Review of Matrix Algebra

Elementary Operations

- Matrix Addition and Subtraction

For two matrices **A** and **B**, both of the same size ($m \times n$), the addition and subtraction are defined by

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \quad \text{with} \quad c_{ij} = a_{ij} + b_{ij}$$

$$\mathbf{D} = \mathbf{A} - \mathbf{B} \quad \text{with} \quad d_{ij} = a_{ij} - b_{ij}$$

- Scalar Multiplication

$$\lambda \mathbf{A} = \left[\lambda a_{ij} \right]$$

- Matrix Multiplication

For two matrices **A** (of size $l \times m$) and **B** (of size $m \times n$), the product of **AB** is defined by

Elementary Operations

- Matrix Multiplication

$$\mathbf{C} = \mathbf{AB} \qquad \text{with } c_{ij} = \sum_{k=1}^{m} a_{ik} b_{kj}$$

$$i = 1, 2, \ldots, l; \; j = 1, 2, \ldots, n.$$

Note that, in general, **AB** ≠ **BA**, but (**AB**)**C** = **A**(**BC**)

- Transpose of a Matrix

If **A** = [a_{ij}], then the transpose of **A** is $\mathbf{A}^T = [a_{ji}]$

and $(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T$

- Determinant of a Matrix

The determinant of square matrix **A** is a scalar number denoted by det **A** or |**A**|. For 2×2 and 3×3 matrices, their determinants are given by

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13}$$

$$- a_{13}a_{22}a_{31} - a_{12}a_{21}a_{33} - a_{23}a_{32}a_{11}$$

Elementary Operations

- Singular Matrix

A square matrix **A** is singular if det **A** = 0, which indicates problems in the systems (nonunique solutions, degeneracy, etc.)

- Matrix Inversion

For a square and nonsingular matrix **A** ($\det \mathbf{A} \neq 0$), its inverse $\mathbf{A}^{-1}$ is constructed in such a way that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

$$(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Positive Definite Matrix

A square ($n \times n$) matrix **A** is said to be positive definite, if for any nonzero vector **x** of dimension n,

$$\mathbf{x}^T \mathbf{A} \mathbf{x} > 0$$

Note that positive definite matrices are nonsingular.

Differentiation and Integration of a Matrix

$$\mathbf{A}(t) = \left[a_{ij}(t) \right]$$

The differentiation is defined by

$$\frac{d}{dt}\mathbf{A}(t) = \left[\frac{da_{ij}(t)}{dt} \right]$$

the integration by

$$\int \mathbf{A}(t)dt = \left[\int a_{ij}(t)dt \right]$$

- Linear System of Algebraic Equations

$$a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2$$

$$\ldots\ldots\ldots$$

$$a_{n1}x_1 + a_{n2}x_2 + \ldots + a_{nn}x_n = b_n$$

where $x_1, x_2, \ldots, x_n$ *are the unknowns.*

In matrix form: $\mathbf{A}\mathbf{x} = \mathbf{b}$

Review of Matrix Algebra

$$\mathbf{A} = \left[a_{ij}\right] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad \mathbf{x} = \{x_i\} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{Bmatrix}$$

$$\mathbf{Ax} = \mathbf{b} \qquad \mathbf{b} = \{b_i\} = \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{Bmatrix}$$

- **A** is called a $n \times n$ (square) matrix, and **x** and **b** are (column) vectors of dimension n.

Review of Matrix Algebra

If det **A** = 0 (i.e., **A** is singular), then $\mathbf{A}^{-1}$ does not exist!

matrix **A** is nonsingular: $$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

- Solution Techniques for Linear Systems of Equations

  Gauss elimination methods

  Iterative methods

MATLAB Fundamentals

Why MATLAB?

- Industry standard software application
- Wealth of built-in functions and libraries
- Toolboxes (add-on software modules) – optimization, neural network, image and signal processing, control systems design, fuzzy logic, etc.
- Has own structured programming language
- Ease of application and testing (pre- and post-processing without lots of programming and formatting)
- Platform independent

What is MATLAB?

- Both a computer programming language and a software
- Began as a set of tools to solve linear algebraic equations. Has grown to a complete scientific programming suite
- Interpretive programming language: Read script files and perform operations; generally not compiled
- Enhanced calculator / spreadsheet – much more flexible
- Complete visualization package and post-processing analysis suite

MATLAB

- MATLAB is a numerical analysis system
- Can write "programs", but they are not formally compiled
- Should still use structured programming
- Should still use comments
- Comments are indicated by "%" at the beginning of the line

MATLAB Windows

- Command Window

  -- enter commands and data

  -- print results

- Graphics Window

  -- display plots and graphs

- Edit Window

  -- create and modify m-files

Managing MATLAB Environment

- who or whos -- See the current runtime environment
- clear -- remove all variables from memory
- clc -- clear the command window
- clf -- clear the graphics window
- save -- save the workspace environment
- load -- restore workspace from a disk file
- abort -- CTRL-C
- help -- help "command"

Really good "help" command

MATLAB Syntax

- No complicated rules
- Perhaps the most important thing to remember is semicolons (;) at the end of a line to suppress output
- diary “filename” saves a text record of session
- diary off turns it off

MATLAB

- MATLAB’s basic component is a Vector or Matrix
- Even single value variables (Scalars)
- All operations are optimized for vector use
- Loops run slower in MATLAB than in Fortran (not a vector operation)
- “size” command gives size of the matrix

Scalars, Vectors, Matrices

- MATLAB treat variables as "matrices"
- Matrix (m × n) - a set of numbers arranged in rows (m) and columns (n)
- Scalar: 1 × 1 matrix
- Row Vector: 1 × n matrix
- Column Vector: m × 1 matrix

$$A = \begin{bmatrix} 5.27 \end{bmatrix} \qquad B = \begin{bmatrix} 5.02 & -2.3 & 7.21 \end{bmatrix}$$

$$C = B' = \begin{bmatrix} 5.02 \\ -2.3 \\ 7.21 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 3 & -2 & 5 \\ -2 & 4 & 3.2 & 9.5 \\ -0.5 & -1 & 7.2 & -2 \end{bmatrix}$$

Scalars, Vectors, Matrices

```
>> pi
ans =
  3.1416

>> size(pi)
ans =
   1     1
```

```
>> a=[1 2 3; 4 5 6]
a =
  1     2     3
  4     5     6

>> size(a)
ans =
   2     3
```

$$pi = 3.1416$$

$$a = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Complex variables

- MATLAB handles complex arithmetic automatically
- No need to compute real and imaginary parts separately
- The unit imaginary number $i = \sqrt{-1}$ is preassigned

```
» x=5+2*i
x =
   5.0000 + 2.0000i
» y=5*x+3
y =
  28.0000 +10.0000i
```

MATLAB Example

```
» x=3+5-0.2
x =
    7.8000
» y=3*x^2+5
y =
  187.5200
» z=x*sqrt(y)
z =
  106.8116
» A=[1 2 3; 4 5 6]
A =
     1     2     3
     4     5     6
» b=[3;2;5]
b =
     3
     2
     5
» C=A*b
C =
    22
    52
```

```
» who

Your variables are:

A         b         y
C         x         z

» whos
  Name      Size         Bytes  Class

  A         2x3             48  double array
  C         2x1             16  double array
  b         3x1             24  double array
  x         1x1              8  double array
  y         1x1              8  double array
  z         1x1              8  double array

» save

Saving to: matlab.mat

» save matrix1
```

← **default filename** (Saving to: matlab.mat)

← **filename "matrix1"** (» save matrix1)

Data types

- All numbers are double precision
- Text is stored as arrays of characters
- You don’t have to declare the type of data (defined when running)
- MATLAB is case-sensitive!!!

Variable Names

- Usually, the name is identified with the problem
- Variable names may consist of up to 31 characters
- Variable names may be alphabetic, digits, and the underscore character (_)
- Variable names must start with a letter

ABC, A1, C56, CVEN_302

day, year, iteration, max

time, velocity, distance, area, density, pressure

Time, TIME, time (case sensitive!!)

