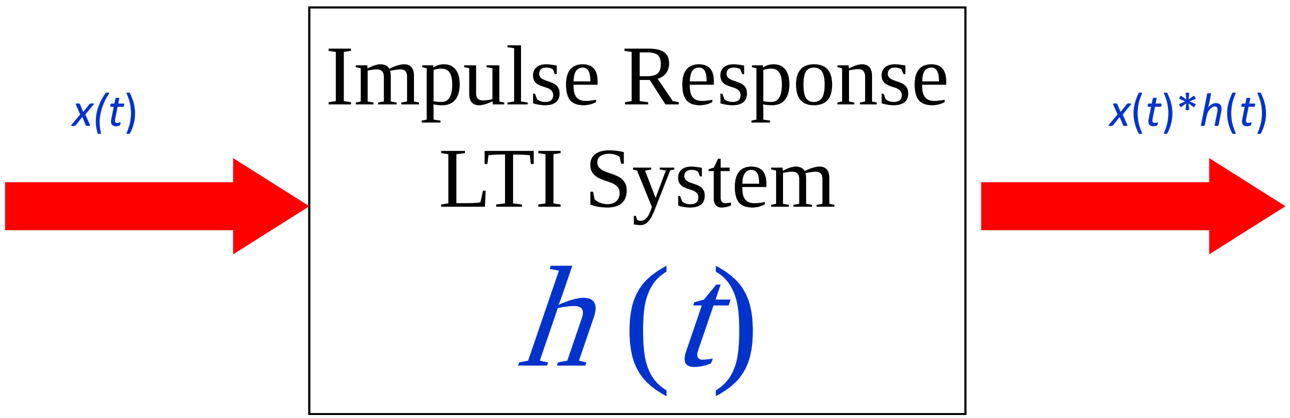
CT Fourier transform

- Here: CTFT

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$



$$f_1(t) * f_2(t) \overset{\mathcal{F}}{\longleftrightarrow} F_1(j\omega)F_2(j\omega)$$

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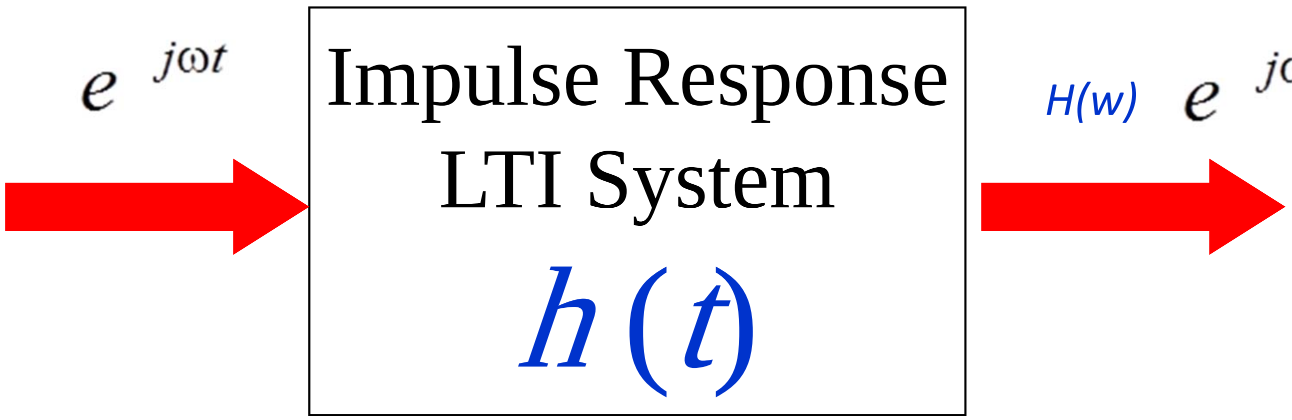
CT Fourier transform

- Eigen-functions $e^{j\omega t}$



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CT Fourier transform

- Convergence of the integral
  - Similar to DTFT (sufficient conditions)
  - The integral converges for energy signals

$$\int_{-\infty}^{\infty} [f(t)]^2 dt < \infty$$

  - Dirichlet conditions

$$\int_{-\infty}^{\infty} \left| f(t) \right| dt < \infty$$

    - And some other mild conditions

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CT Fourier transform

- Examples: $x(t) = e^{-at}u(t)\,,\ a > 0$
  - For the FT:

$$X(j\omega) = \int_0^{\infty} e^{-at} e^{-j\omega t} dt = -\frac{1}{a + j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty} = \frac{1}{a + j\omega}, \qquad a > 0$$

  - As expected, it is complex!

