



دانشگاه صنعتی اصفهان  
دانشکده مکانیک

# Modeling Fracture with Abaqus

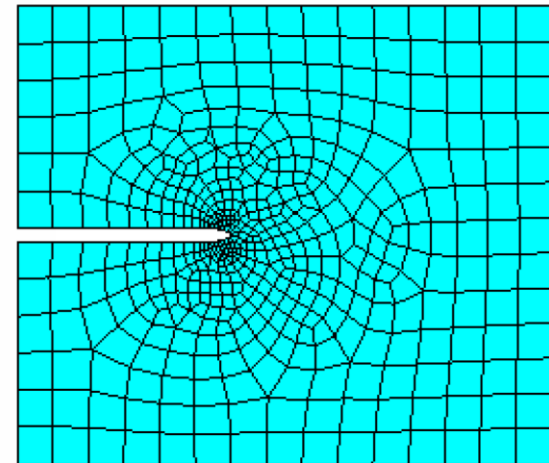
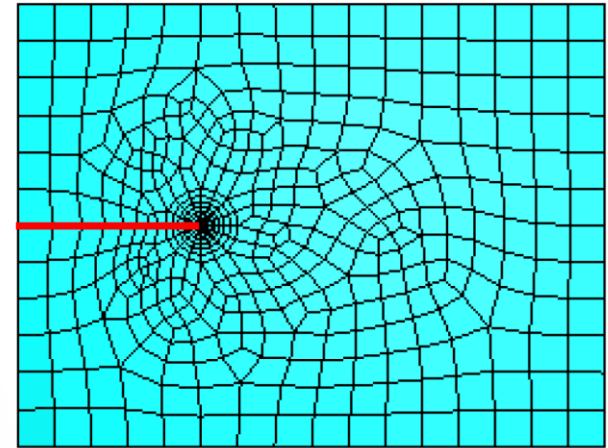


# Modeling Fracture with Abaqus

- ❖ Modeling Cracks
- ❖ Calculation of Contour Integrals
- ❖ Creating an XFEM Fracture Model

# Crack Modeling Overview

- A crack can be modeled as either
  - Sharp
    - Small-strain analysis
    - Singular behavior at the crack tip
      - Requires special attention
    - In Abaqus, a sharp crack is modeled using *seam* geometry
  - Blunted
    - Finite-strain analysis
    - Non-singular behavior at crack tip
    - In Abaqus, a blunted crack is modeled using *open* geometry
      - For example, a notch





# Crack Modeling Overview

## •The crack-tip singularity in small-strain analysis

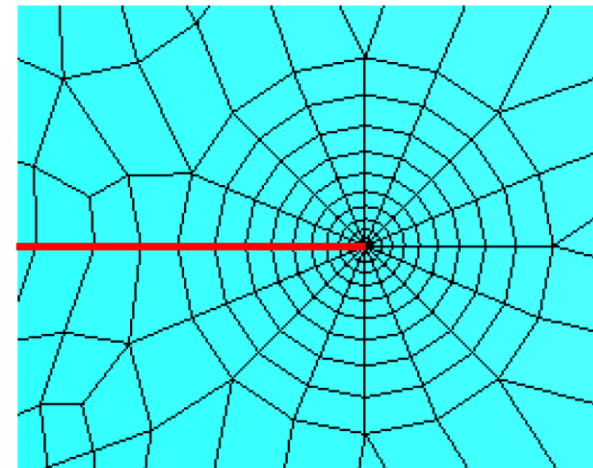
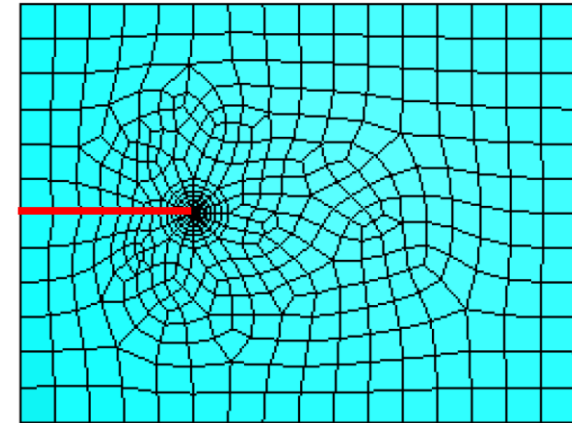
- For mesh convergence in a small-strain analysis, the singularity at the crack tip must be considered.
  - $J$  values are more accurate if some singularity is included in the mesh at the crack tip than if no singularity is included.
  - The stress and strain fields local to the crack tip will be modeled more accurately if singularities are considered.
- In small-strain analysis, the strain singularity is:

- Linear elasticity  $\varepsilon \propto \frac{1}{\sqrt{r}}$
- Perfect plasticity  $\varepsilon \propto \frac{1}{r}$
- Power-law hardening  $\varepsilon \propto \frac{1}{r^{\frac{n}{n+1}}}$

# Modeling Sharp Cracks in Two Dimensions

## In two dimensions...

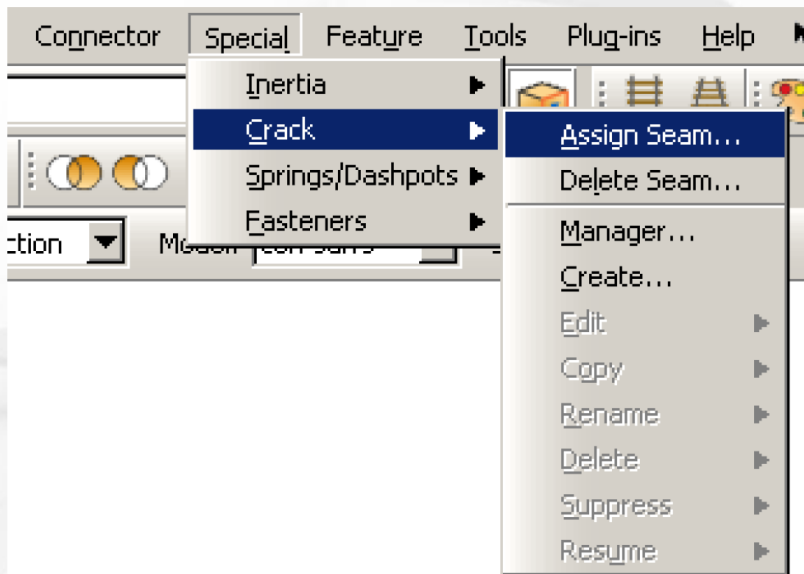
- The crack is modeled as an internal edge partition embedded (partially or wholly) inside a face.
  - This is called a *seam crack*
  - The edge along the seam will have duplicate nodes such that the elements on the opposite sides of the edge will not share nodes.
- Typically, the entire 2D part is filled with a quad or quad-dominated mesh.
  - At the crack tip, a ring of triangles are inserted along with concentric layers of structured quads.
  - All triangles in the contour domains must be represented as degenerated quads.



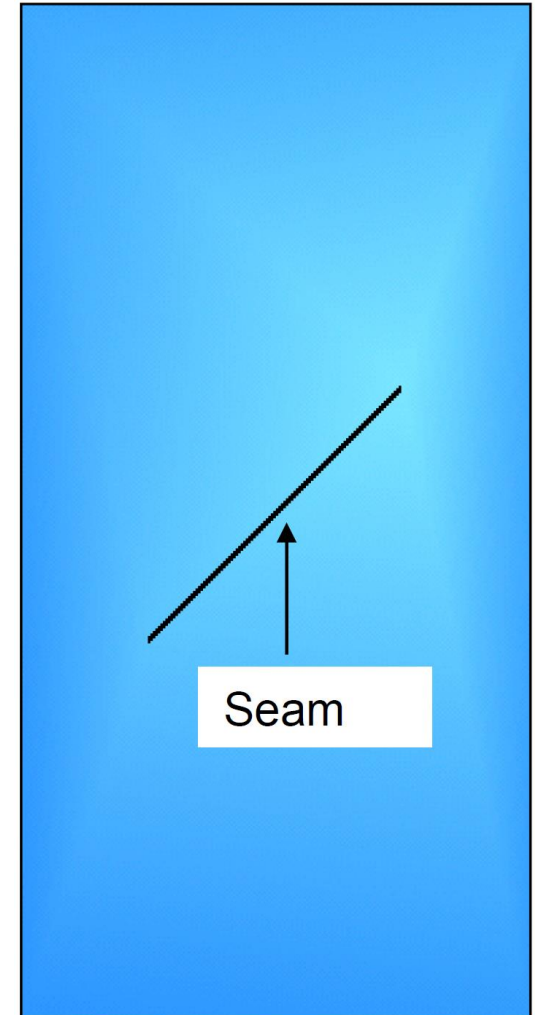
# Modeling Sharp Cracks in Two Dimensions

## Example: Slanted crack in a plate

- In Abaqus/CAE a seam is defined by through the **Crack** option underneath the **Special** menu of the Interaction module.
  - The seam will generate duplicate nodes along the edge.