Initializing Variables

- Explicitly list the values
- reads from a data file
- uses the colon (:) operator
- reads from the keyboard

A = [1; 3; 5; 10]; B = [1 3 5; -6 4 -1]

C = [2 3 5 1; 0 1 … (continuation)

1 -2; 3 5 1 -3]

E = [A; 1; A]; F = [C(2,3); A]

Matrix Concatenation

$$x = [1 \quad 2 \quad 3] \; ; \qquad y = [7 \quad 9 \quad 4]$$

$$z = [x \quad y] = [1 \quad 2 \quad 3 \quad 7 \quad 9 \quad 4]$$

$$u = [x \; ; \; y] = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 9 & 4 \end{bmatrix}$$

$$v = [x \;\; y \; ; \; y \;\; x] = \begin{bmatrix} 1 & 2 & 3 & 7 & 9 & 4 \\ 7 & 9 & 4 & 1 & 2 & 3 \end{bmatrix}$$

Colon Operator

- **Creating new matrices from an existing matrix**

C = [1,2,5; -1,0,1; 3,2,-1; 0,1,4]

F = C(:, 2:3) = [2,5; 0,1; 2,-1; 1,4]

$$C = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 0 & 1 \\ 3 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix} \qquad F = \begin{bmatrix} 2 & 5 \\ 0 & 1 \\ 2 & -1 \\ 1 & 4 \end{bmatrix}$$

Colon Operator

- **Creating new matrices from an existing matrix**

  ***C* = [1,2,5; -1,0,1; 3,2,-1; 0,1,4]**

  ***E* = C(2:3,:) = [-1 0 1; 3 2 -1]**

$$C = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 0 & 1 \\ 3 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix} \qquad E = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}$$

Colon Operator

- **Creating new matrices from an existing matrix**

C **= [1,2,5; -1,0,1; 3,2,-1; 0,1,4]**

G **= C(3:4,1:2) = [3,2; 0,1]**

$$C = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 0 & 1 \\ 3 & 2 & -1 \\ 0 & 1 & 4 \end{bmatrix} \qquad G = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$$

Colon Operator

- Variable_name = a:step:b

  time = 0.0:0.5:2.5

  time = [0.0, 0.5, 1.0, 1.5, 2.0, 2.5]

- Negative increment

  values = 10:-1:2

  values = [10, 9, 8, 7, 6, 5, 4, 3, 2]

linspace Function

- **linspace(x1, x2) gives 100 evenly spaced values between x1 and x2**

  x = linspace(x1,x2)

- **linspace(a,b,n) generate n equally spaced points between a and b**

  x = linspace(a,b,n)

```
» linspace(0,2,11)
ans =
  Columns 1 through 7
         0    0.2000    0.4000    0.6000    0.8000    1.0000    1.2000
  Columns 8 through 11
    1.4000    1.6000    1.8000    2.0000
```

Special Matrices

$$\mathbf{eye(3)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{zeros(3,2)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{ones(3)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \mathbf{ones(2,4)} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Scalar Arithmetic Operations

- **In order of priority**

Symbol	Operation	MATLAB Form
^	Exponentiation a^b	a^b
–	Negation $-a$	$-a$
* /	Multiplication and division $ab; a \div b$	$a*b;\ a/b$
\	Left division $a \backslash b = \frac{b}{a}$ (Matrix inverse)	$a \backslash b$
+ –	Addition and subtraction $a \pm b$	$a+b; a-b$

Example: x = (a + b*c)/d^2

count = count + 1

Order of Precedence of Arithmetic Operations

1. Parentheses, starting with the innermost pair
2. Exponentiation, from left to right
3. Multiplication and division with equal precedence, from left to right
4. Addition and subtraction with equal precedence, from left to right

Examples: factor = 1 + b/v + c/v^2

slope = (y2 - y1)/(x2 - x1)

loss = f * length/dia * (1/2 * rho * v^2)

func = 1 + 0.5*(3*x^4 + (x + 2/x)^2)

Order of Precedence of Arithmetic Operations

- The priority order can be overridden with parentheses

```
» a=3; b=5; c=2;
» s1 = a-b*c
s1 =
    -7
» s2=(a-b)*c
s2 =
    -4
```

Multiplication has higher priority than subtraction

```
» y = -7.3^2
y =
  -53.2900
» y=(-7.3)^2
y =
   53.2900
```

Exponentiation has higher priority than negation

Array Operations

- An array operation is performed element-by-element

$$C(1) = A(1)*B(1);$$
$$C(2) = A(2)*B(2);$$
$$C(3) = A(3)*B(3);$$
$$C(4) = A(4)*B(4);$$
$$C(5) = A(5)*B(5);$$

MATLAB: C = A.*B;

Element-by-Element Operations

Symbol	Operation	Form	Example
+	Scalar-array addition	$A+b$	$[4,6]+3=[7,9]$
-	Scalar-array subtraction	$A-b$	$[8,3]-6=[2,-3]$
+	Array addition	$A+B$	$[4,6]+[8,3]=[12,9]$
-	Array subtraction	$A-B$	$[4,6]-[8,3]=[-4,3]$
.*	Array multiplication	$A.*B$	$[3,6].*[2,-3]=[6,-18]$
./	Array right division	$A./B$	$[3,7]./[8,5]=[3/8,7/5]=[0.375,1.400]$
.\	Array left division	$A.\backslash B$	$[3,7].\backslash[8,5]=[3\backslash 8,7\backslash 5]=[2.667,0.7143]$
.^	Array exponentiation	$A.\wedge B$	$[4,2].\wedge 3=[4\wedge 3,2\wedge 3]=[64,8]$
			$3.\wedge[2,5]=[3\wedge 2,3\wedge 5]=[9,243]$
			$[5,3].\wedge[2,4]=[5\wedge 2,3\wedge 4]=[25,81]$

Vector and Matrix operations

$$a = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad a + b = \begin{bmatrix} 3 \\ 7 \\ 11 \end{bmatrix}$$

But a*b gives an error (undefined) because dimensions are incorrect. Need to use .*

$$a.*b = \begin{bmatrix} 1*2 \\ 3*4 \\ 5*6 \end{bmatrix} = \begin{bmatrix} 2 \\ 12 \\ 30 \end{bmatrix}$$

Vectorized Matrix Operations

$$A = \begin{bmatrix} 2 & 3 & 8 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 5 & 2 \end{bmatrix}$$

$$C = A.*B = \begin{bmatrix} 2 & 12 & 40 & 2 \end{bmatrix}$$

$$D = A./B = \begin{bmatrix} 2 & 0.75 & 1.6 & 0.5 \end{bmatrix}$$

$$E = A.\wedge 3 = \begin{bmatrix} 8 & 27 & 512 & 1 \end{bmatrix}$$

$$F = (3).\wedge B = \begin{bmatrix} 3 & 81 & 243 & 9 \end{bmatrix}$$

Array Operations for m × n Matrices

$$A = [1:4; -1:-1:-4; 3\ 1\ 2\ -1] = \begin{bmatrix} 1 & 2 & 3 & 4 \\ -1 & -2 & -3 & -4 \\ 3 & 1 & 2 & -1 \end{bmatrix}$$

$$B = A.*5 = \begin{bmatrix} 5 & 10 & 15 & 20 \\ -5 & -10 & -15 & -20 \\ 15 & 5 & 10 & -5 \end{bmatrix}$$

$$C = A.\text{^}3 = \begin{bmatrix} 1 & 8 & 27 & 64 \\ -1 & -8 & -27 & -64 \\ 27 & 1 & 8 & -1 \end{bmatrix}$$

Matrix Transpose

$$x = \begin{bmatrix} 4 & -2 & 3 \end{bmatrix} \; ; \qquad y = \begin{bmatrix} 3 & 1 & -2 \end{bmatrix}$$

$$x' = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} \; ; \qquad y' = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$$

$$x' * y = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} \begin{bmatrix} 3 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -8 \\ -6 & -2 & 4 \\ 9 & 3 & -6 \end{bmatrix}$$

$$x * y' = \begin{bmatrix} 4 & -2 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = (4)(3) + (-2)(1) + 3(-2) = 4$$

Built-in Functions

- All the standard operators +, –, *, /, ^
- Sqrt( ), abs( ), sin( ), cos( ), exp( ), tanh( ), acos( ), log( ), log10( ), etc.
- These operators are vectorized

$$a = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} \; ; \quad \sin(a) = \begin{bmatrix} \sin(3) \\ \sin(5) \\ \sin(4) \end{bmatrix} \; ; \quad \exp(a) = \begin{bmatrix} \exp(3) \\ \exp(5) \\ \exp(4) \end{bmatrix}$$