$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}, \qquad \angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

www.site.uottawa.ca/~jpyao/courses/...files/ch 4.pdf

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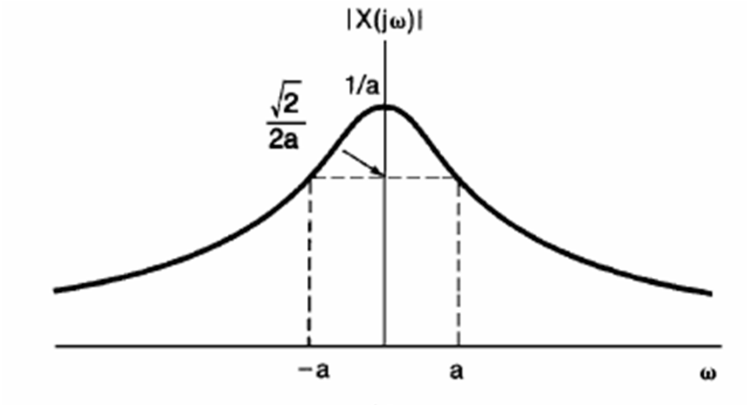
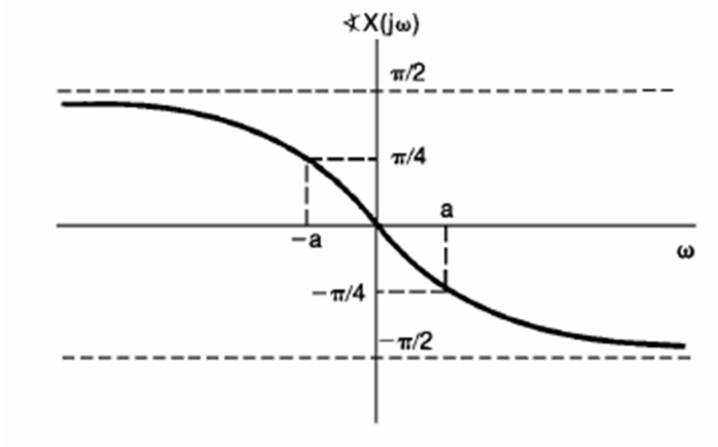
CT Fourier transform

- Magnitude/phase of FT



(a)



(b)

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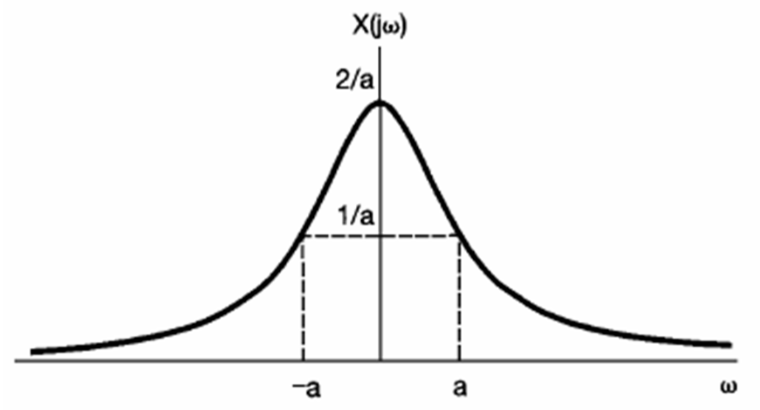
CT Fourier transform

- Example: $x(t) = e^{-a|t|}, \quad a > 0$
  - FT:

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \frac{1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{2a}{a^2 + \omega^2}$$



