دانشگاه صنعتی اصفهان

دانشکده مکانیک

Crack Tip Plasticity

Crack Tip Plasticity

Linear elastic stress analysis of sharp cracks predicts infinite stresses at the crack tip. In real materials, however, stresses at the crack tip are finite because the crack tip radius must be finite. Inelastic material deformation, such as plasticity in metals leads to further relaxation of crack tip stresses.

بررسی محدوده پلاستیک جلوی ترک

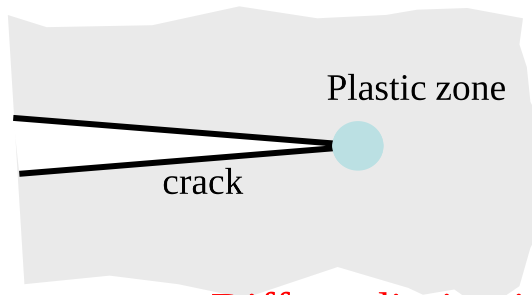
Linear elastic fracture mechanics (LEFM)

Applies when non-linear deformation is confined to a *small* region surrounding the crack tip

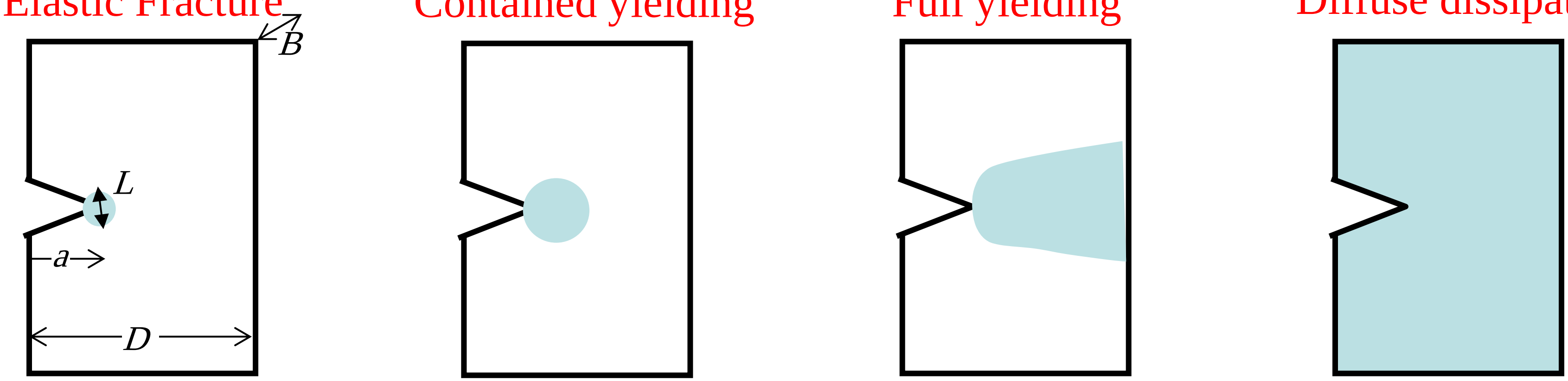
Effects of the plastic zone negligible, linear asymptotic mechanical field.

Elastic-Plastic fracture mechanics (EPFM) :

Generalization to materials with a non-negligible plastic zone size: elastic-plastic materials



Elastic Fracture	Contained yielding	Full yielding	Diffuse dissipation

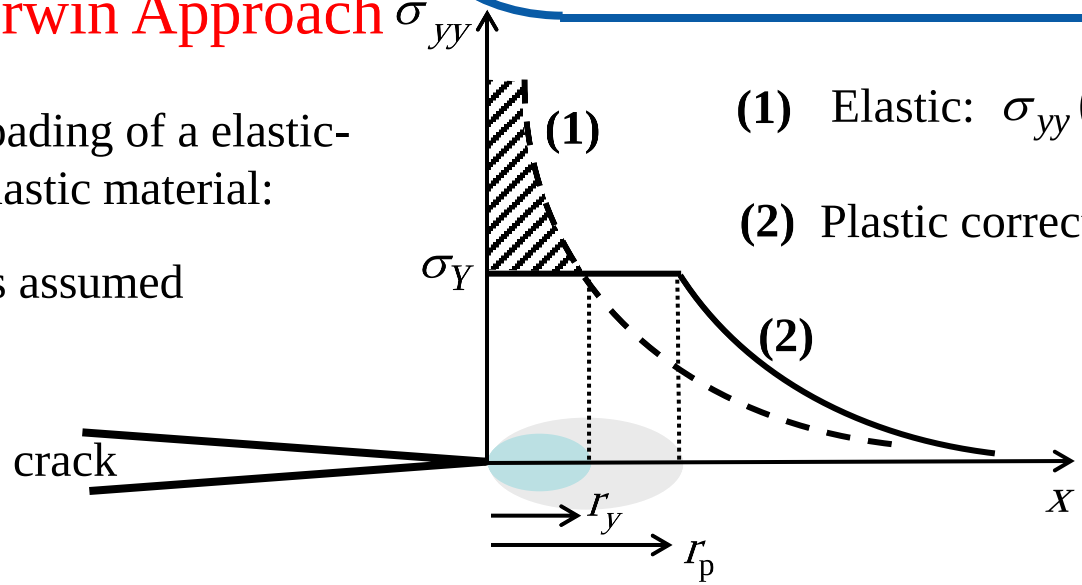


$L << a, D, B$	$L < D - a$	$L \approx D - a$	
LEFM, K_{IC} or G_{IC} fracture criterion	EPFM, J_C fracture criterion	Catastrophic failure, large deformations	