Built-in Functions

- Certain functions, such as exponential and square root, have matrix definition also
- Use "`help expm`" and "`help sqrtm`" for details

```
>> A = [1 3 5; 2 4 6; -3 2 -1]
A =
     1     3     5
     2     4     6
    -3     2    -1
>> B = sqrt(A)
B =
   1.0000               1.7321               2.2361
   1.4142               2.0000               2.4495
        0 + 1.7321i     1.4142                    0 + 1.0000i
>> C = sqrtm(A)
C =
   2.1045 + 0.0000i   0.1536 - 0.0000i   1.8023 + 0.0000i
   1.7141 - 0.0000i   1.1473 + 0.0000i   1.7446 + 0.0000i
  -2.0484 + 0.0000i   1.3874 + 0.0000i   0.5210 - 0.0000i
```

MATLAB Graphics

- One of the best things about MATLAB is interactive graphics
- "plot" is the one you will be using most often
- Many other 3D plotting functions -- plot3, mesh, surfc, etc.
- Use "help plot" for plotting options
- To get a new figure, use "figure"
- logarithmic plots available using semilogx, semilogy and loglog

Plotting Commands

plot(x,y) defaults to a blue line

plot(x,y,’ro’) uses red circles

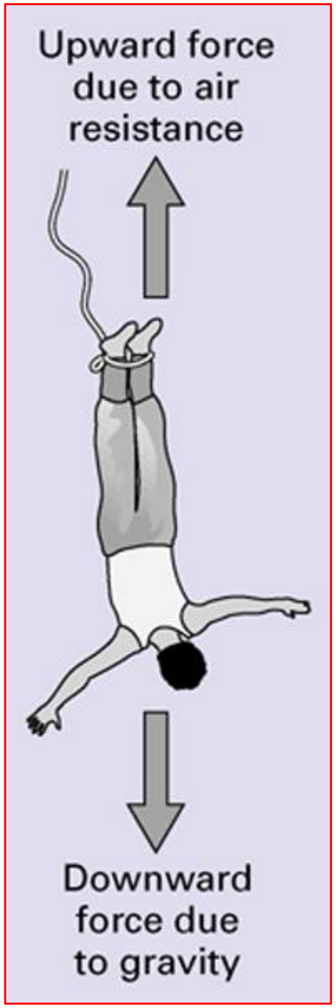
plot(x,y,’g*’) uses green asterisks

If you want to put two plots on the same graph, use “hold on”

```
plot(a,b,’r:’)      (red dotted line)
hold on
plot(a,c,’ko’)      (black circles)
```

Bungee Jumper

- You are asked to plot the velocity of a bungee jumper as a function of time during the free-fall part of the jump



Exact (Analytic) Solution

- **Newton's Second Law**

$$m\frac{dv}{dt} = mg - c_d v^2$$

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

- Exact Solution

$$v(t) = \sqrt{\frac{mg}{c_d}}\tanh\left(\sqrt{\frac{gc_d}{m}}\,t\right)$$

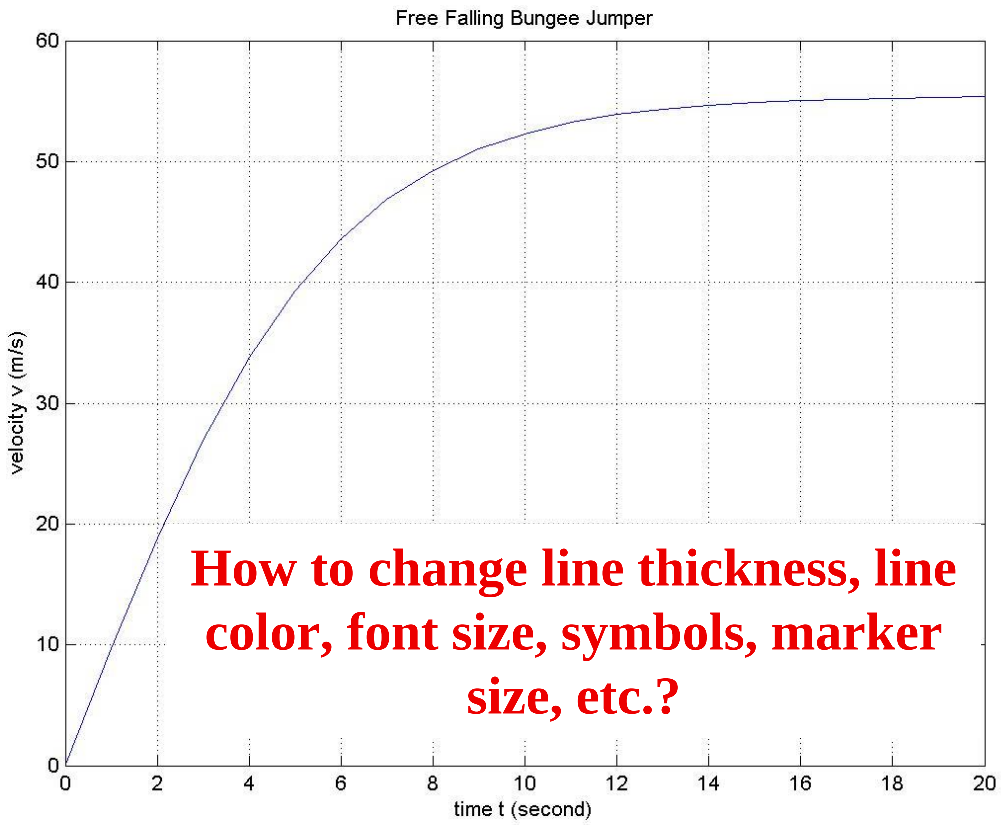
Free-Falling Bungee Jumper

- Use built-in functions **sqrt** & **tanh**

```
>> g = 9.81; m = 75.2; cd = 0.24;

>> t = 0:1:20

t =

  Columns 1 through 15

     0     1     2     3     4     5     6     7     8     9    10    11    12    13    14

  Columns 16 through 21

    15    16    17    18    19    20

>> v=sqrt(g*m/cd)*tanh(sqrt(g*cd/m)*t)

v =

  Columns 1 through 9

         0    9.7089   18.8400   26.9454   33.7794   39.2956   43.5937   46.8514   49.2692

  Columns 10 through 18

   51.0358   52.3119   53.2262   53.8772   54.3389   54.6653   54.8956   55.0579   55.1720

  Columns 19 through 21

   55.2523   55.3087   55.3484
```

Plot the velocity versus time curve

```
>> plot(t,v); grid on
>> title('Free Falling Bungee Jumper')
>> xlabel('time t (second)'); ylabel('velocity v (m/s)')
>> print -djpeg bungee.jpg
```