  - Real even; real even

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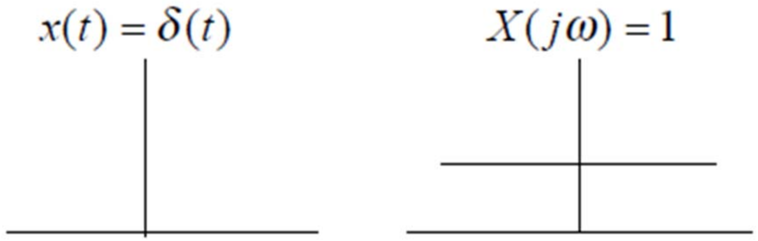
CT Fourier transform

- Example: $x(t) = \delta(t)$
  - FT:

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1$$



  - Compacted in time; expanded in frequency

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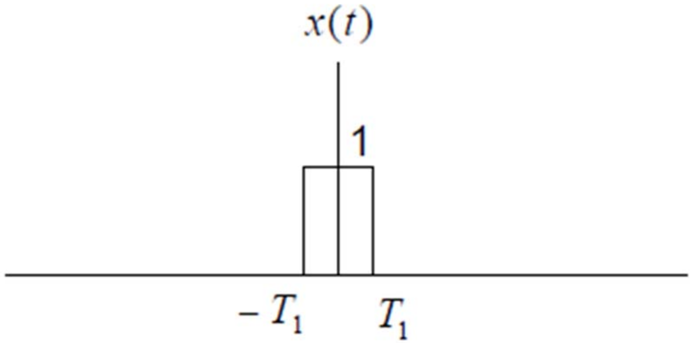
CT Fourier transform

- An important example (pulse-sinc relationship)

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & |t| > T_1 \end{cases}.$$



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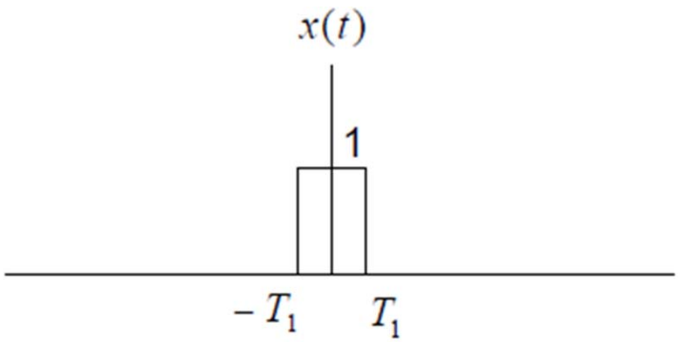
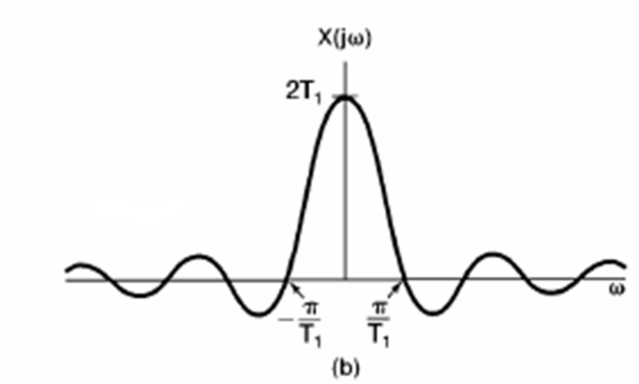
CT Fourier transform

- We obtain:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-T_1}^{T_1} 1e^{-j\omega t}dt = 2\frac{\sin \omega T_1}{\omega}$$





- Observation with varying T_1

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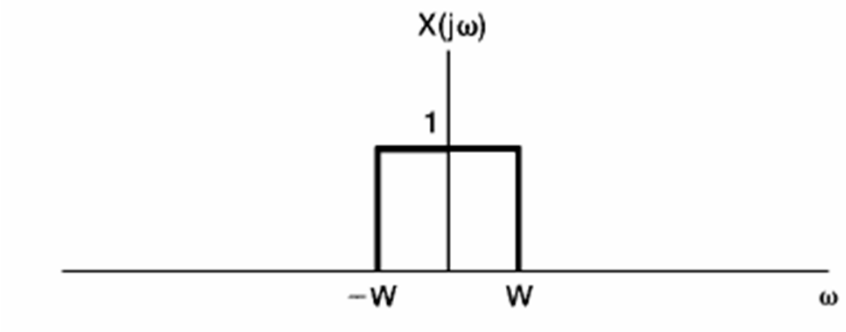
CT Fourier transform