➢ The Irwin Approach

▪ Mode I loading of a elastic-perfectly plastic material:

Plane stress assumed



(1) Elastic: $\sigma_{yy}(\theta=0)=\dfrac{K_I}{\sqrt{2\pi r}}$

(2) Plastic correction $\sigma_{yy}=\sigma_Y\ , \quad r<r_p$

r_p ?

r_y : Intersection between the elastic distribution σ_{yy} and the horizontal line σ_{YS}

$$\frac{K_I}{\sqrt{2\pi r_y}}=\sigma_{YS} \quad \Rightarrow \quad r_y=\frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

To equilibrate the two stresses distributions (cross-hatched region)

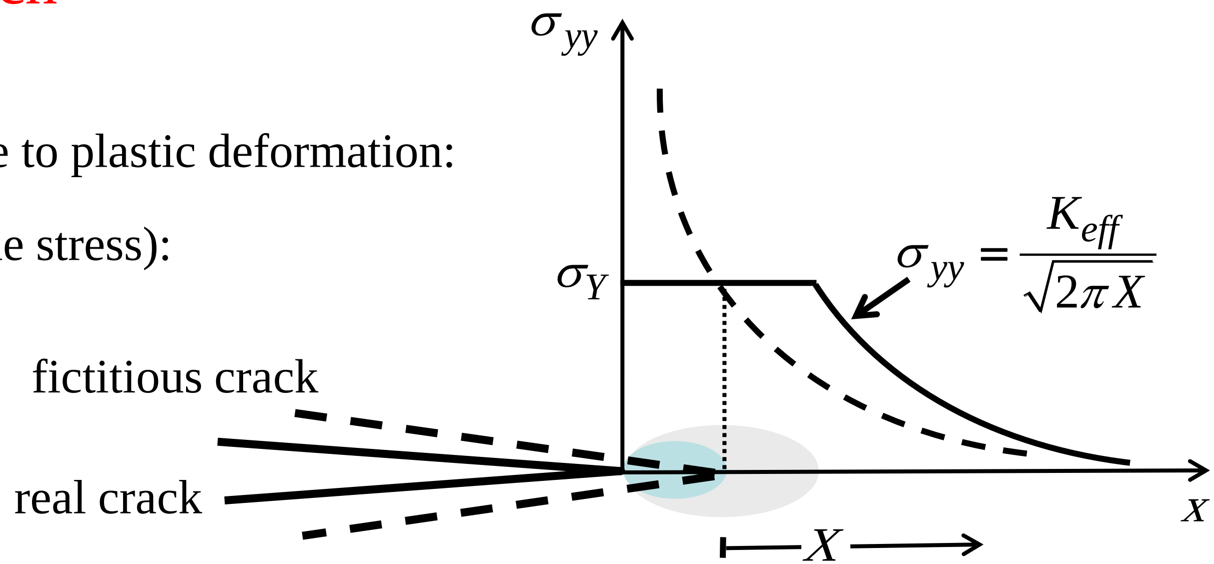
$$\sigma_{YS}\, r_p=\int_0^{r_y}\sigma_{yy}\,dr=\int_0^{r_y}\frac{K_I}{\sqrt{2\pi r}}\,dr \qquad r_p=\frac{1}{\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

➢ The Irwin Approach

➢Redistribution of stress due to plastic deformation:

Plastic zone length (plane stress):

$$r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$



➢Irwin's model = simplified model for the extent of the plastic zone:

- Focus only on the *extent* of the plastic zone *along the crack axis*, not on *its shape.*
- Equilibrium condition along the *y*-axis not respected.

➢In plane strain, increasing of $\sigma_{YS.}$: Irwin suggested $\sqrt{3}\,\sigma_{YS}$ in place of σ_{YS}

$$r_y = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

➢ The Irwin Approach

The effective crack: $a_{eff} = a + r_y$

For plane stress: $r_y = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$ *For plane strain:* $r_y = \frac{1}{6\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$

The effective stress intensity is obtained by inserting a_{eff} into the K expression for the geometry of interest:

$$K_{eff} = Y(a_{eff})\sigma\sqrt{\pi a_{eff}}$$

Since the effective crack size is taken into account in the geometry correction factor, Y, an iterative solution is usually required to solve for K_{eff}.

➢ The Irwin Approach

Algorithm:

Initial:

$$K_I = Y(a)\sigma\sqrt{\pi a}$$

$$a_0 = a + \frac{1}{2\pi m}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$

$$m = \begin{cases} 1 & \text{plane stress} \\ 3 & \text{plane strain} \end{cases}$$

↓

Do $i = 1, imax$ **:** ← $i = i + 1$

↓

$$K_I^{(i)} = Y(a_{i-1})\sigma\sqrt{\pi a_{i-1}}$$

$$a_i = a + \frac{1}{2\pi m}\left(\frac{K_I^{(i)}}{\sigma_{YS}}\right)^2$$

↓

$$\left|K_I^{(i)} - K_I^{(i-1)}\right| \le \varepsilon \ ?$$

→ No: $i = i + 1$ (back to Do loop)

→ Yes → $K_{eff} = K_I^{(i)} = Y(a_i)\sigma\sqrt{\pi a_i}$

Convergence after a few iterations…

Application: Through-crack in an infinite plate (plane stress):