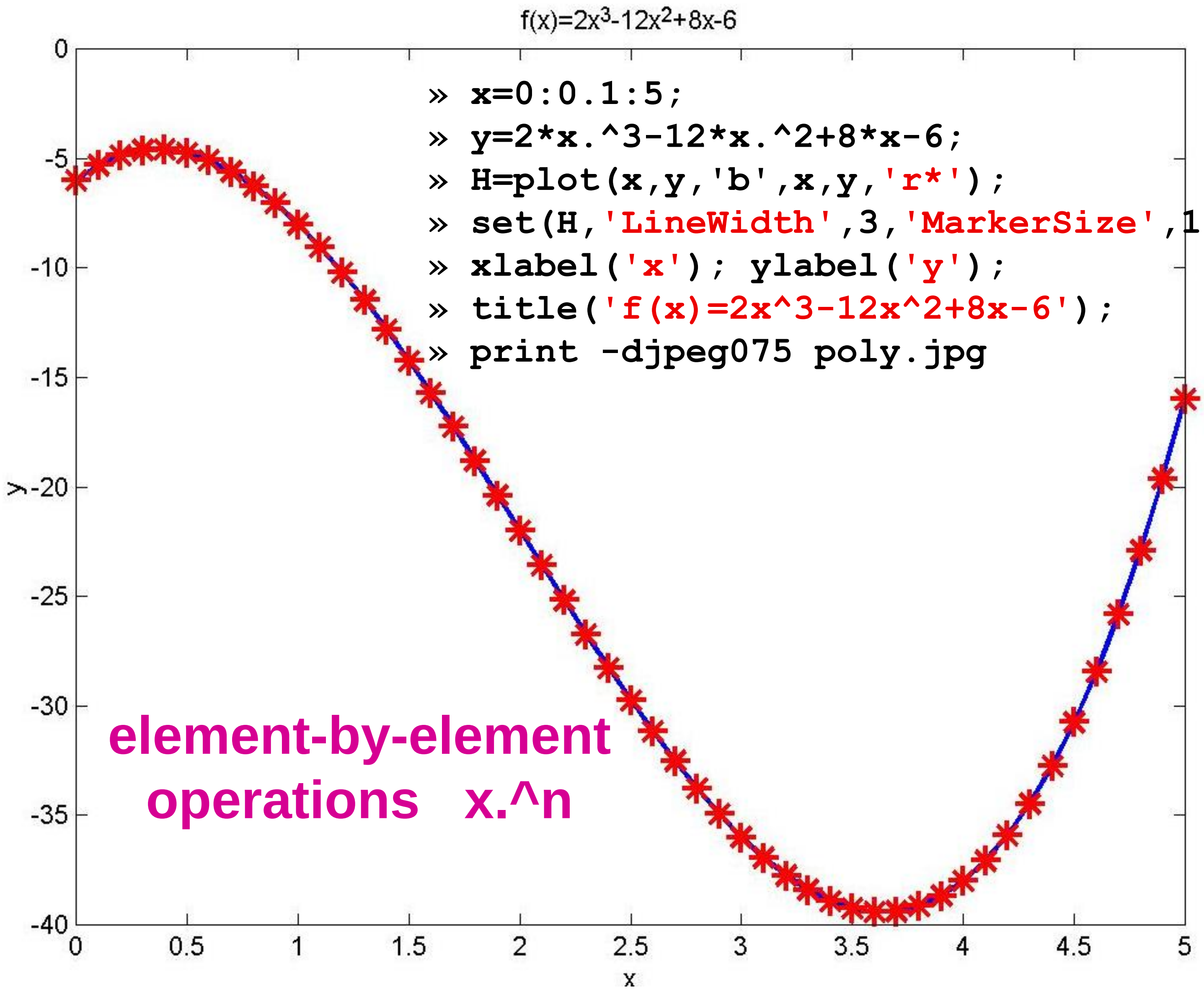
How to change line thickness, line color, font size, symbols, marker size, etc.?

Color, Symbols, and Line Types

- Use "**help plot**" to find available Specifiers

Colors		Symbols		Line Types	
b	blue	.	point	-	solid
g	green	o	circle	:	dotted
r	red	x	x-mark	-.	dashdot
c	cyan	+	plus	--	dashed
m	magenta	*	star		
y	yellow	s	square		
k	black	d	diamond		
		v	triangle (down)		
		^	triangle (up)		
		<	triangle (left)		
		>	triangle (right)		
		p	pentagram		
		h	hexagram		

Plot



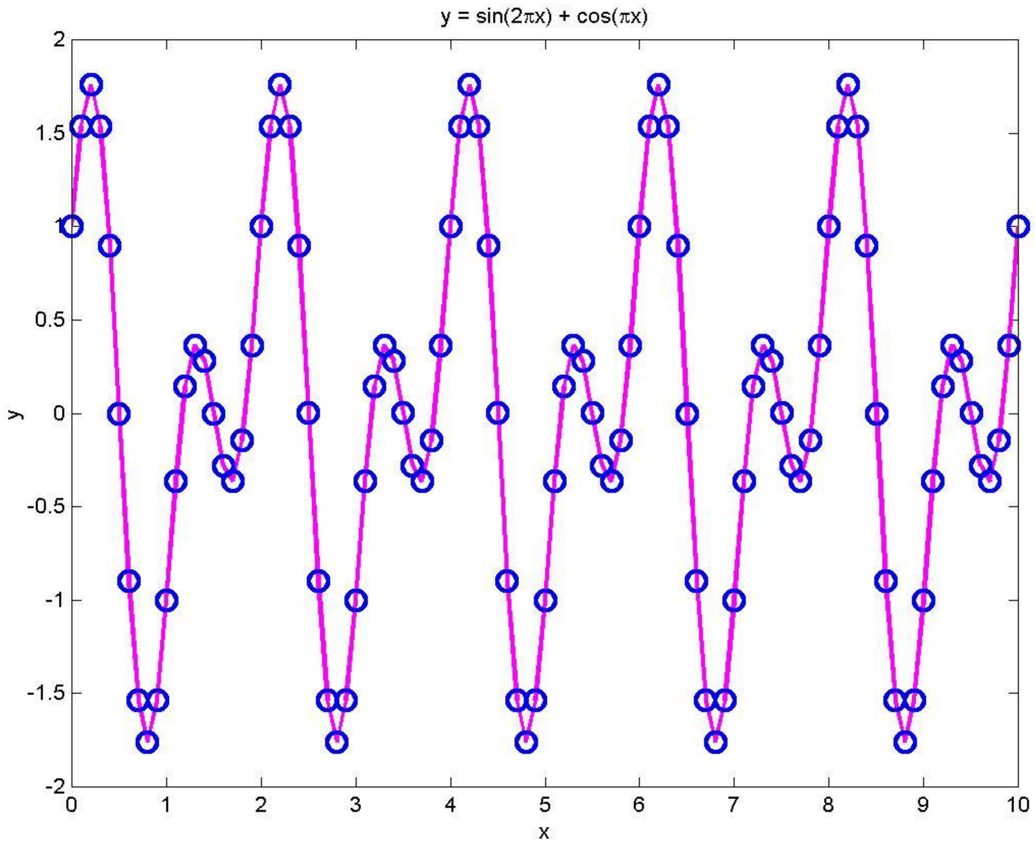

```
» x=0:0.1:5;
» y=2*x.^3-12*x.^2+8*x-6;
» H=plot(x,y,'b',x,y,'r*');
» set(H,'LineWidth',3,'MarkerSize',12)
» xlabel('x'); ylabel('y');
» title('f(x)=2x^3-12x^2+8x-6');
» print -djpeg075 poly.jpg
```

Adjust line thickness, font size, marker size, etc.

element-by-element operations x.^n

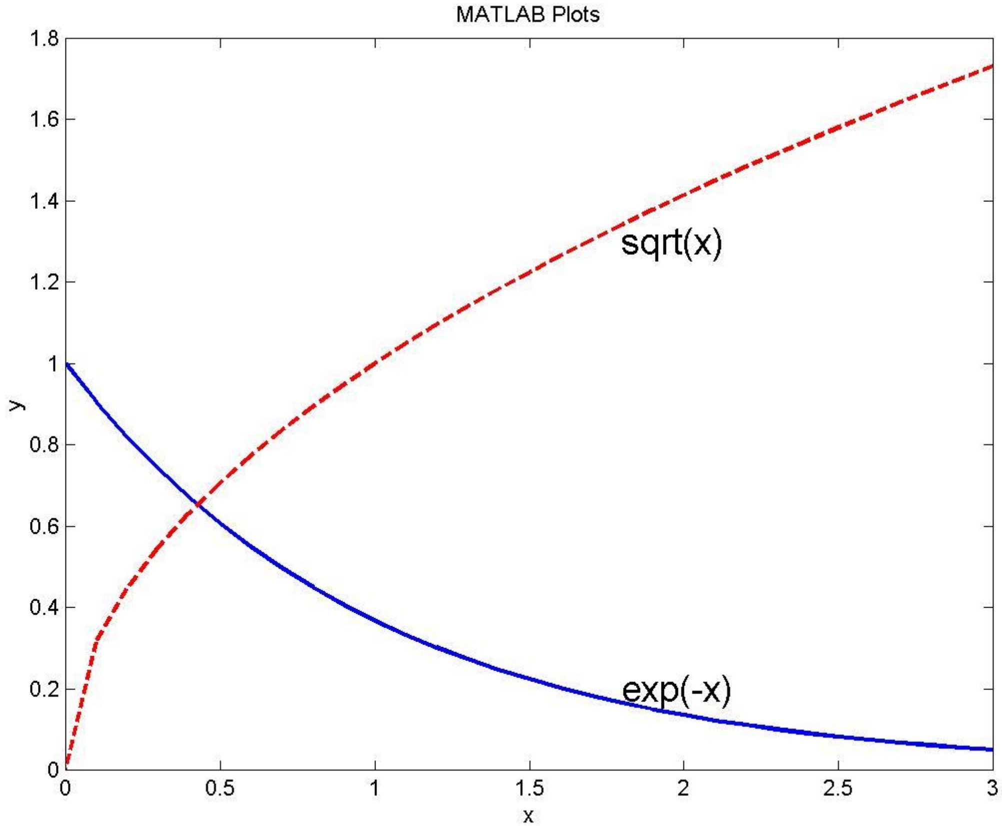
Plot

```
» x=0:0.1:10;
» y=sin(2.*pi*x)+cos(pi*x);
» H1=plot(x,y,'m'); set(H1,'LineWidth',3); hold on;
» H2=plot(x,y,'bO'); set(H2,'LineWidth',3,'MarkerSize',10); hold off;
» xlabel('x'); ylabel('y');
» title('y = sin(2\pix)+cos(\pix)');
» print -djpeg075 function.jpg
```



