دانشگاه صنعتی اصفهان

دانشکده مکانیک

Computational Fracture Mechanics (3)

Computational fracture mechanics

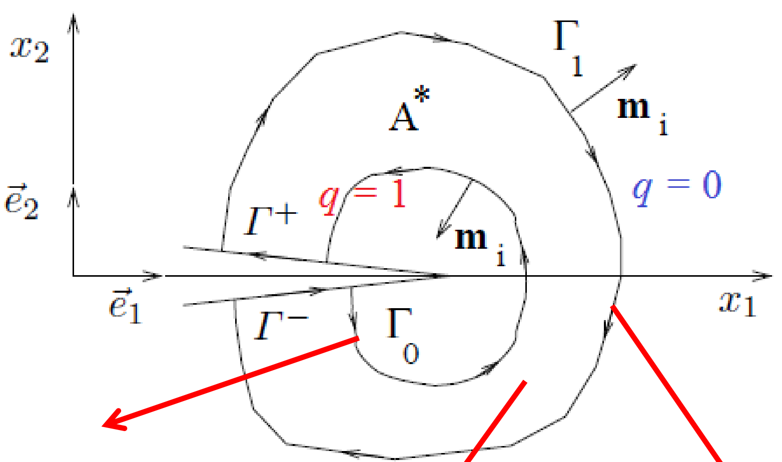
- Introduction to Finite Element method
- Singular Stress Finite Elements
- Extraction of K (SIF), G
- J integral
- Finite Element mesh design for fracture mechanics
- Computational crack growth
- Traction Separation Relations

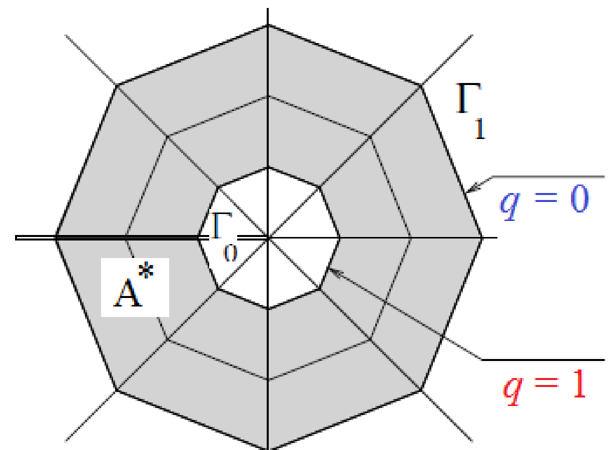
J integral

- Energy Domain Integral

Divergence theorem: Line/Surface (2D/3D) integral → Surface/Volume Integral

Application in FEM meshes





Original J integral contour

Surface integral after using divergence theorem

$\Gamma_0 \to 0$ ➡ 2D mesh covers crack tip

Contour integral added to create closed surface

By using $q = 0$ this integral in effect is zero

J integral

- **Energy Domain Integral**

Divergence theorem: Line/Surface (2D/3D) integral ⟶ Surface/Volume Integral

$$J=\int_{\Gamma_0}\left[w\,\delta_{1i}-\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right]n_i\,d\Gamma$$

$$\int_L f_{ij}\,n_i\,dl=\int_A\frac{\partial}{\partial x_i}(f_{ij})\,dA$$

$$=\int_{\Gamma^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\,\delta_{1i}\right]q m_i\,d\Gamma-\int_{\Gamma^++\Gamma^-}\sigma_{2j}\frac{\partial u_j}{\partial x_1}q\,d\Gamma$$

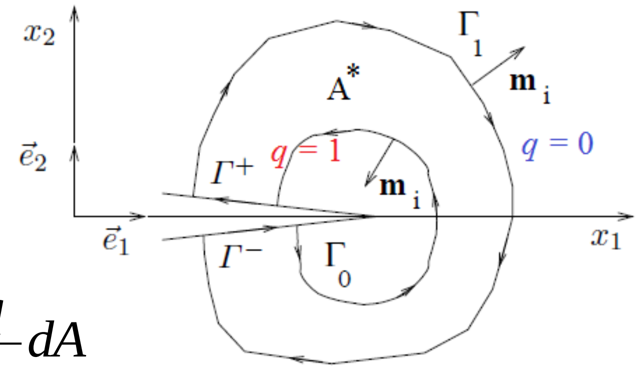
Zero integral on Γ_1 ($q=0$)

$\Gamma^*=\Gamma_1+\Gamma^++\Gamma^-+\Gamma_0$

Note that $m_i=-n_i$ on Γ_0 ; also, $m_1=0$ and $m_2=\pm1$ on Γ^+ and Γ^-.