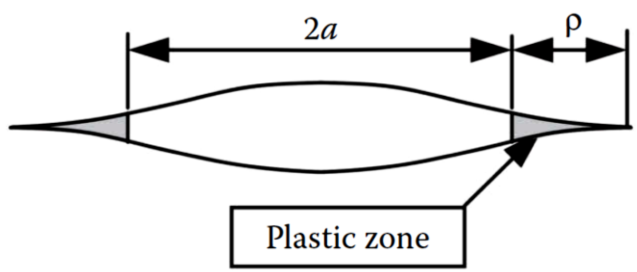
$\sigma = 2$ MPa, $\sigma_{YS} = 50$ MPa, $a = 0.1$ m ➡ $K_I = 1.1209982$

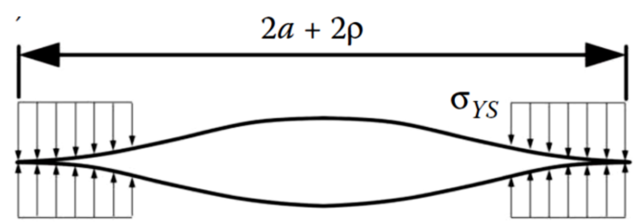
$K_{eff} = 1.1214469$ 4 iterations

➢ The Strip Yield Model

The strip yield model was first proposed by *Dugdale* and Barenblatt.

This model approximates the elastic–plastic behavior by superimposing two elastic solutions: a through crack under remote tension and a through crack with closure stresses at the tip. Thus, the strip yield model is a classical application of the principle of superposition.





➢ The Strip Yield Model

Load σ: $$K_I(\sigma) = \sigma\sqrt{\pi(a + r_p)}$$

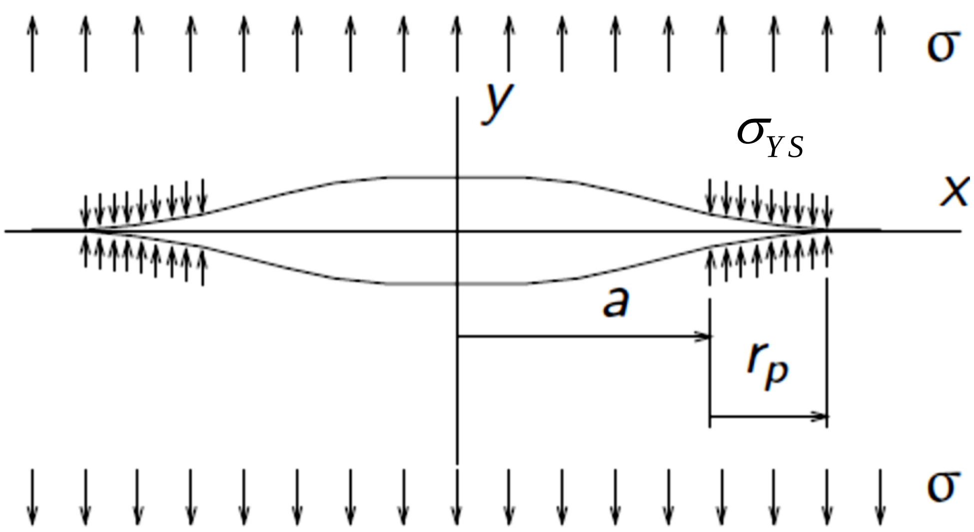
Load σ_{YS}: $$K_I(\sigma_{YS}) = K_{\text{closure}} = -2\sigma_{YS}\sqrt{\frac{a + r_p}{\pi}}\arccos\left(\frac{a}{a + r_p}\right)$$

singular term = 0

$$K_I(\sigma) = K_I(\sigma_{YS})$$

$$\frac{a}{a + r_p} = \cos\left(\frac{\pi\sigma}{2\sigma_{YS}}\right)$$

$$r_p = \frac{\pi}{8}\left(\frac{K_I}{\sigma_{YS}}\right)^2$$



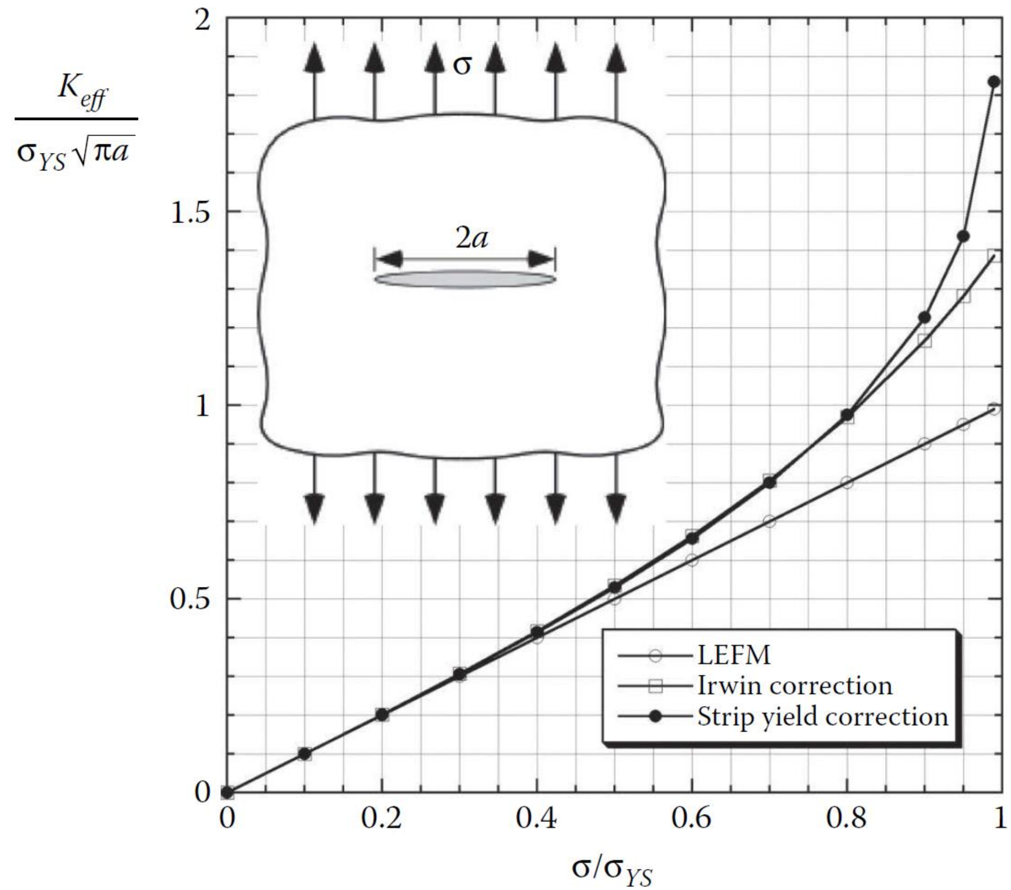
Since $1/\pi = 0.318$ and $\pi/8 = 0.392$, the Irwin and strip yield approaches predict similar plastic zone sizes.