Plot

```
» x=0:0.1:3; y1=exp(-x); y2=sqrt(x);
» H=plot(x,y1,'b-',x,y2,'r--');
» set(H,'LineWidth',3)
» xlabel('x'); ylabel('y'); title('MATLAB Plots');
» H1=text(1.8,0.2,'exp(-x)'); set(H1,'FontSize',18);
» H2=text(1.8,1.3,'sqrt(x)'); set(H2,'FontSize',18);
```



Plotting Commands

plot (x, y)   plot(x1, y1, x2, y2)

plot (x, y, ‘<u>color</u> <u>symbol</u> <u>line style</u>’)

```
» x = linspace(0, 2*pi);
» y = sin (2.*x);
» z = cos (0.5*x);
» plot (x, y)
» plot (x, y, x, z)
» figure (2)
» plot (x, y, 'r o -'); grid on
» hold on
» plot (x, z, 'b * :')
```

figure or figure (#) : open a figure

(red, circle, solid line)

(blue, star, dotted line)

Graphics Commands

- Axis, Labels, and Title

```
xlabel (′ label ′)      ylabel (′ label ′)
title (′ title of the plot ′)
text ( x_location, y_location, ′ text ′ )
axis ( [ x_min  x_max  y_min  y_max ] )
```

′ ′ - text string

```
» xlabel (′Time′)
» ylabel (′Temperature′)
» title (′Temperature Record : 1900 - 2000′)
» text (17, 120, ′Record High′ )
» text (85, -40, ′Record Low′ )
» axis ([0  100  -50  140])
» hold off
```

Programming with MATLAB

M-Files: Scripts and Functions

- You can create and save code in text files using MATLAB Editor/Debugger or other text editors (called m-files since the ending must be .m)
- M-file is an ASCII text file similar to FORTRAN or C source codes (computer programs)
- A script can be executed by typing the file name, or using the “run” command

Difference between scripts and functions

Scripts share variables with the main workspace

Functions do not

Script Files

- Script file – a series of MATLAB commands saved on a file, can be executed by

- typing the file name in the Command Window
- invoking the menu selections in the Edit Window: `Debug, Run`

- Create a script file using menu selection:

`File, New, M-file`

Function File

- Function file: M-file that starts with the word `function`
- `Function` can accept input arguments and return outputs
- Analogous to user-defined functions in programming languages such as Fortran, C, …
- Save the function file as `function_name.m`

- User `help function` in command window for additional information

Functions

- One variable

  function y = function_name(input arguments)

- More than one output variables

  function [y, z] = function_name(input arguments)

Examples:

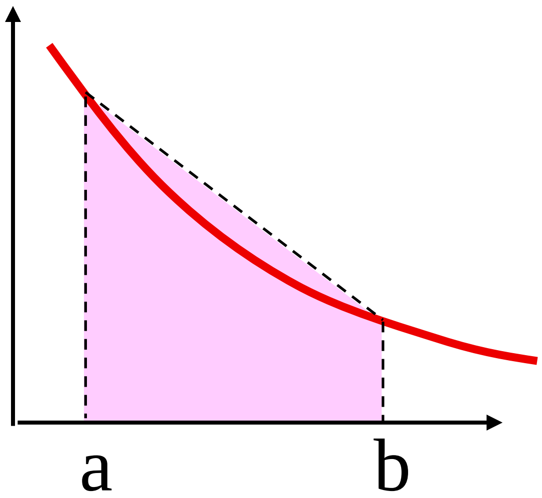
```
function y = my_func (x)
    y = x^3 + 3*x^2 -5 * x +2 ;

function area = integral (f, a, b)
ya = feval (f, a);  yb = feval(f, b);
area = (b-a)*(ya+yb)/2;
```

Functions

Approximate the integral of $f(x) = 1/x^3$

using basic trapezoid rule

$$y = \int_a^b \frac{1}{x^3}\,dx \cong \frac{b-a}{2}\left(\frac{1}{a^3} + \frac{1}{b^3}\right)$$



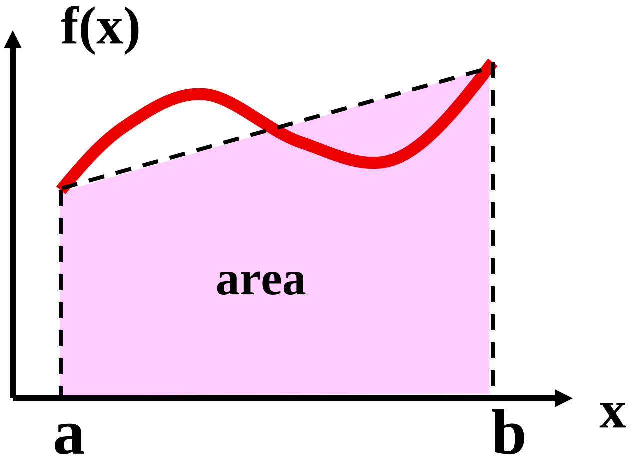
Filename: trap_ex.m

```
function t = trap_ex(a, b)
t = (b - a) * (a^(-3) + b^(-3)) / 2;
```

```
» y = trap_ex (1, 3)
y =
    1.0370
```

Script File for Integral

$$\int_a^b f(x)dx = \frac{b-a}{2}\left[f(a)+f(b)\right]$$



1. Save integral (f, a, b) in script file integral.m

2. Save function my_func(x) in script my_func.m

3. Run script file

>> area = integral('my_func', 1, 10)

>> area = integral('my_func', 3, 6)

Script File for Integral

feval - evaluate function specified by string

my_func.m

```
function y = my_func(x)
% function 1/x^3
y = x.^(-3);
```

basic_trap.m

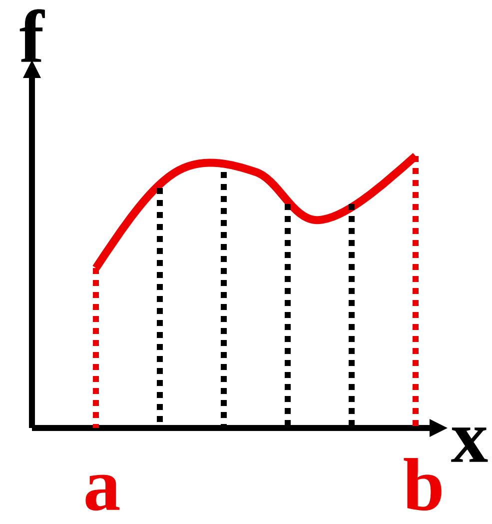
```
function q = basic_trap(f, a, b)
% basic trapezoid rule
ya = feval(f, a);
yb = feval(f, b);
q = (b - a)* (ya + yb)/2;
```

```
» y = basic_trap ('my_func', 1,3)
y =
   1.0370
```

Composite Trapezoid Rule

Filename: comp_trap.m

```
function I = Trap(f, a, b, n)
% find the integral of f using
% composite trapezoid rule
h=(b - a)/n;   S = feval(f, a);
for i = 1 : n-1
    x(i) = a + h*i;
    S = S + 2*feval(f, x(i));
end
S = S + feval(f, b);   I =h*S/2;
```