Divergence theorem (assume that the crack faces are traction free):

$$J=\int_{A^*}\frac{\partial}{\partial x_i}\left\{\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\,\delta_{1i}\right]q\right\}dA$$

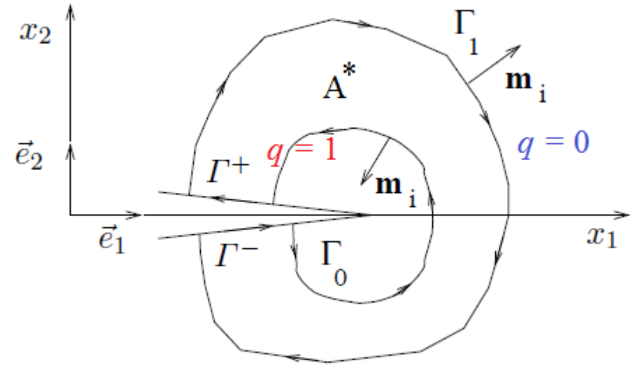


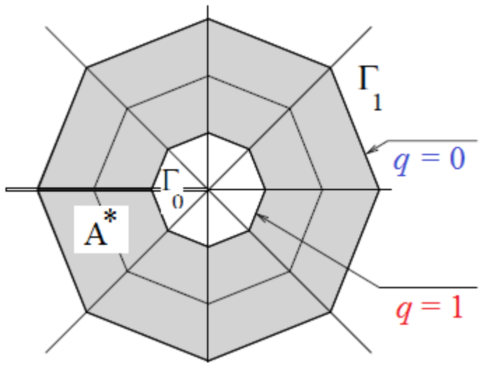
$$J=\int_{A^*}\left[\frac{\partial}{\partial x_i}\left(\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)-\frac{\partial w}{\partial x_i}\right]q\,dA+\int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1}-w\,\delta_{1i}\right]\frac{\partial q}{\partial x_i}dA$$

- Energy Domain Integral

Referring to Appendix 3A.2: $\frac{\partial}{\partial x_i}\left(\sigma_{ij}\frac{\partial u_j}{\partial x_1}\right)-\frac{\partial w}{\partial x_i}=0$ (Page 23 Lesson 10)

$$w = w^e + w^p$$
$$= \int_0^{\varepsilon_{kl}^e} \sigma_{ij}\, d\varepsilon_{ij}^e + \int_0^{\varepsilon_{kl}^p} \sigma_{ij}{}^D\, d\varepsilon_{ij}^p$$





General form of J

$$J = \int_{A^*}\left\{\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\,\delta_{1i}\right]\frac{\partial q}{\partial x_i} + \left[\sigma_{ij}\frac{\partial \varepsilon_{ij}^p}{\partial x_1} - \frac{\partial w^p}{\partial x_1} + \alpha\sigma_{ii}\frac{\partial \Theta}{\partial x_1} - F\frac{\partial u_j}{\partial x_1}\right]q\right\}dA$$
$$-\int_{\Gamma^+ + \Gamma^-} \sigma_{2j}\frac{\partial u_j}{\partial x_1}\, q\, d\Gamma$$

Plasticity effects

Thermal effects

Body force

Nonzero crack surface traction

- Energy Domain Integral

Simplified Case:

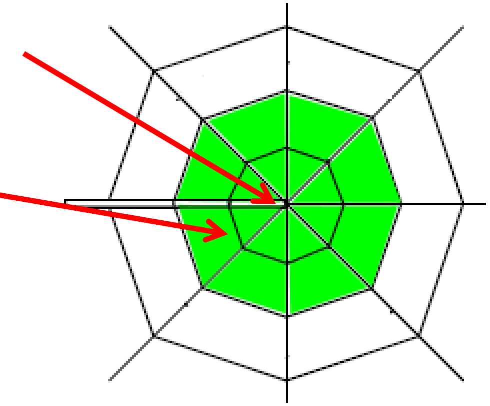
(Nonlinear) elastic, no thermal strain, no body force, traction free crack surfaces

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\,\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