The effective crack: $a_{eff} = a + r_p$

$$K_{eff} = \sigma\sqrt{\pi a \sec\left(\frac{\pi\sigma}{2\sigma_{YS}}\right)}$$

Comparison of Plastic Zone Corrections

Both the Irwin and strip yield corrections deviate from LEFM theory at stresses greater than $0.5\sigma_{YS}$. The two plasticity corrections agree with each other up to approximately $0.85\sigma_{YS}$. According to the strip yield model, K_{eff} is infinite at yield; the strip yield zone extends completely across the plate, which has reached its maximum load capacity.



$$LEFM: \quad K_I = \sigma\sqrt{\pi a}$$

Plastic Zone Shape

The von Mises equation: $\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2\right]^{1/2}$ (*)

The principal stresses: $\sigma_1,\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \left[\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2+\tau_{xy}{}^2\right]$ (**)

For plane stress $\sigma_3 = 0$ and $\sigma_3 = \nu(\sigma_1+\sigma_2)$ for plane strain.

Substituting the Mode I stress fields into Eq. (**):

$$\sigma_1 = \frac{K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2}\left[1+Sin\frac{\theta}{2}\right]$$

$$\sigma_2 = \frac{K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2}\left[1-Sin\frac{\theta}{2}\right] \qquad (***)$$

$$\sigma_3 = 0 \qquad \text{(plane stress)}$$

$$= \frac{2\nu K_I}{\sqrt{2\pi r}} Cos\frac{\theta}{2} \qquad \text{(plane strain)}$$

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Plastic Zone Shape

By substituting Eq. (***) into Eq. (*) , setting $\bar{\sigma}_e = \sigma_{YS}$, and solving for r,

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \left[1 + Cos\,\theta + \frac{3}{2}Sin^2\theta\right] \quad \text{for plane stress}$$

$$r_y(\theta) = \frac{1}{4\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2 \left[(1-2\nu)^2(1 + Cos\,\theta) + \frac{3}{2}Sin^2\theta\right] \quad \text{for plane strain}$$