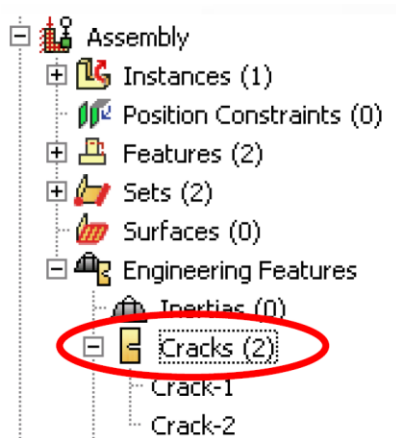
Seam Create face partition to represent the seam; assign a seam to the partition.



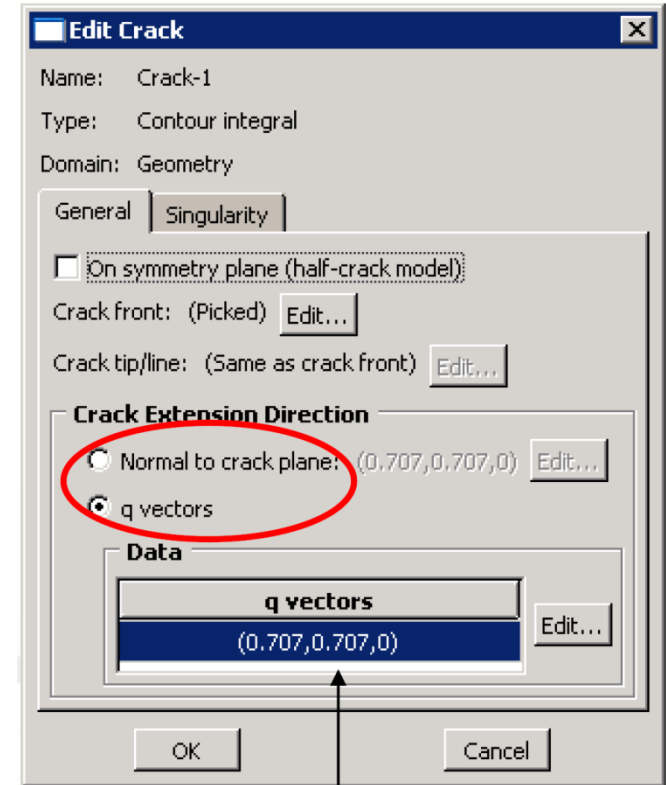
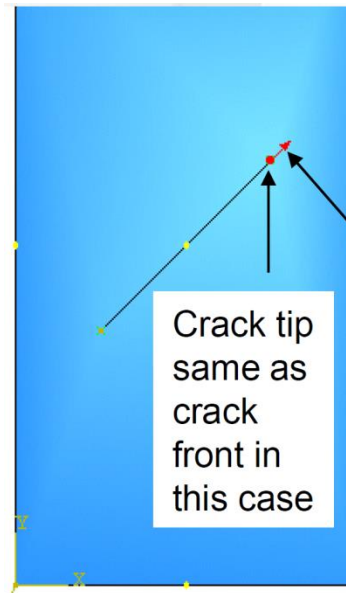


# Modeling Sharp Cracks in Two Dimensions

- To define the crack, you must specify
  - Crack front and the crack-tip
  - Normal to the crack plane or the direction of crack advance
    - ❖ The crack advance direction is called the  $q$  vector.



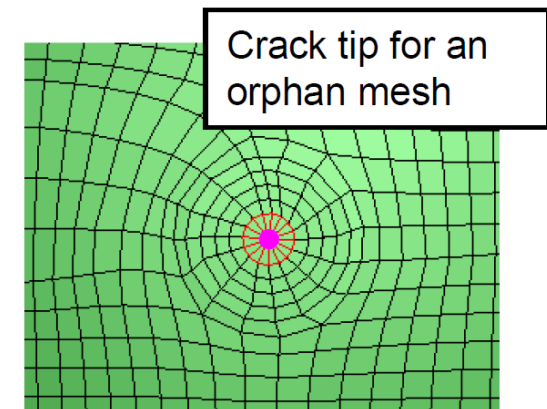
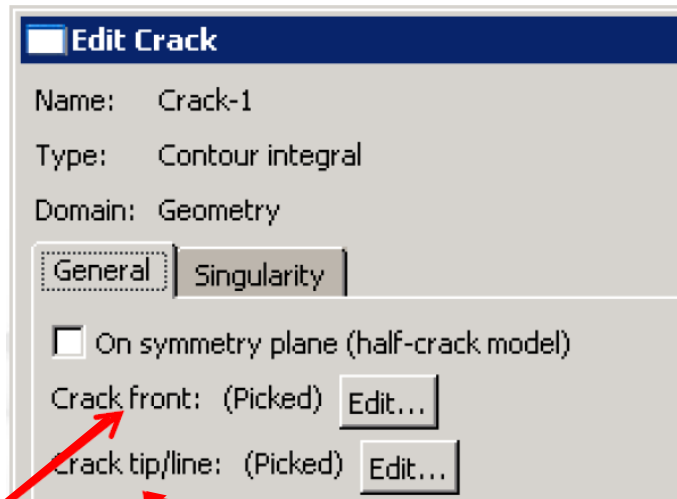
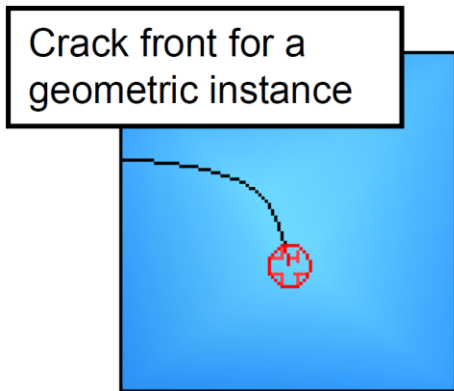
Select the vertex at either end as the crack front. (Repeat for the other end.)



The crack extension direction ( $q$  vector) defines the direction in which the crack would extend if it were growing. It is used for contour integral calculations.

# Modeling Sharp Cracks in Two Dimensions

- Other options for defining the crack front and crack tip



**Crack front may be:**  
 Vertex/Node  
 Edges/Element edges  
 Faces/Elements

Geometric  
Instances

Orphan  
Mesh

**Crack tip may be:**  
 Vertex/Node

Geometric  
Instances

Orphan  
Mesh

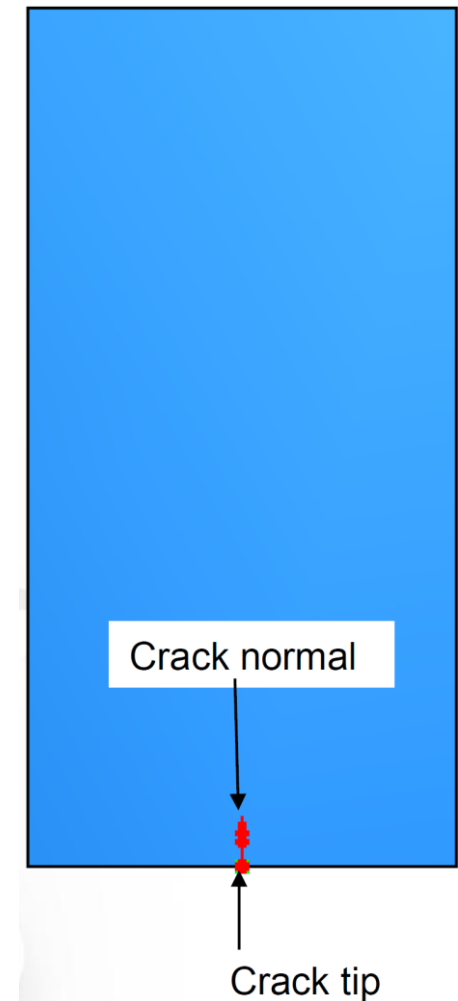
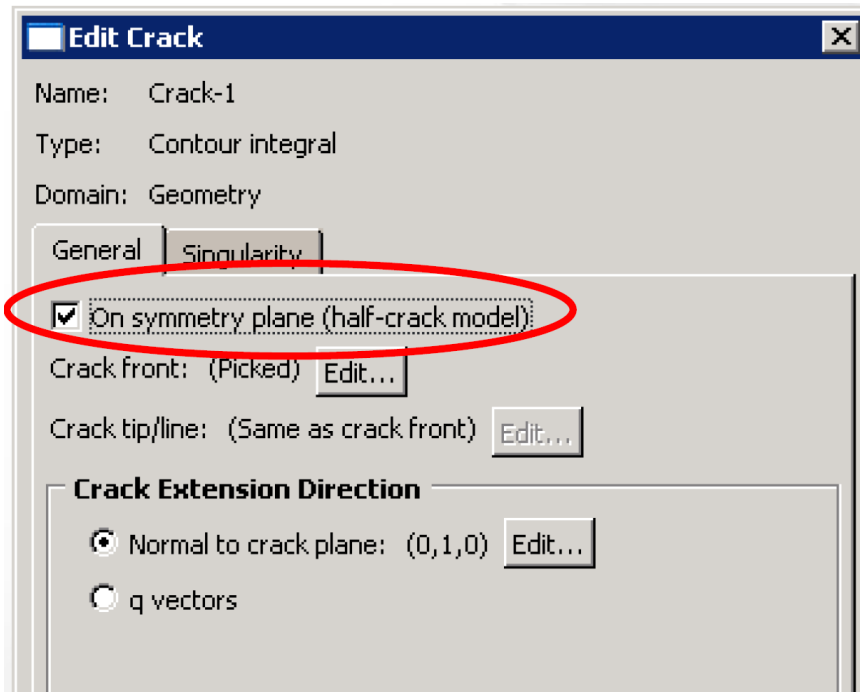