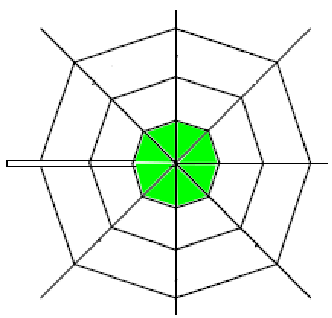
J integral

Energy Domain Integral FEM Aspects

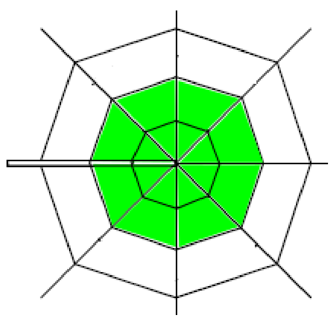
- Since $J_0 \longrightarrow 0$ the inner J_0 collapses to the crack tip (CT)
- J_1 will be formed by element edges
- By using spider web (rozet) meshes any reasonable number of layers can be used to compute J:



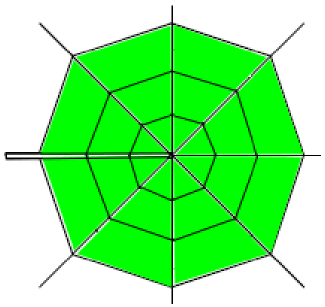
1 layer



2 layer



3 layer

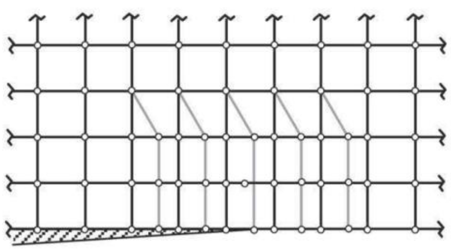


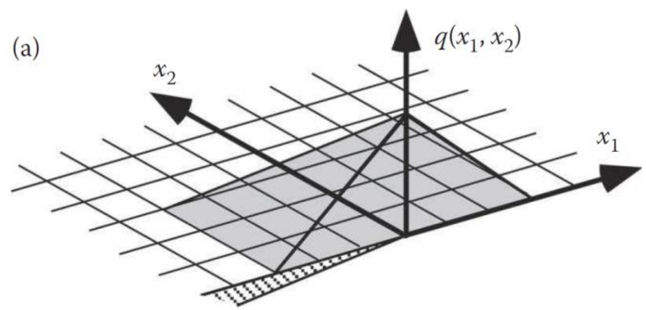
- Spider web (rozet) mesh:
  - One layer of triangular elements (preferably singular, quadrature point elements)
  - Surrounded by quad elements

J integral

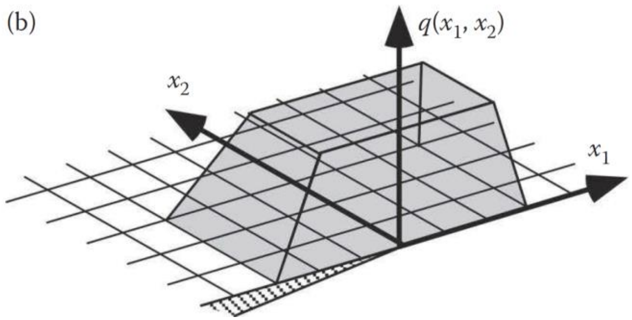
Energy Domain Integral FEM Aspects

- Shape of decreasing function *q*:



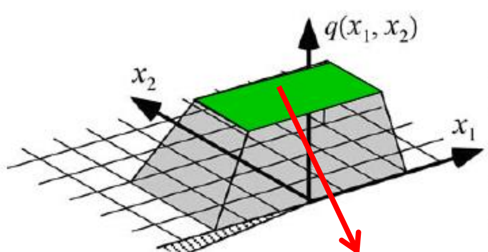


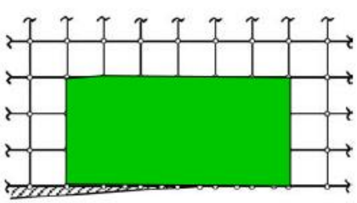
Pyramid *q* function



Plateau *q* function

- Plateau *q* function useful when inner elements are not very accurate: e.g. when singular/quarter point elements are not used





$$J = \int_{A^*}\left[\sigma_{ij}\frac{\partial u_j}{\partial x_1} - w\,\delta_{1i}\right]\frac{\partial q}{\partial x_i}\,dA$$