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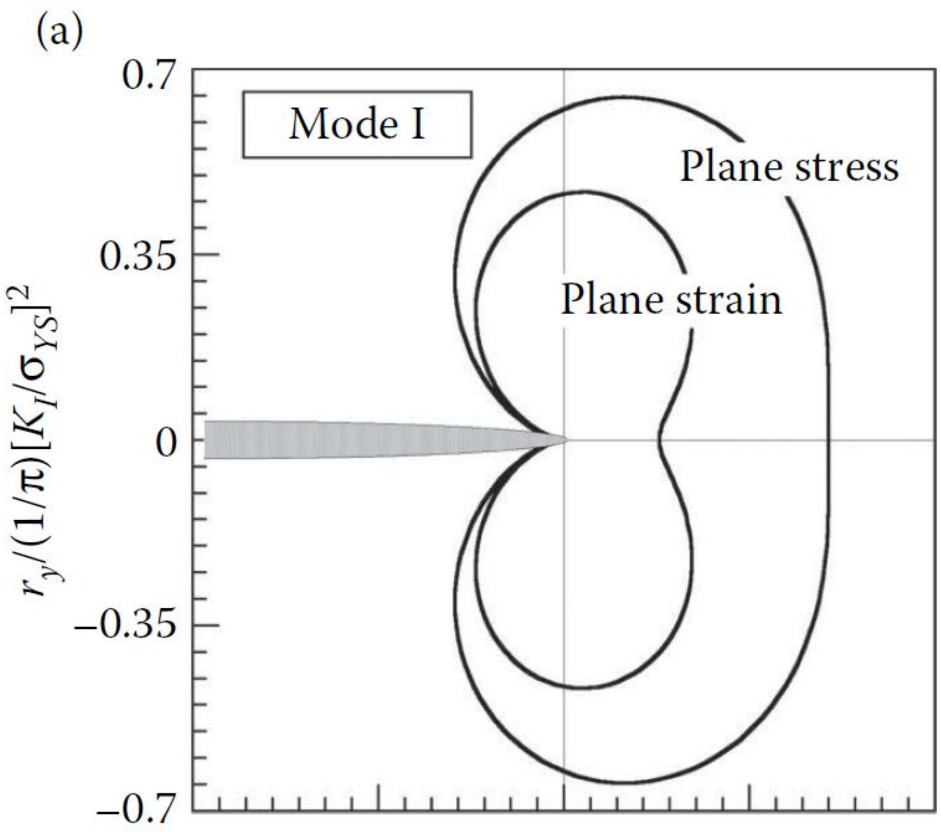
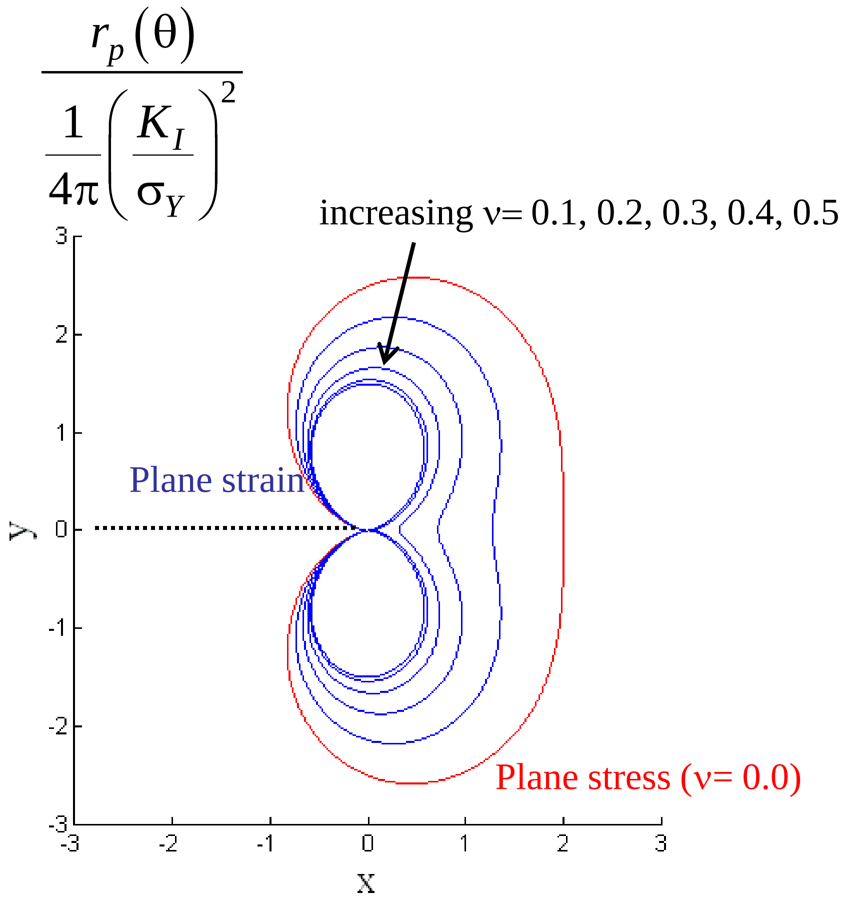
Plastic Zone Shape





Crack tip plastic zone shapes estimated from the elastic solutions and the von Mises yield criterion: (a) Mode I, (define the approximate boundary between elastic and plastic behavior).

Plane strain or plane stress?

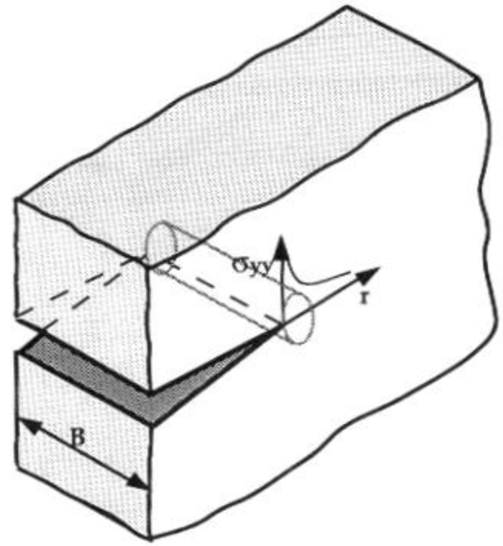
In general, the conditions ahead of a crack tip are neither plane stress nor plane strain. There are limiting cases where a two dimensional assumptions are valid, or at least provides a good approximation.

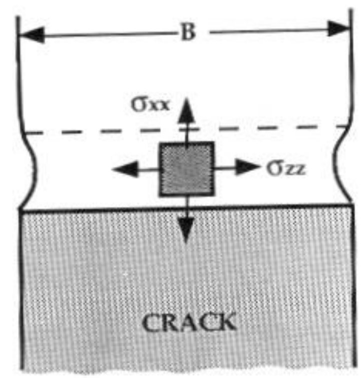
The nature of the plastic zone that is formed ahead of a crack tip plays a very important role in the determination of the type of failure that occurs. Since the plastic region is larger in PSS than in PSN, plane stress failure will, in general, be ductile, while, on the other hand, plane strain fracture will be brittle, even in a material that is generally ductile. This phenomenon explains the peculiar thickness effect, observed in all fracture experiments, that thin samples exhibit a higher value of fracture toughness than thicker samples made of the same material and operating at the same temperature. From this it can be surmised that the plane stress fracture toughness is related to both metallurgical parameters and specimen geometry while the plane strain fracture toughness depends more on metallurgical factors than on the others.