Composite Trapezoid Rule

```
» I=comp_trap('my_func',1,3,1)
I =
    1.0370
» I=comp_trap('my_func',1,3,2)
I =
    0.6435
» I=comp_trap('my_func',1,3,4)
I =
    0.5019
» I=comp_trap('my_func',1,3,8)
I =
    0.4596
» I=comp_trap('my_func',1,3,16)
I =
    0.4483
» I=comp_trap('my_func',1,3,100)
I =
    0.4445
» I=comp_trap('my_func',1,3,500)
I =
    0.4444
» I=comp_trap('my_func',1,3,1000)
I =
    0.4444
```

one segment

two segments

four segments

eight segments

16 segments

100 segments

500 segments

1000 segments

Function M-Files

- Function M-file can **return more than one result**
- Example – mean and standard deviation of a vector

```
function [mean, stdev] = stats(x)
% calculate the mean and standard deviation of a vector x
n = length(x);
mean = sum(x)/n;
stdev = sqrt(sum((x-mean).^2/(n-1)));
```

```
>> x=[1.5  3.7  5.4  2.6  0.9  2.8  5.2  4.9  6.3  3.5];
>> [m,s] = stats(x)
m =
    3.6800
s =
    1.7662
```

- Textbook refers function M-files as simply **M-files**

Data Files

- MAT Files

  -- memory efficient binary format

  -- preferable for internal use by MATLAB program

- ASCII files

  -- in ASCII characters

  -- useful if the data is to be shared (imported or exported to other programs)

MATLAB Input

To read files in

- if the file is an ascii table, use "load"
- if the file is ascii but not a table, file I/O needs "fopen" and "fclose"
- Reading in data from file using fopen depends on type of data (binary or text)
- Default data type is "binary"

Save Files

- 8-digit text format (variable list)
  save <fname> <vlist> - ascii
- 16-digit text format
  save <fname> <vlist> - double
- Delimit elements with tabs
  save <fname> <vlist> - double - tabs

Example: **Vel = [1 3 5; -6 2 -3]**

save velocity.dat Vel -ascii

```
 1.0000000e+000   3.0000000e+000    5.0000000e+000
-6.0000000e+000   2.0000000e+000  -3.0000000e+000
```

Load Files

- Read velocity into a matrix “velocity.dat”

```
>> load velocity.dat
>> velocity
    velocity =  1   3    5
               -6   2   -3
```

```
 1.0000000e+000   3.0000000e+000    5.0000000e+000
-6.0000000e+000   2.0000000e+000  -3.0000000e+000
```

Load Files

- Create an ASCII file temperature.dat

% Time	Temperature
0.0	75.0
0.5	73.2
1.0	72.6
1.5	74.8
2.0	79.3
2.5	83.2

- read “Time” and “Temperature” from temperature.dat

```
>> load temperature.dat
>> temperature
```

Note: temperature is a 6×2 matrix

MATLAB Output

Matlab automatically prints the results of any calculation (unless suppressed by semicolon ;)

Use “disp” to print out text to screen

```
disp (x.*y)
disp (′Temperature =′)
```

sprintf - display combination

Make a string to print to the screen

```
output = sprintf(‘Pi is equal to %f ’, pi)
```

Formatted Output

fprintf (format-string, var, ….)

%[flags] [width] [.precision] type

Examples of "type" fields

%d display in integer format

%e display in lowercase exponential notation

%E display in uppercase exponential notation

%f display in fixed point or decimal notation

%g display using %e or %f, depending on which is shorter

%% display "%"

Numeric Display Format

MATLAB Command	Display	Example
format short	default	3.1416
format long	14 decimals	3.14159265358979
format bank	2 decimals	3.14
format short e	4 decimals	3.1416e + 00
format long e	15 decimals	3.141592653589793e + 00
format +	+, – , blank	+

x = [5 -2 3 0 1 -2]; format +

x = [+ – +   + –]   (+/– sign only)

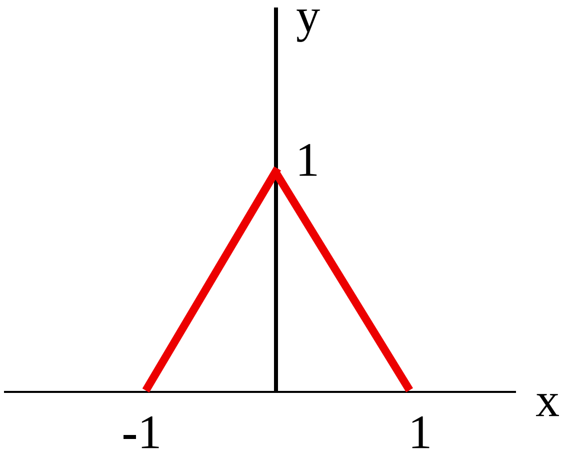
Selection (IF) Statements

- The most common form of selection structure is simple if statement
- The if statement will have a condition associated with it
- The condition is typically a logical expression that must be evaluated as either "true" or "false"
- The outcome of the evaluation will determine the next step performed

Logical IF Statements

- If (condition) executable_statements
  end

```
if (x < = -1.0 | x > = 1.0) y = 0.
end
if (x > -1.0 & x < 0.) y = 1. + x
end
if (x > = 0. & x < 1.0) y = 1.- x
end
```



Relation Operators

<table>
<tr><th><u>MATLAB</u></th><th><u>Interpretation</u></th></tr>
<tr><td>==</td><td>is equal to</td></tr>
<tr><td>~=</td><td>is not equal to</td></tr>
<tr><td><</td><td>is less than</td></tr>
<tr><td><=</td><td>is less than or equal to</td></tr>
<tr><td>></td><td>is greater than</td></tr>
<tr><td>>=</td><td>is greater than or equal to</td></tr>
<tr><td>&</td><td>and, true if both are true</td></tr>
<tr><td>|</td><td>or, true if either one is true</td></tr>
<tr><td>~</td><td>not</td></tr>
</table>

Logical Conditions

- **~ (not) – logical negation of an expression**

  `~ expression`

  - If the `expression` is true, the result is false. Conversely, if the `expression` is false, the result is true.

- **& (and) – logical conjunction on two expressions**

  $\texttt{expression}_1$ `&` $\texttt{expression}_2$

  - If both `expressions` are true, the result is true. If either or both `expressions` are false, the result is false.

- **| (or) – logical disjunction on two expressions**

  $\texttt{expression}_1$ `|` $\texttt{expression}_2$

  - If either or both `expressions` are true, the result is true

Logical Operators

- **0 - 1 matrix**
- **0: false ;   1: True**

$$a = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \qquad b = \begin{bmatrix} 3 & 5 & 1 \end{bmatrix} \qquad c = \begin{bmatrix} 4 & 3 & 2 \end{bmatrix}$$

$$a < b \qquad ans = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

$$a \sim= b \qquad ans = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

$$a < b \;\&\; b < c \qquad ans = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$a <= b \;|\; b >= c \qquad ans = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Nested IF Statement

- **Structures can be nested within each other**

```
if (condition)
   statement block
elseif (condition)
   another statement block
else
   another statement block
end
```

How to use Nested IF

- If the condition is true the statements following the statement block are executed.
- If the condition is not true, then the control is transferred to the next

  else, elseif, or end statement at the same if level.

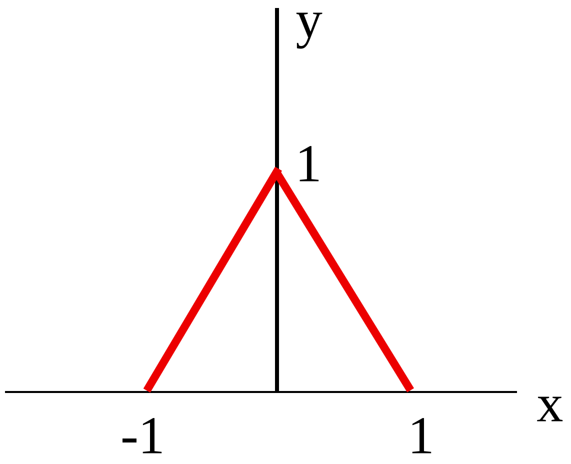
Else and Elseif

```
if  temperature > 100
    disp('Too hot - equipment malfunctioning.')
elseif  temperature > 75
    disp('Normal operating range.')
elseif  temperature > 60
    disp('Temperature below desired operating range.')
else
    disp('Too Cold - turn off equipment.')
end
```

Nested IF Statements

- nested if (if, if else, if elseif)

```
if (x < = -1.0)
    y = 0.
elseif (x < = 0.)
    y = 1. + x
elseif (x < = 1.0)
    y = 1. - x
else
    y=0.
end
```



Repetition

```
for i=1:m
    for j=1:n
        a(i,j)=(i+1)^2*sin(0.2*j*pi);
    end
end
```

Use “script file” for all do loops

For Loops

```
for  index = start :  step : finish
      statements
end
```

Ends after a specified number of repetitions

```
for k = 1:length(d)
    if d(k) < 30
        velocity(k) = 0.5 - 0.3*d(k).^2;
    else
        velocity(k) = 0.6 + 0.2*d(k)-0.01*d(k).^2
    end
end
```

For Loops

- M-file for computing the factorial **n!**
- MATLAB has a built-in function **factorial(n)** to compute **n!**

```
function fout = factor(n)
% factor(n):
%  Computes the product of all the integers from 1 to n.
x=1;
for i = 1:n
    x = x*i;
end
fout = x;
```

```
>> factor(12)

ans =

   479001600
```

```
>> factor(100)

ans =

    9.332621544394410e+157
```

While Loops

```
while  expression
        statements
end
```

Ends on the basis of a logical condition

- The statements are executed while the real part of the expression has all non-zero elements.
- If the statement is true, the statements are executed
- If the statement is always true, the loop becomes an "infinite loop"
- The "break" statement can be used to terminate the "while" or "for" loop prematurely.