# Modeling Sharp Cracks in Two Dimensions

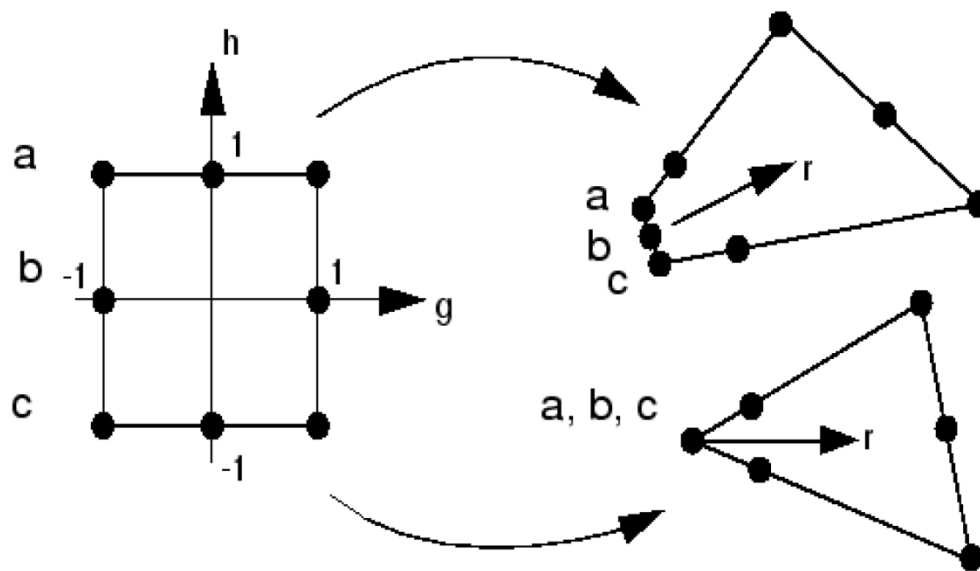
**Example:** crack on a symmetry plane

- If the crack is on a symmetry plane, you do not need to define a seam.
  - This feature can be used only for Mode I fracture.



# Modeling Sharp Cracks in Two Dimensions

- Modeling the crack-tip singularity with second-order quad elements
  - To capture the singularity in an 8-node isoparametric element:
    - Collapse one side (e.g., the side made up by nodes a, b, and c) so that all three nodes have the same geometric location at the crack tip.
    - Move the midside nodes on the sides connected to the crack tip to the  $\frac{1}{4}$  point nearest the crack tip.





# Modeling Sharp Cracks in Two Dimensions

- If nodes  $a$ ,  $b$ , and  $c$  are free to move independently, then

$$\varepsilon \rightarrow \frac{A}{r} + \frac{B}{\sqrt{r}} \text{ as } r \rightarrow 0$$

everywhere in the collapsed element.

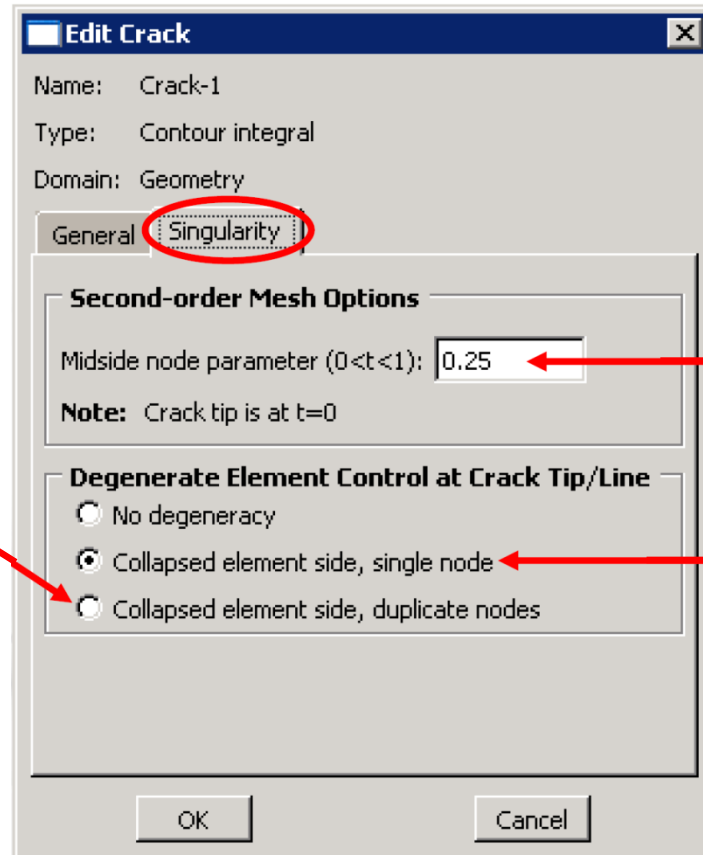
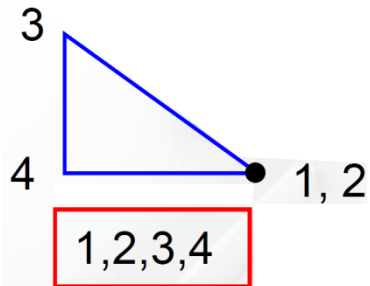
- If nodes  $a$ ,  $b$ , and  $c$  are constrained to move together,  $A = 0$ :
  - The strains and stresses are square-root singular (suitable for linear elasticity).
- If nodes  $a$ ,  $b$ , and  $c$  are free to move independently and the midside nodes remain at the midsides,  $B = 0$ :
  - The singularity in strain is correct for the perfectly plastic case.
- For materials in between linear elastic and perfectly plastic (most metals), it is better to have a stronger singularity than necessary.
  - The numerics will force the coefficient of this singularity to be small.



# Modeling Sharp Cracks in Two Dimensions

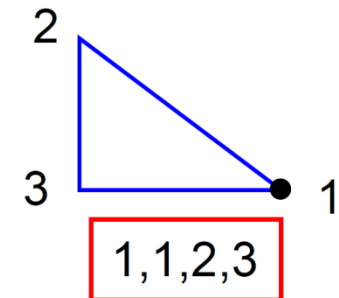
- Usage:

The crack tip nodes are independent:  $r^{-1}$  singularity



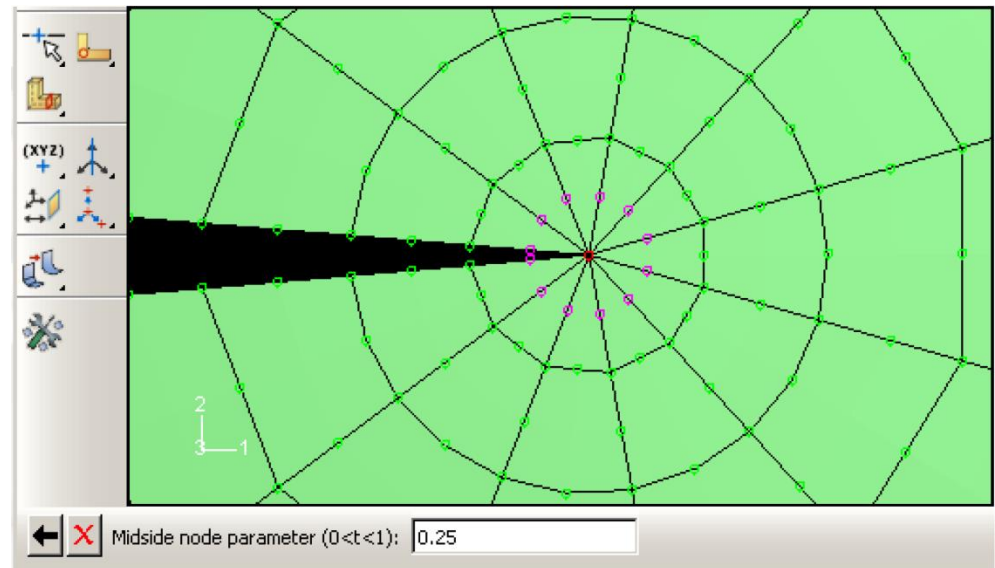
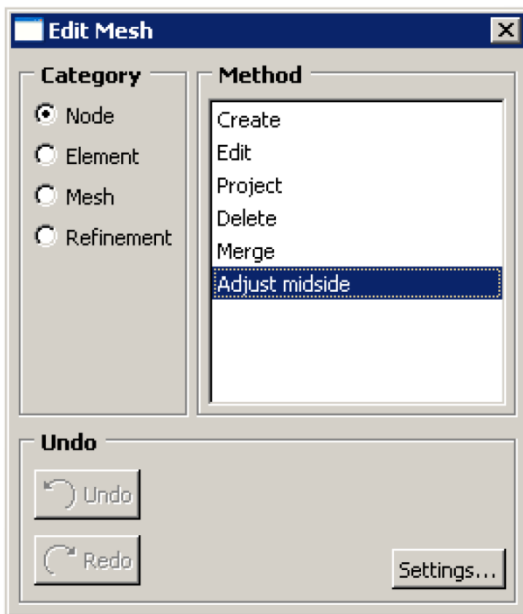
Quarter-point midside nodes on the sides connected to the crack tip

The crack tip nodes are constrained:  $r^{-1/2}$  singularity



# Modeling Sharp Cracks in Two Dimensions

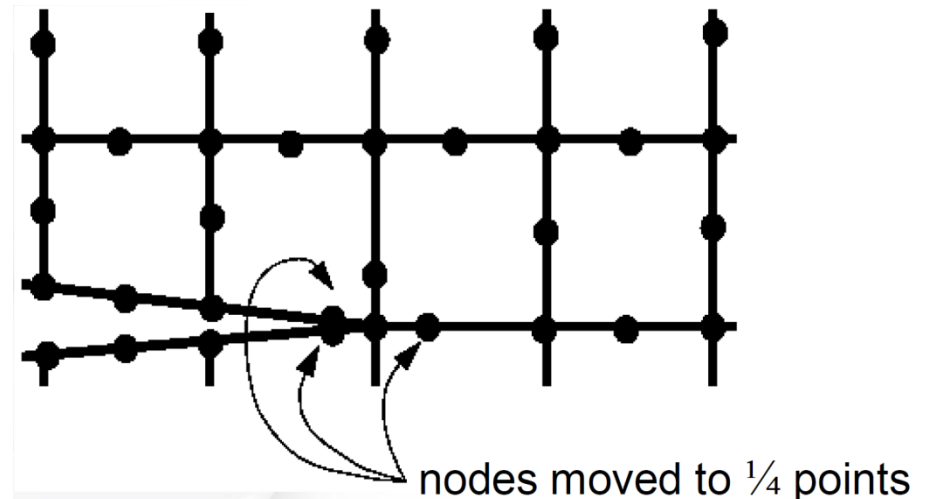
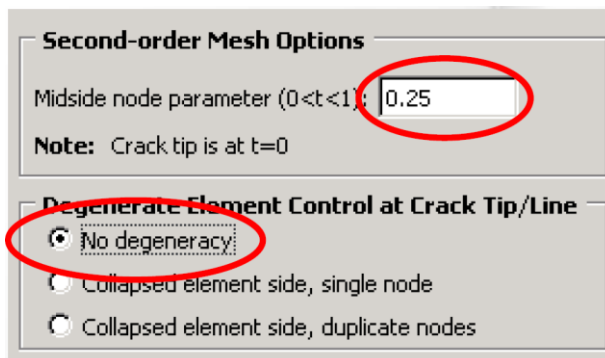
- Aside: Controlling the position of midside nodes for orphan meshes
  - Singularity controls cannot be applied to orphan meshes.
  - Use the **Mesh Edit** tools to adjust their position.





# Modeling Sharp Cracks in Two Dimensions

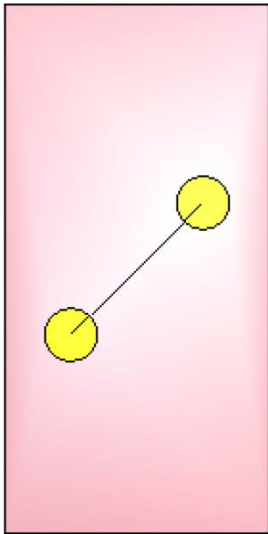
- If the side of the element is **not collapsed** but the midside nodes on the sides of the element connected to the crack tip are moved to the  $\frac{1}{4}$  point:
  - The strain is square root singular along the element edges but not in the interior of the element.
  - This is better than no singularity but not as good as the collapsed element.



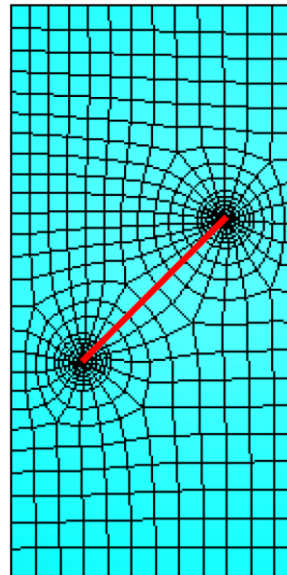
# Modeling Sharp Cracks in Two Dimensions

- Example: Slanted crack in a plate

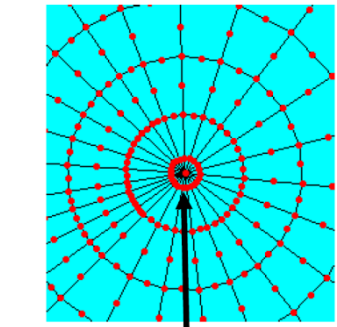
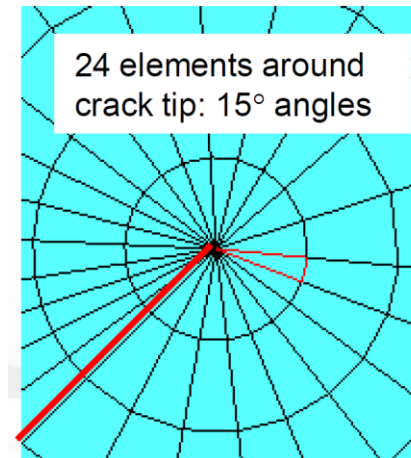
- To enable the creation of degenerate quads, you must create swept meshable regions around the crack tips (using partitions) and specify a quad-dominated mesh.



Quad-dominated mesh + swept technique for the circular regions surrounding the crack tips



Quadratic element type assigned to part



Quarter-point nodes

CPE8R elements; typical nodal connectivity shows repeated node at crack tip:

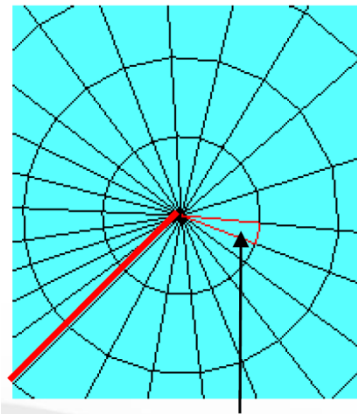
8, 8, 583, 588, 8, 1969, 1799, 1970

All crack-tip elements repeat node 8 in this example (nodes are constrained).