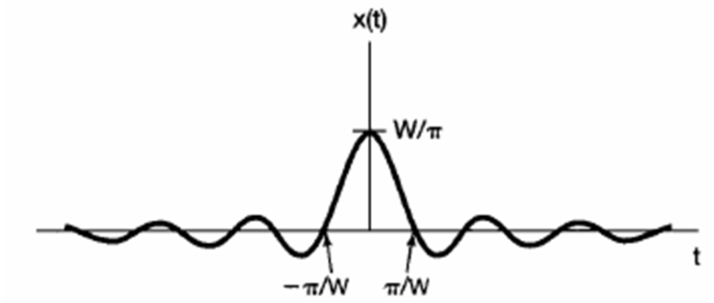
- Example: an ideal LPF

$$X(j\omega) = \begin{cases} 1, & |\omega| < W \\ 0, & |\omega| > W \end{cases}$$



- The inverse FT

$$x(t) = \frac{1}{2\pi}\int_{-W}^{W} e^{j\omega t}\,d\omega = \frac{\sin Wt}{\pi t}$$



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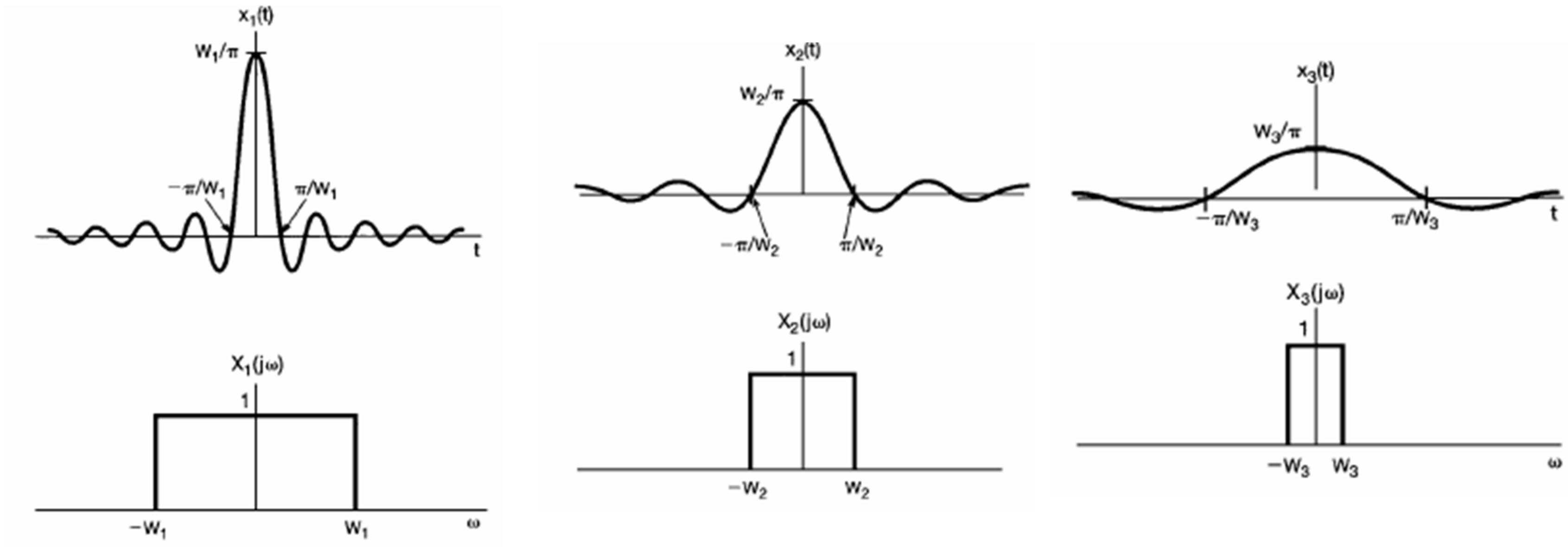
CT Fourier transform

- Observations
  - 1)
  - 2)

$$\text{Square wave} \underset{FT^{-1}}{\overset{FT}{\rightleftarrows}} \text{Sinc function}$$