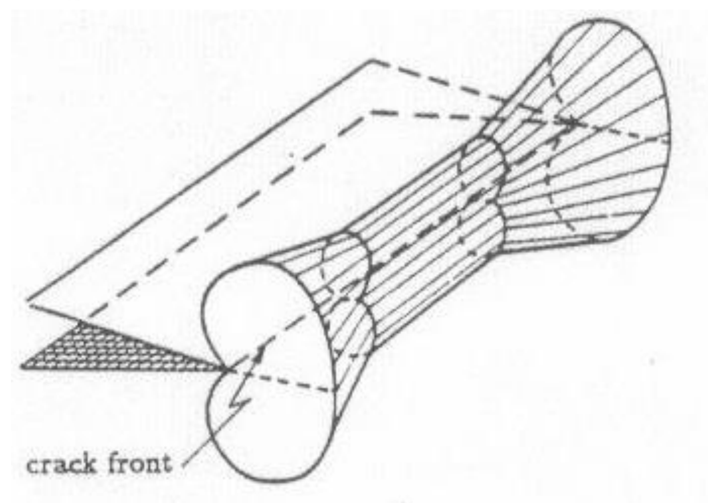
بروز تنش سه‌محوره

Plane strain or plane stress?

Due to presence of crack tip, stress in a direction to normal to crack plane σ_{yy} will be large near the crack tip. This stress would in turn tries to contract in x and z direction. But the material surrounding it will constraint it, inducing stresses in x and z direction, there by a triaxial state of stress exists near the crack tip. This leads to plane strain condition at interior. At the plate surface σ_{zz} is zero and ε_{zz} is maximum. This leads to plane stress condition at exterior.



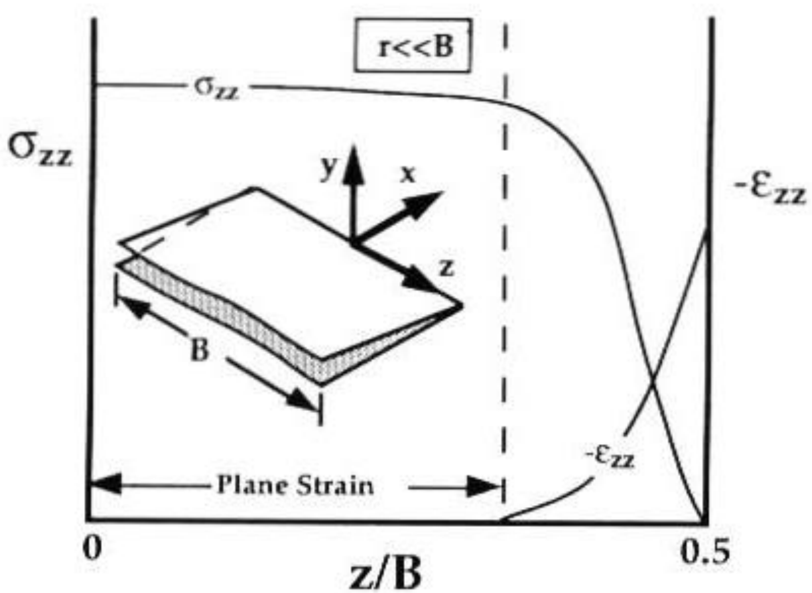


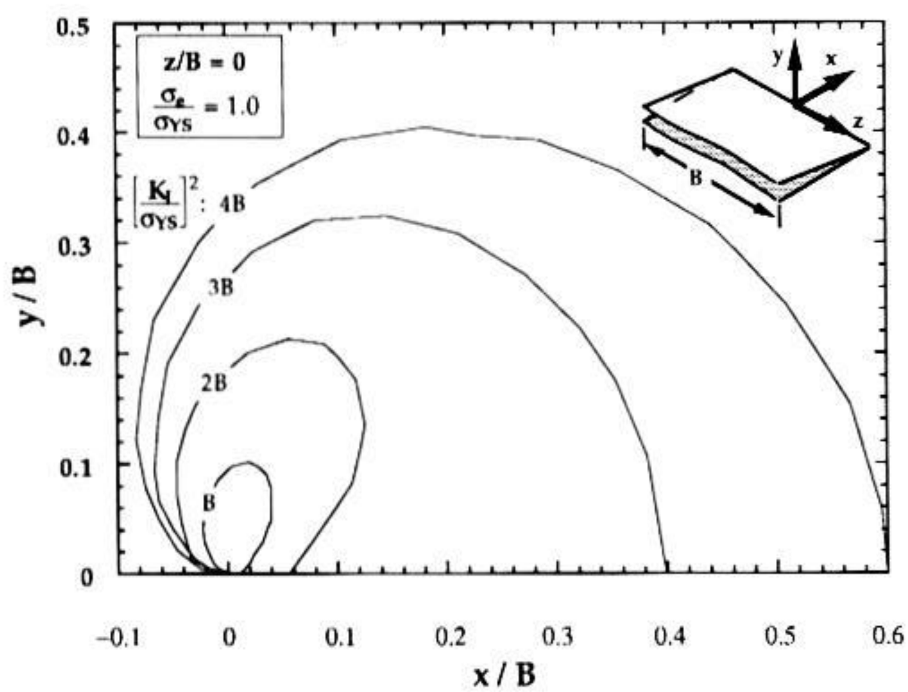


Plane strain or plane stress?