Nesting and Indentation

- Example: Roots of a Quadratic Equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If a=0, b=0, no solution (or trivial sol. c=0)
- If a=0, b≠0, one real root: x=-c/b
- If a≠0, d=b^2 – 4ac ≥ 0, two real roots
- If a≠0, d=b^2 – 4ac <0, two complex roots

Nesting and Indentation

```
function quad = quadroots(a,b,c)
% Computes real and complex roots of quadratic equation
% a*x^2 + b*x + c = 0
% Output: (r1,i1,r2,i2) - real and imaginary parts of the
%                         first and second root
if a == 0                         % weird cases
    if b ~= 0                     % single root
        r1 = -c/b
    else                          % trivial solution
        error('Trivial or No Solution.  Try again')
    end                           % quadratic formula
else
d = b^2 - 4*a*c;                  % discriminant
    if d >= 0                     % real roots
        r1 = (-b + sqrt(d)) / (2*a)
        r2 = (-b - sqrt(d)) / (2*a)
    else                          % complex roots
        r1 = -b / (2*a)
        r2 = r1
        i1 = sqrt(abs(d)) / (2*a)
        i2 = -i1
    end
end
```

Nesting and Indentation

```
>> quad = quadroots(5,3,-4)
r1 =
     0.6434
r2 =
    -1.2434

>> quad = quadroots(5,3,4)
r1 =
    -0.3000
r2 =
    -0.3000
i1 =
     0.8426
i2 =
    -0.8426

>> quad = quadroots(0,0,5)
??? Error using ==> quadroots
Trivial or No Solution.  Try again
```

(two real roots)

(two complex roots)

(no root)

Passing Functions to M-File

- Use built-in "feval" and "inline" functions to perform calculations using an arbitrary function

$$outvar = \text{feval}(\text{'}funcname\text{'}, arg_1, arg_2, \ldots)$$

$$Funcname = \text{inline}(\text{'}expression\text{'}, var_1, var_2, \ldots)$$

```
>> fx=inline('exp(-x)*cos(x)^2*sin(2.*x)')
fx =
     Inline function:
     fx(x) = exp(-x)*cos(x)^2*sin(2.*x)
>> y = fx(2/3*pi)
y =
   -0.0267
```

No need to store in separate M-file

Summary

Vector and Matrix Manipulations

Once we get into Matlab, we meet a prompt >> called Matlab prompt.

Define a matrix/vector

```
>> A=[1 3 6; 2 7 8; 0 3 9]

A =

     1     3     6

     2     7     8

     0     3     9

>> size(A)

ans =

     3     3
```

Transpose of a matrix

```
>> A`

ans =

1     2     0
3     7     3
6     8     9
```

Vector and Matrix Manipulations

Column or row components

```
>> A(:,3)
ans =
      6
      8
      9

>> A(1,:)

ans =

      1       3       6


>> A(1,:)+A(3,:)

ans =

      1       6      15
```

Matrix Addition

```
>> B=[3 4 5; 6 7 2; 8 1 10];
>> B
B =
       3       4       5
       6       7       2
       8       1      10
>> C=A+B

C =
       4       7      11
       8      14      10
       8       4      19

>> C=A-B

C =
      -2      -1       1
      -4       0       6
      -8       2      -1
```

Vector and Matrix Manipulations

Matrix Multiplication

```
>> C=A*B

C =
     69      31      71
    112      65     104
     90      30      96
>> C=A*B';
>> C=A′*B;
```

Matrix Function

```
>> inv(A)

ans =
     1.8571    -0.4286    -0.8571
    -0.8571     0.4286     0.1905
     0.2857    -0.1429     0.0476
>> A*inv(A)

ans =
     1.0000     0.0000          0
     0.0000     1.0000          0
          0          0     1.0000
```

Basic matrix Functions

Symbol	Explanation
inv	Inverse of a matrix
det	Determinate of a matrix
rank	Rank of a matrix
cond	Condition number of a matrix
eye(n)	The n by n identity matrix
trace	Summation of diagonal elements of a matrix
zeros(n,m)	The n by m matrix consistings of all zeros
ones(n,m)	The n by m matrix consistings of all ones

Examples:

inv(A);
det(A);
rank(A);
eye(5);
ones(6,7);

Basic Matrix Function

Matrix of random number

```
>> A=rand(3,3)

A =

    0.3529    0.1389    0.6038
    0.8132    0.2028    0.2722
    0.0099    0.1987    0.1988
```

Matrix exponential

```
>> expm(A)

ans =

    1.5267    0.2680    0.8472
    1.1116    1.3533    0.6651
    0.1191    0.2521    1.2783
```

Eigenvalus

The eigenvalus problem of a matrix is defined

$$A\phi = \lambda\phi$$

```
>> A=[5 3 2; 1 4 6; 9 7 2]

A =
     5     3     2
     1     4     6
     9     7     2
>> e=eig(A)

e =

   12.5361
    1.7486
   -3.2847
```

Basic Matrix Function

Eigenvalues and Eignvectors

```
>> [V,D]=eig(A)

V =

   -0.4127   -0.5992    0.0459
   -0.5557    0.7773   -0.6388
   -0.7217   -0.1918    0.7680


D =

   12.5361         0         0
         0    1.7486         0
         0         0   -3.2847
```

LU decomposition

```
>> A=[1 3 5; 2 4 8; 4 7 3];
>> [L,U]=lu(A)

L =
    0.2500    1.0000         0
    0.5000    0.4000    1.0000
    1.0000         0         0

U =
    4.0000    7.0000    3.0000
         0    1.2500    4.2500
         0         0    4.8000

>> L*U
ans =

     1     3     5
     2     4     8
     4     7     3
```

Basic Matrix Function

Singular Value Decomposition (SVD)

$$A = U\Sigma V'$$

```
>> D=[1 3 7; 2 9 5; 2 8 5];
>> [U,sigma,V]=svd(D)

U =
   -0.4295    0.8998   -0.0775
   -0.6629   -0.3723   -0.6495
   -0.6133   -0.2276    0.7564

sigma =
   15.6492         0         0
         0    4.1333         0
         0         0    0.1391

V =
   -0.1905   -0.0726    0.9790
   -0.7771   -0.5982   -0.1956
   -0.5999    0.7980   -0.0576
```

QR Decomposition

$$A = QR$$

Q is a matrix with orthonormal Column and **R** is the upper triangular Matrix.

```
>>A=[1 3 5; 2 4 8; 4 7 3];
>> [Q,R]=qr(A)

Q =
   -0.2182    0.9117    0.3482
   -0.4364    0.2279   -0.8704
   -0.8729   -0.3419    0.3482

R =
   -4.5826   -8.5105   -7.2012
         0    1.2536    5.3561
         0         0   -4.1779
```

Basic Matrix Function

Solution of linear equations

$$Ax = y$$

```
>> A=[1 3 5; 2 4 8; 4 7 3];
>> A=[1 3 4; 5 7 8; 2 3 5];
>> y=[10; 9; 8];
>> x=inv(A)*y

x =

   -4.2500
    1.7500
    2.2500
```

Or

```
>> x=A\y

x =

   -4.2500
    1.7500
    2.2500
```

Vector componentwise operation

```
>> v1=[1 5 6 7]; v2=[0 2 3 5];
>> v3=v1.*v2
v3 =
     0    10    18    35
>> v4=v2./v1
v4 =
         0    0.4000    0.5000    0.7143
```