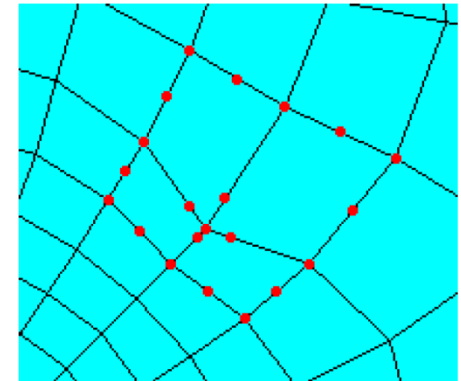
# Modeling Sharp Cracks in Two Dimensions

Example: Slanted crack in a plate; **Alternate meshes**

- No degeneracy:

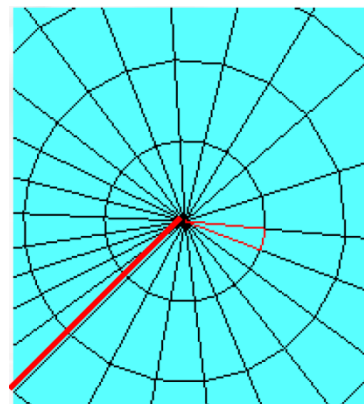
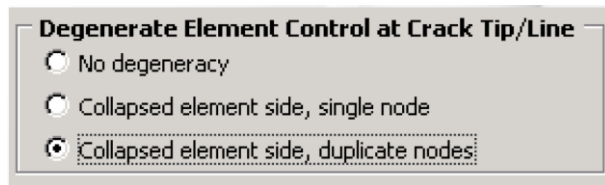


With swept meshable region: CPE6M elements at crack tip cannot be used for fracture studies in Abaqus.



With arbitrary mesh, singularity only along edges connected to crack tip.

- Degenerate with duplicate nodes:



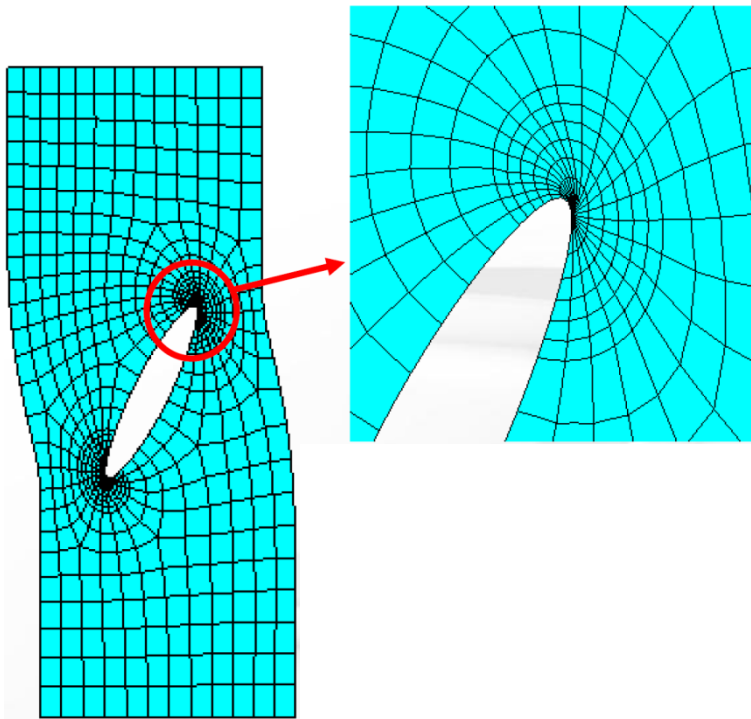
CPE8R elements at crack tip but no repeated nodes: 1993, 1992, 583, 588, 2016, ...

Coincident nodes located at crack tip

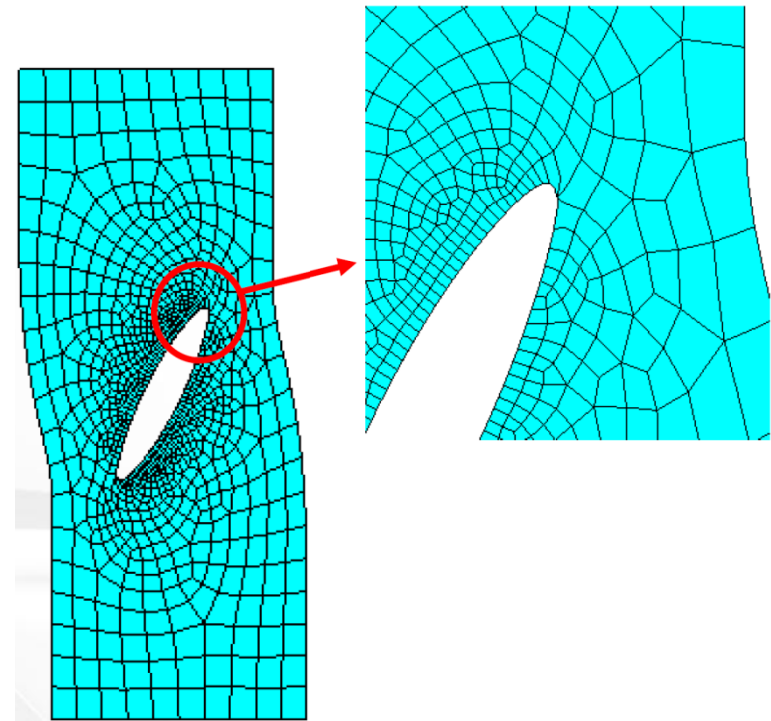


# Modeling Sharp Cracks in Two Dimensions

- Example: Slanted crack in plate; Deformed shape



• Focused mesh

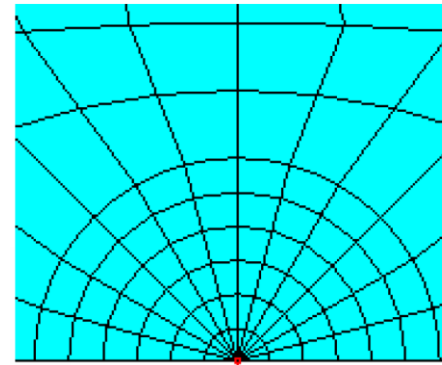


• Arbitrary mesh

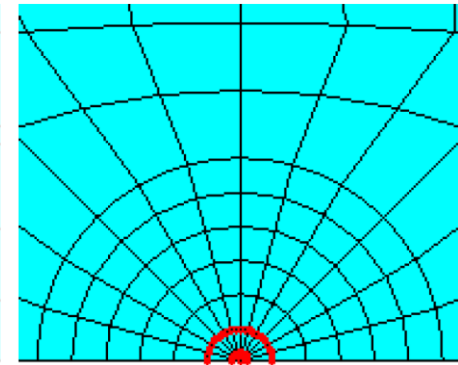
# Calculation of Contour Integrals

- Different contours (domains) are created automatically by Abaqus.
  - The first contour consists of the crack front and one layer of elements surrounding it.

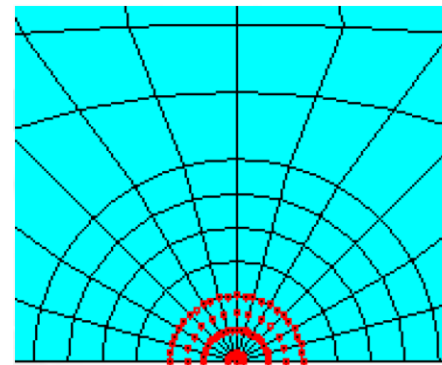
Ring of elements from one crack surface to the other (or the symmetry plane).
  - The next contour consists of the ring of elements in contact with the first contour as well as the elements in the first contour.
  - Each subsequent contour is defined by adding the next ring of elements in contact with the previous contour.