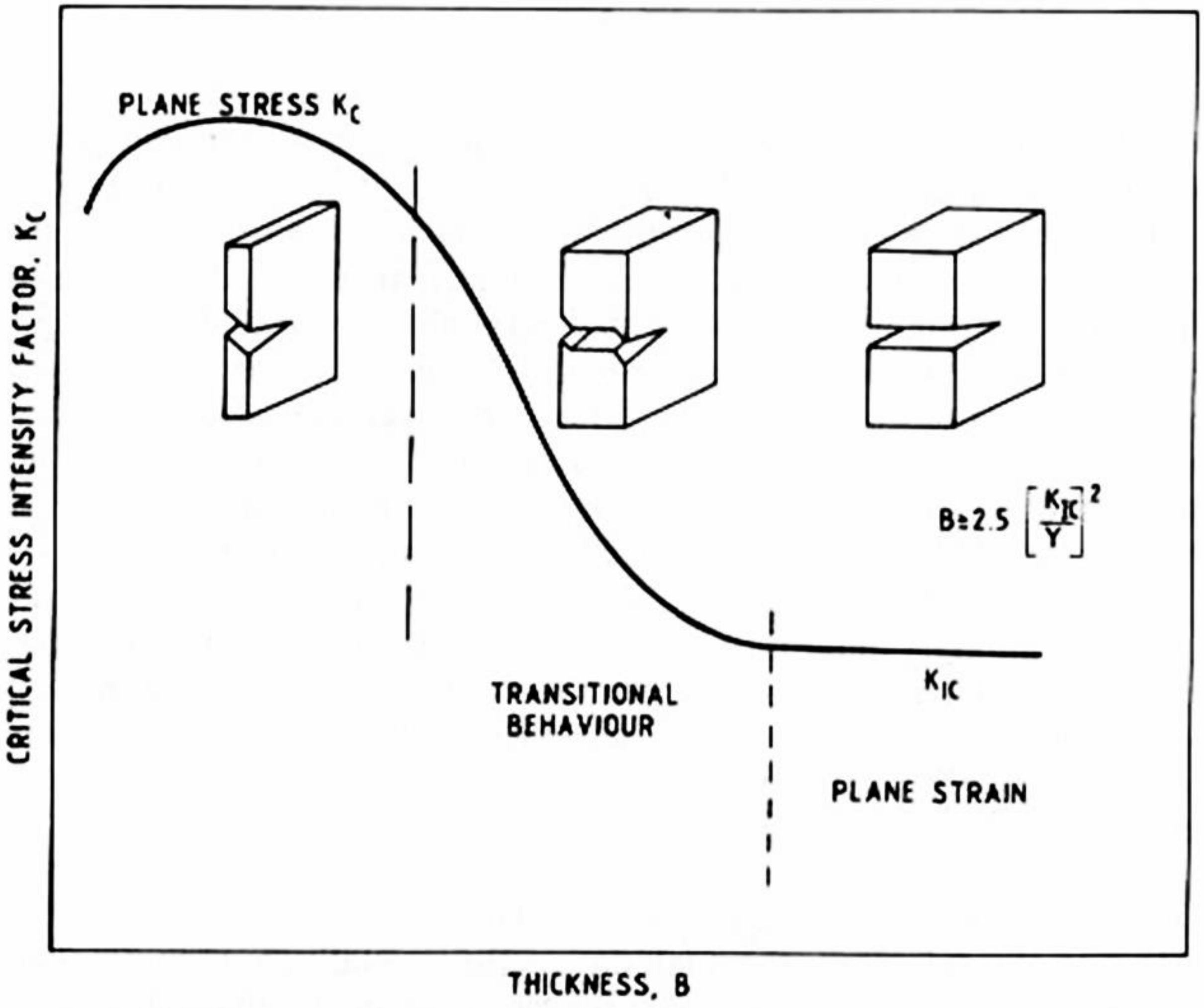
The state of stress is also dependent on size of plate thickness.

If the plastic zone size is small compared to the plate thickness, plane strain condition exists. If the plastic zone size is larger than the plate thickness, plane stress condition prevails. As the loading is increased, plastic zone size also increases leading to plane stress conditions.





Effect of plate thickness on fracture toughness



Limits of LEFM

As per ASTM standard LEFM is applicable for components of size

$$a \geq 2.5\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

$$t \text{ or B} \geq 2.5\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

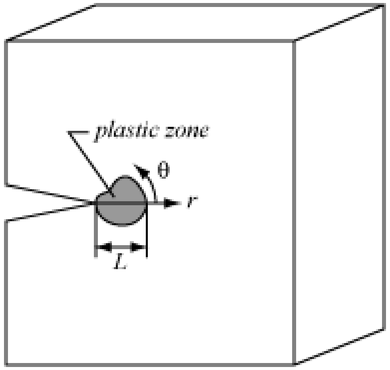
$$W \text{ or b } \geq 2.5\left(\frac{K_I}{\sigma_{ys}}\right)^2$$

As per ASTM standard fracture toughness testing can be done on Specimens of size

$$a, B, W \geq 2.5\left(\frac{K_{Ic}}{\sigma_{ys}}\right)^2$$

- **Remarks:**

1) 1D approximation [3] L corresponding to $r_p(\theta = 0)$



Thus, in plane strain: $L = \frac{(1-2\nu)^2}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$ in plane stress: $L = \frac{1}{2\pi}\left(\frac{K_I}{\sigma_{YS}}\right)^2$