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CT Fourier transform

- Properties
  - 1) linearity

If $x(t) \stackrel{F}{\longleftrightarrow} X(j\omega)$ and $y(t) \stackrel{F}{\longleftrightarrow} Y(j\omega)$

Then

$$ax(t) + by(t) \stackrel{F}{\longleftrightarrow} aX(j\omega) + bY(j\omega)$$

  - 2) time-shift

$$x(t - t_0) \stackrel{F}{\longleftrightarrow} e^{-j\omega t_0} X(j\omega)$$

    - Phase change

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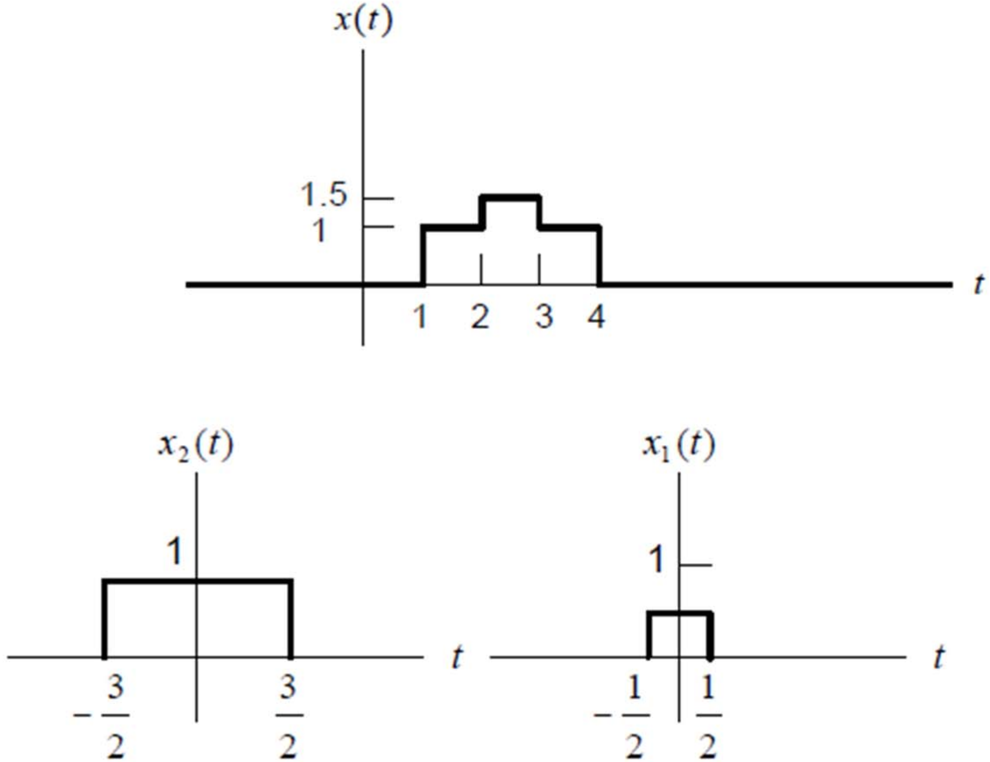
CT Fourier transform

- Example:



$$x(t) = \frac{1}{2} x_1(t - 2.5) + x_2(t - 2.5)$$

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CT Fourier transform

– It is checked that

$$X_1(j\omega)=\frac{2\sin(\omega/2)}{\omega} \text{ and } X_2(j\omega)=\frac{2\sin(3\omega/2)}{\omega}$$

– and hence:

$$X(j\omega)=e^{-j5\omega/2}\left\{\frac{\sin(\omega/2)+2\sin(3\omega/2)}{\omega}\right\}$$

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CT Fourier transform

– 3) Conjugate symmetry

$$x^*(t) \stackrel{F}{\longleftrightarrow} X^*(-j\omega)$$

– For real signals

$$X(-j\omega) = X^*(j\omega)$$

– Develop transform pairs for even-odd parts

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CT Fourier transform

– 4) differentiation

$$\frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(j\omega)$$

– 5) time-scaling

$$x(at) \xleftrightarrow{F} \frac{1}{|a|} X(\frac{j\omega}{a})$$

– useful

$$x(-t) \xleftrightarrow{F} X(-j\omega)$$

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