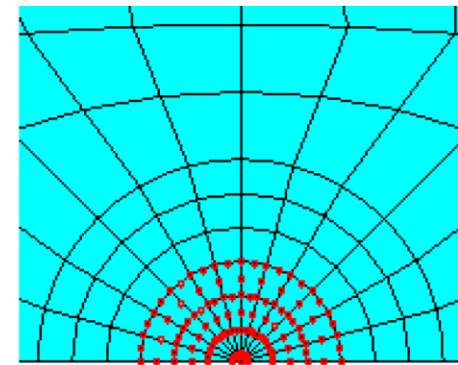
Contour 1



Contour 2

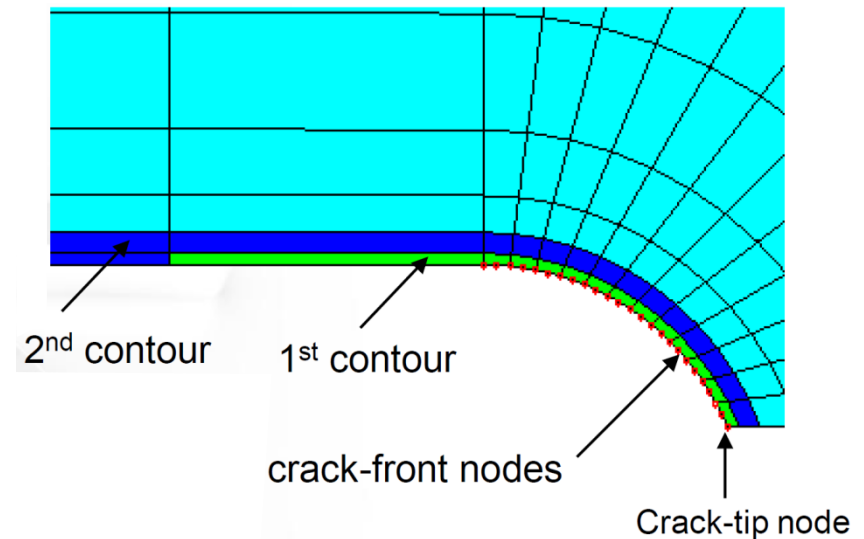
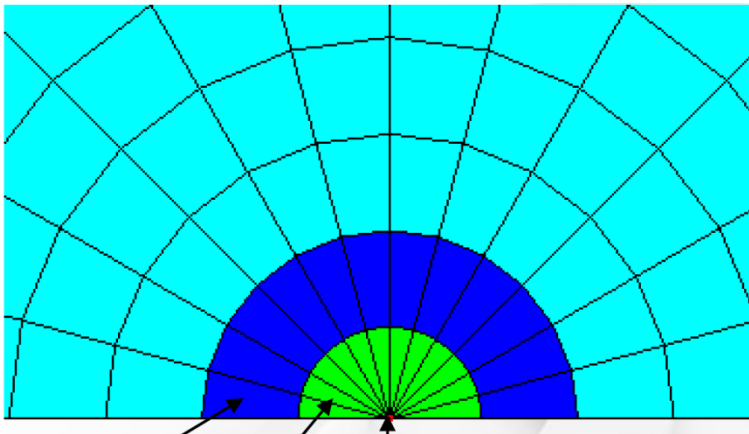


Contour 3



Contour 4

- The J-integral and the  $C_t$ -integral at steady-state creep should be path (domain) independent.
  - The value for the first contour is generally ignored.
- Examples of contour domains:



- Each subsequent contour is defined by adding the next ring of elements in contact with the previous contour.

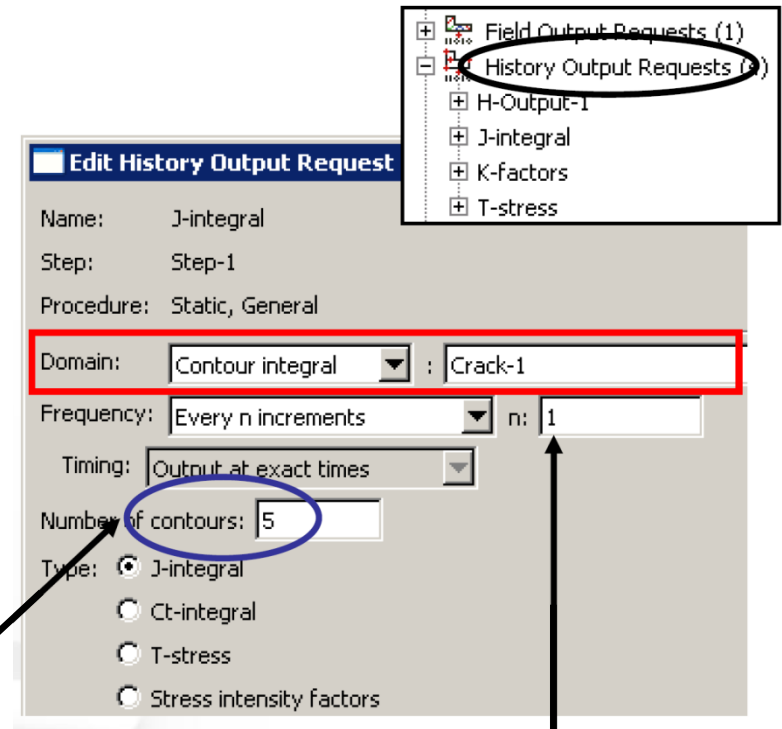


# Calculation of Contour Integrals

## • Usage:

**\*CONTOUR INTEGRAL, CONTOURS=** $n$   
**TYPE={J, C, T STRESS, K FACTORS}**  
**DIRECTION = {MTS, MERR, KII0}**

Specifies the number of contours (domains) on which the contour integral will be calculated



This is the output frequency in increments

- Note: In this lecture, we focus on the output-specific parameters of the \*CONTOUR INTEGRAL option. The crack-specific parameters SYMM and NORMAL were discussed in the previous lecture.



# Calculation of Contour Integrals

## •Usage:

**\*CONTOUR INTEGRAL, CONTOURS=  $n$ ,  
TYPE={J, C, T STRESS, K FACTORS},  
DIRECTION = {MTS, MERR, KII0}**

- J for J-integral output,
- C for  $C_t$ -integral output.
- T STRESS to output T-stress calculations
- K FACTORS for stress intensity factor output

**Edit History Output Request**

Name: J-integral  
Step: Step-1  
Procedure: Static, General

Domain: Contour integral : Crack-1

Frequency: Every n increments n: 1

Timing: Output at exact times

Number of contours: 5

Type:  J-integral  
 Ct-integral  
 T-stress  
 Stress intensity factors



# Calculation of Contour Integrals

## •Usage:

```
*CONTOUR INTEGRAL, CONTOURS= n,  
TYPE={J, C, T STRESS, K FACTORS},  
DIRECTION = {MTS, MERR, KII0}
```

Three criteria to calculate the crack propagation direction at initiation

Step: Step-1  
Procedure: Static, General  
Domain: Contour integral : Crack-1  
Frequency: Every n increments n: 1  
Timing: Output at exact times  
Number of contours: 5  
Type:  J-integral  
 Ct-integral  
 T-stress  
 Stress intensity factors  
Crack initiation criterion:  Maximum tangential stress  
 Maximum energy release rate  
 KII=0

Use with TYPE=KFACTORS to specify the criterion to be used for estimating the crack propagation direction in homogenous, isotropic, linear elastic materials:

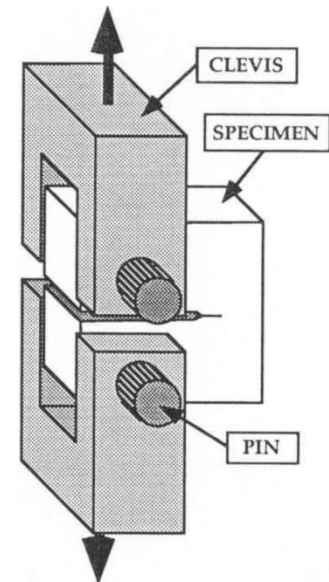
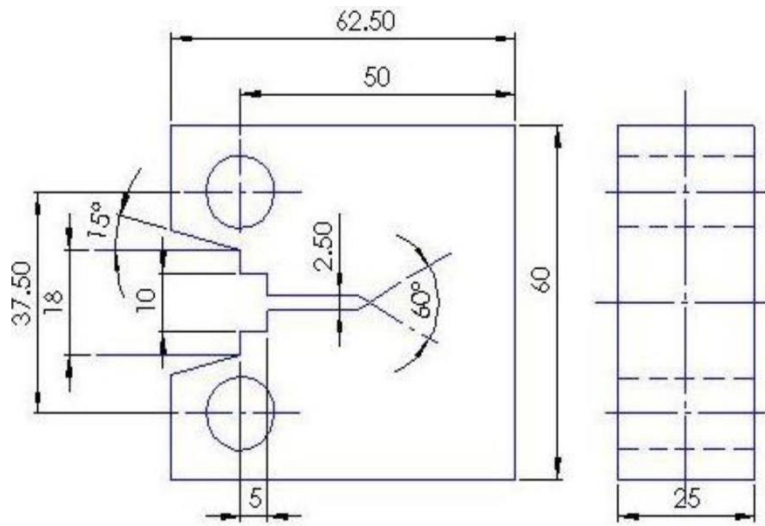
- Maximum tangential stress criterion (MTS)
- Maximum energy release rate criterion (MERR)
- $K_{II}=0$  criterion (KII0)



# Calculation of Contour Integrals

- Loads
  - Loads included in contour integral calculations:
    - Thermal loads.
    - Crack-face pressure and traction loads on continuum elements as well as those applied using user subroutines **DLOAD** and **UTRACLOAD**.
    - Surface traction and crack-face edge loads on shell elements as well as those applied using user subroutine **UTRACLOAD**.
    - Uniform and nonuniform body forces.
    - Centrifugal loads on continuum and shell elements.
  - Not all types of distributed loads (e.g., hydrostatic pressure and gravity loads) are included in the contour integral calculations.
  - The presence of these loads will result in a warning message.