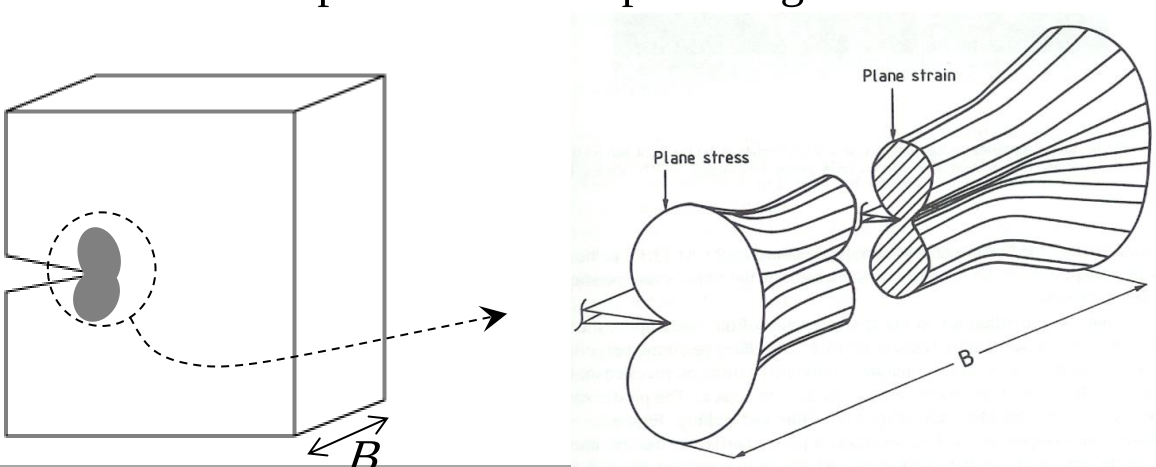
2) Significant difference in the size and shape of mode I plastic zones.

For a *cracked* specimen with *finite* thickness *B*, effects of the boundaries:

➡ Triaxial state of stress near the crack tip:

- Essentially plane strain in the in the central region.
- Pure plane stress state only at the free surface.

➡ Evolution of the plastic zone shape through the thickness:



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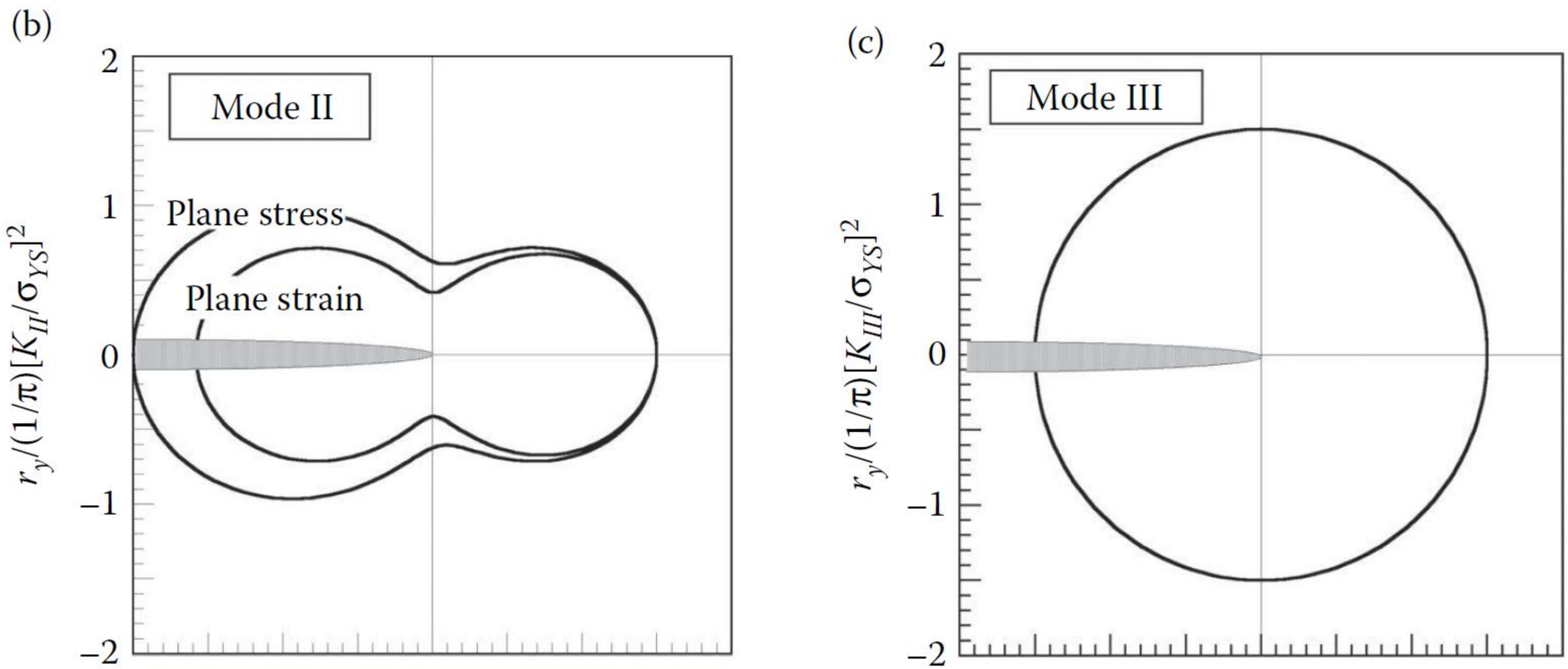
3) Similar approach to obtain mode II and III plastic zones:



4) Solutions for r_p not strictly correct, because they are based on a purely elastic:

➡ Stress equilibrium not respected.

➡ Alternatively, *Irwin plasticity correction* using an effective crack length …