Data Analysis Functions

Symbol	Explanations
min (max)	minimum (maximum) of a vector

MIN(X) is a row vector containing the minimum element from each
[Y,I] = MIN(X) returns the indices of the minimum values in vector I.

sum	sum of elements of a vector
std	standard deviation of a data collection
sort	sort of element of a vector

[Y,I] = SORT(X) also returns an index matrix I. If X is a vector, then Y = X(I).

mean	mean value of a vector
A(i,j)	element i and j of matrix A
V(i)	element I of vector V
A(:,i)	all elements of matrix A in column i
A(i,:)	all elements of matrix A in row i

Polynomial Functions

- **MATLAB** — **Interpretation**
- **Poly** — convert collection of roots into a polynomial equation
- **roots** — finds the roots of a polynomial equation
- **polyval** — evaluate a polynomial for a given value
- **conv** — multiply two polynomial
- **deconv** — decompose a polynomial into a dividend and a residual
- **Polyfit** — curve fitting of a given polynomial

Example:

If C has N+1 components, the polynomial is C(1)*X^N + ... + C(N)*X + C(N+1).

```
>> C=[1 15 136 498 968 592];
>> roots(C)
ans =
  -5.0000 + 7.0000i
  -5.0000 - 7.0000i
  -2.0000 + 2.0000i
  -2.0000 - 2.0000i
  -1.0000
```

Complex Number

MATLAB	Interpretation
abs	the magnitude of a number
angle	the phase angle
real	the real part of a complex number
imag	the imaginary part
conj	the complex conjugate

Example:

In order to make a complex number for example 2+3*I

```
>> 2+3*i
ans =
    2.0000 + 3.0000i

>> 2+3*j
ans =
    2.0000 + 3.0000i
>> i=sqrt(-1)
i =
0 + 1.0000i
```

Note: Matlab takes i and j as a pure complex number

```
>> c=-1+i
c =
   -1.0000 + 1.0000i
>> [ abs(c) angle(c) real(c) imag(c)]
ans =
      1.4142     2.3562    -1.0000    1.0000
>> conj(c)
ans =
   -1.0000 - 1.0000i
```

Basic Function

MATLAB	**Interpretation**
fminbnd	find minimum of a function of one variable
fzero	solves a nonlinear algebric equation of one variable
ode23	solution using the 2nd/3rd order Runge-Kutta algorithm
ode45	solution using the 4th/5th order Runge-Kutta algorithm

```
>> fminbnd('x*cos(x)',-2,2)

ans =
   -0.8603

>> x=fzero('tan(x)',2)

x =
    1.5708
```

Example:

[t,y]=ode45(‘func’,[t0 tf],[x0,v0]);

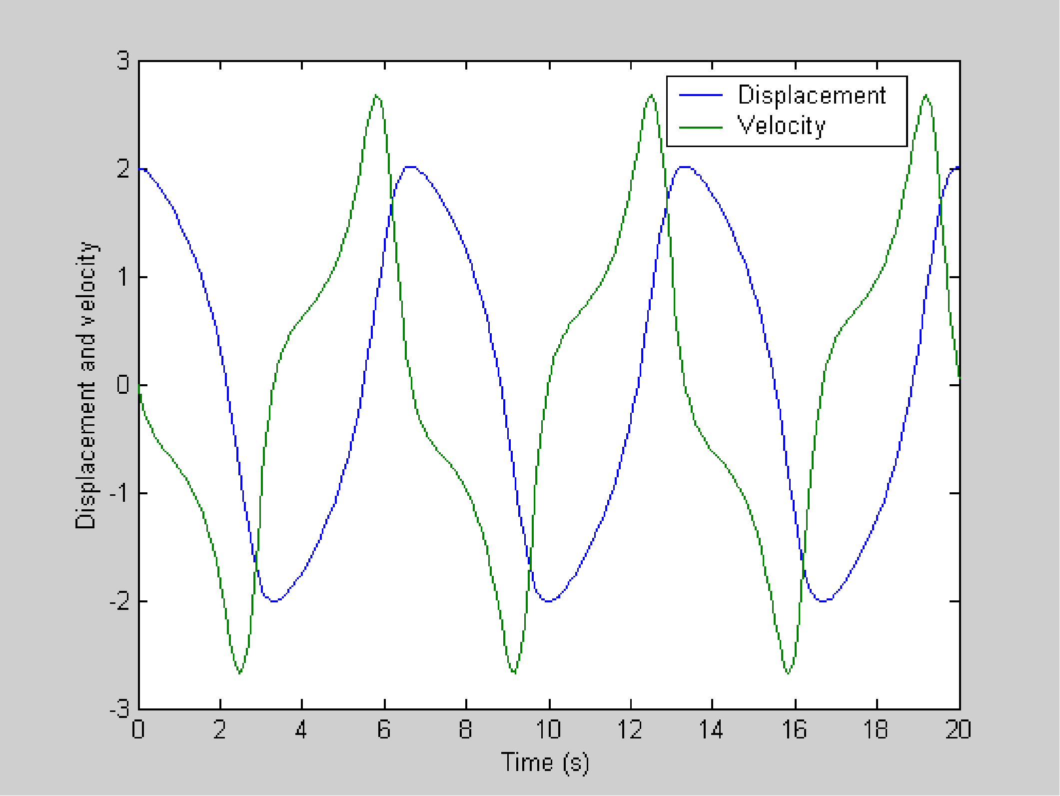
```
>> [t,y]=ode45('vdp1',[0 20],[2 0]);
>> [size(t) size(y)]
ans =
   237      1    237      2
```

Plotting Tools

Example:

Assume that t , y are the solution of ODE of previous slide

```
>> plot(t,y)                         >> xlabel('Time (s)')
>> plot(t,y(:,1))                    >> ylabel('Displacement and velocity')
>> plot(t,y(:,1),t,y(:,2))           >> legend('Displacement','Velocity')
```



Loop and logical statement

- **MATLAB** — **Interpretation**
- **for** — loop command similar to other language
- **while** — used for a loop combined with conditional statement
- **if** — produce a conditional statement
- **elseif, else** — used in conjugate with if command
- **break** — breaks a loop when a condition is satisfied
- == — two conditions are equal
- ~= — two conditions are not equal
- <=(>=) — one is less (greater) than the other
- **&** — and operator
- | — or operator
- ~ — not operator

Writing Function Subroutine

Function[ov1,ov2,…]=func1(iv1,iv2,…)

iv1, iv2, … are input variables

ov1, ov2, … are output variables

the file then will save as func1.m and can be called inside Matlab by

```
>> [ov1,ov2, …]=func1(iv1,iv2,…)
```

Writing Function Subroutine

- Example: Roots of a Quadratic Equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If a=0, b=0, no solution (or trivial sol. c=0)
- If a=0, b≠0, one real root: x=-c/b
- If a≠0, $d=b^2 - 4ac \geq 0$, two real roots
- If a≠0, $d=b^2 - 4ac < 0$, two complex roots

Writing Function Subroutine

```
function quad = quadroots(a,b,c)
% Computes real and complex roots of quadratic equation
% a*x^2 + b*x + c = 0
% Output: (r1,i1,r2,i2) - real and imaginary parts of the
%                         first and second root
if a == 0                         % weird cases
    if b ~= 0                     % single root
        r1 = -c/b
    else                          % trivial solution
        error('Trivial or No Solution.  Try again')
    end                           % quadratic formula
else
d = b^2 - 4*a*c;                  % discriminant
    if d >= 0                     % real roots
        r1 = (-b + sqrt(d)) / (2*a)
        r2 = (-b - sqrt(d)) / (2*a)
    else                          % complex roots
        r1 = -b / (2*a)
        r2 = r1
        i1 = sqrt(abs(d)) / (2*a)
        i2 = -i1
    end
end
```

Writing Function Subroutine

```
>> quad = quadroots(5,3,-4)
r1 =
     0.6434
r2 =
    -1.2434

>> quad = quadroots(5,3,4)
r1 =
    -0.3000
r2 =
    -0.3000
i1 =
     0.8426
i2 =
    -0.8426

>> quad = quadroots(0,0,5)
??? Error using ==> quadroots
Trivial or No Solution.  Try again
```

(two real roots)

(two complex roots)

(no root)