- **Example**
- Compact Tension Specimen
- This is one of five standardized specimens defined by the ASTM for the characterization of fracture initiation and crack growth.
- The ASTM standardized testing apparatus uses a clevis and a pin to hold the specimen and apply a controlled displacement.

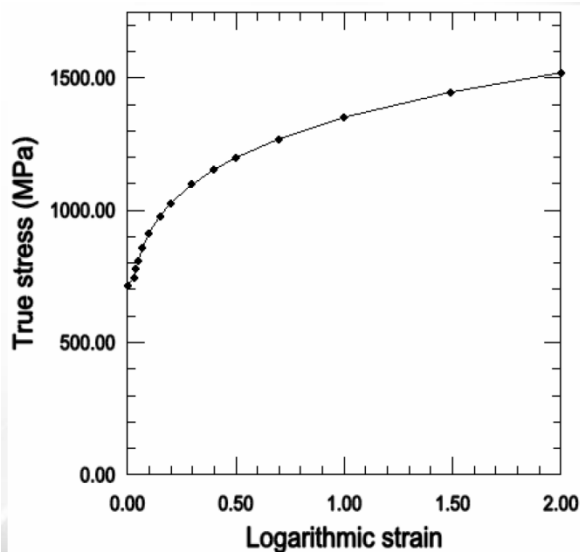




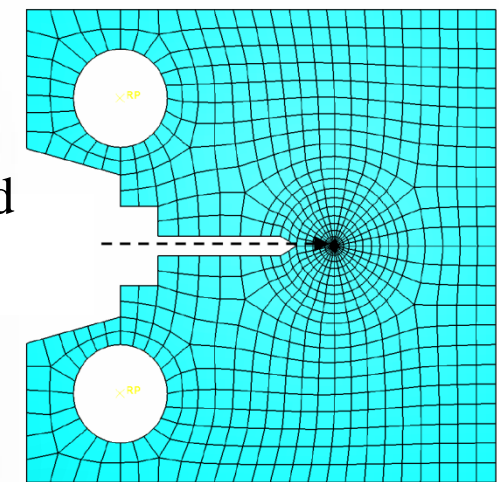
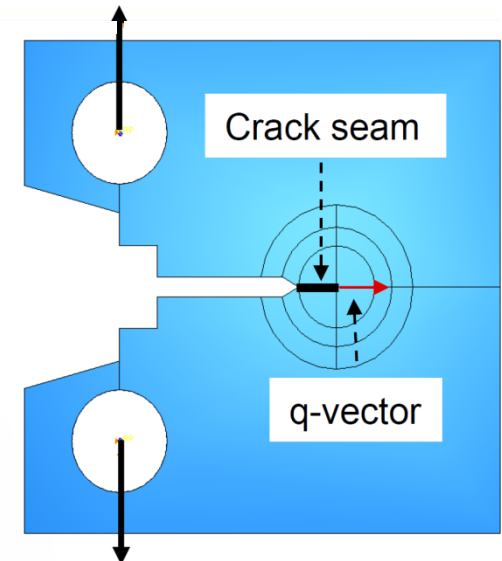
# Calculation of Contour Integrals

- Example**

- Plane strain conditions assumed.
- The initial crack length is 5 mm.
- Elastic-plastic material

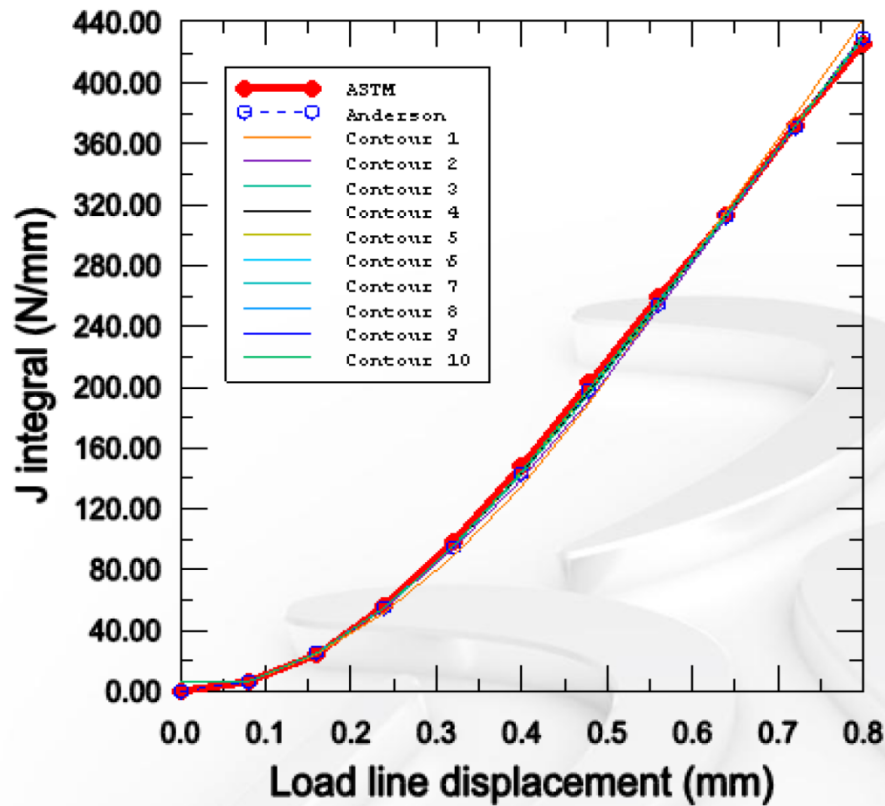


Prescribed load line displacement

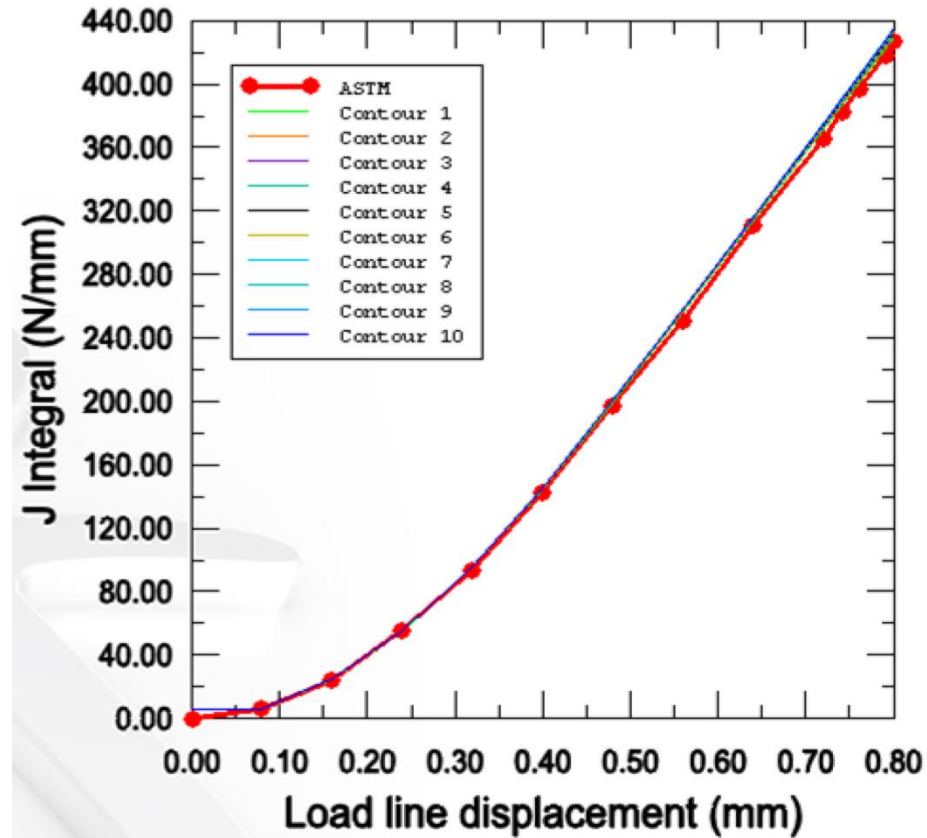


$1/\sqrt{r}$  singularity modeled  
in the crack-tip elements

- Results

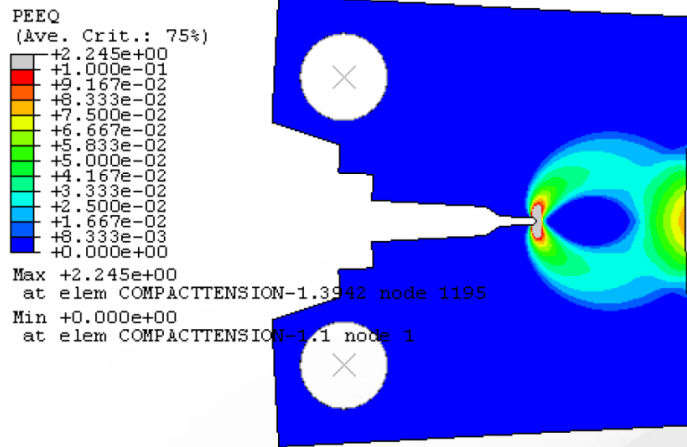


- Small strain analysis

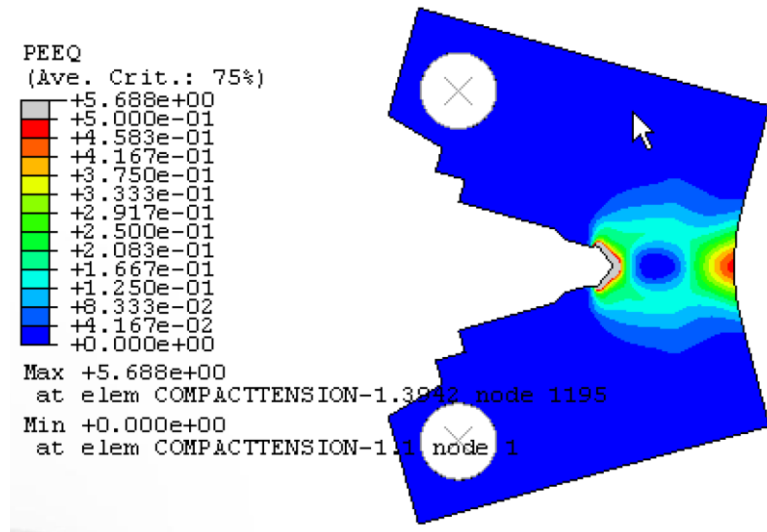


- Finite strain analysis

## • Results



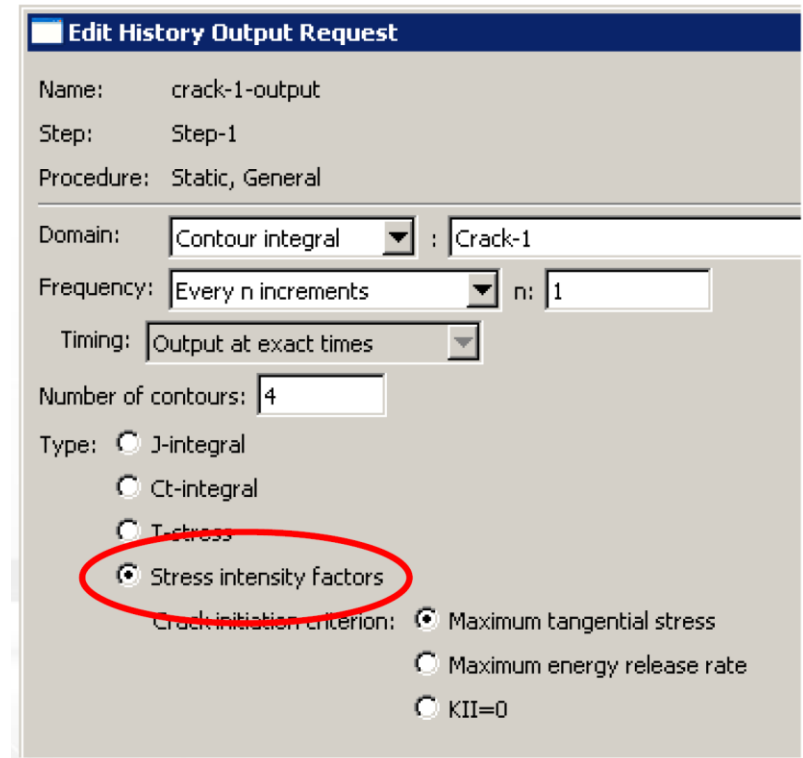
- At small to moderate strain levels, the small and finite strain models yield similar results.



- Finite strain effects must be considered to represent this level of deformation and strain accurately.

# Mixed-Mode Fracture

- Abaqus uses interaction integrals to compute the stress intensity factors.
- This approach accounts for mixed-mode loading effects.
- Note that the J-or  $C_t$ -integrals do not distinguish between modes of loading.
- Usage:
  - \*CONTOUR INTEGRAL,  
TYPE=K FACTORS
- Stress intensity factors can only be calculated for linear elastic materials.



**Edit History Output Request**

Name: crack-1-output  
Step: Step-1  
Procedure: Static, General

Domain: Contour integral : Crack-1

Frequency: Every n increments n: 1

Timing: Output at exact times

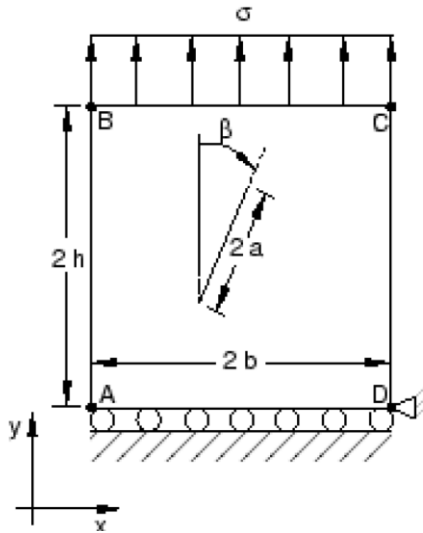
Number of contours: 4

Type:  J-integral  
 Ct-integral  
 T-stress  
 Stress intensity factors

Crack initiation criterion:  Maximum tangential stress  
 Maximum energy release rate  
 KII=0

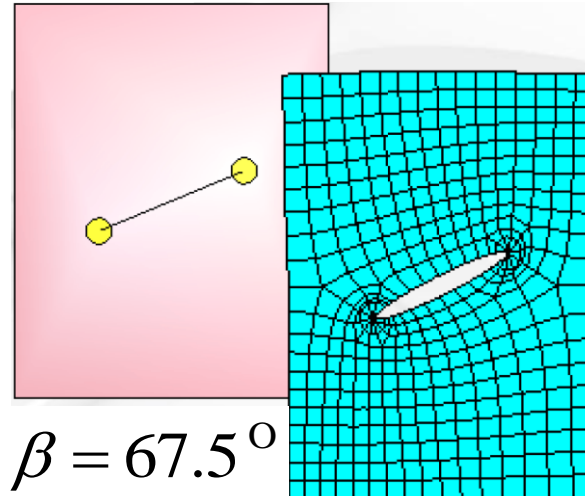
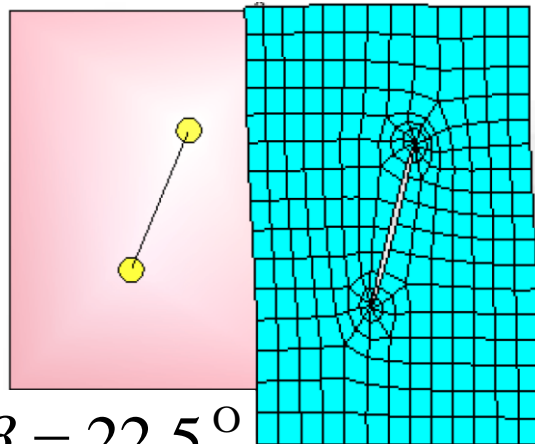
# Mixed-Mode Fracture

- Example:** Center slant cracked plate under tension



$\beta = 22.5^\circ, 67.5^\circ$   
 $a/b = 0.5$   
 $h/b = 1.25$   
 $b = 50.0 \text{ mm}$

$\beta$	Element type	$\frac{K_I}{K_0}$	$\frac{K_{II}}{K_0}$
22.5°	CPE8	0.185 (-2.9%)*	0.403 (-0.2%)
22.5°	CPE8R	0.185 (-2.9%)	0.403 (-0.2%)
67.5°	CPE8	1.052 (+3.6%)	0.373 (+1.0%)
67.5°	CPE8R	1.053 (+3.8%)	0.374 (+1.3%)



$$K_0 = \sigma \sqrt{\pi a}$$

\*Values enclosed in parentheses are percentage differences with respect to the reference solution. See Abaqus Benchmark Problem 4.7.4 for more information.