$$\frac{\partial q}{\partial x_i} = 0$$

These elements do not contribute to J

Computational fracture mechanics

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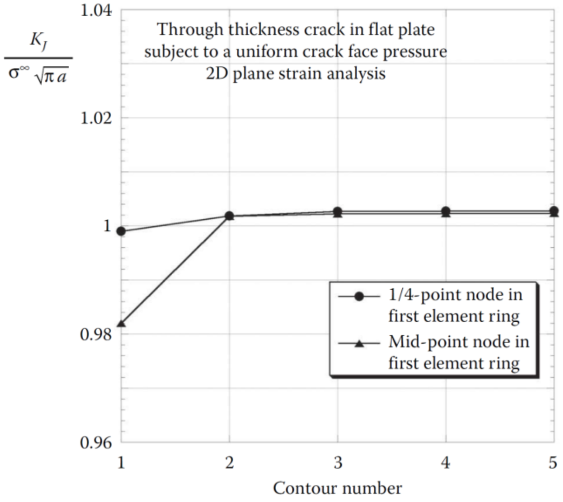
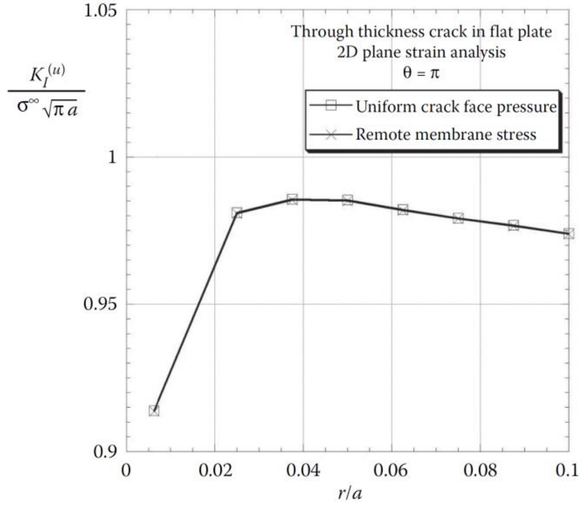
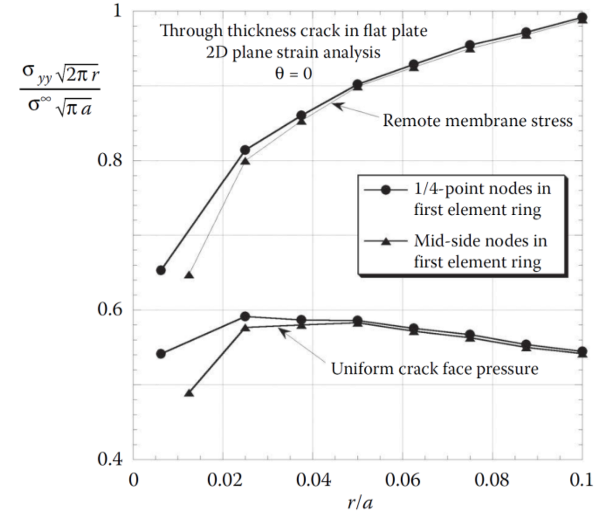
Different elements/methods to compute K

J integral EDI







J integral EDI

$$J = \int_{A^*} \left[\sigma_{ij} \frac{\partial u_j}{\partial x_1} - w\,\delta_{1i} \right] \frac{\partial q}{\partial x_i} dA$$

K from displacement *u*

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \qquad (\theta = \pi)$$

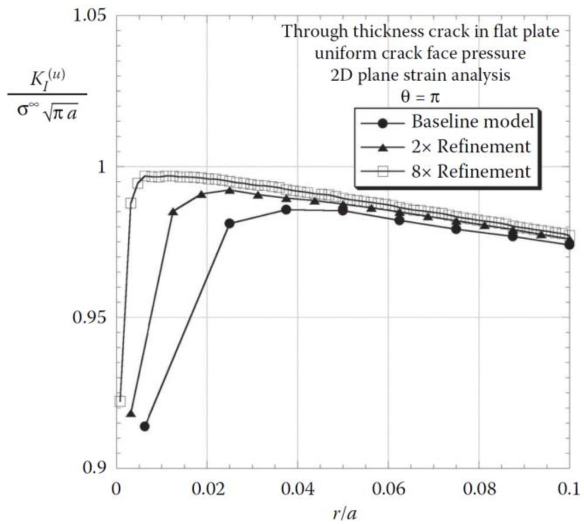
K from stress σ

$$K_I = \lim_{r \to 0} \left(\sqrt{2\pi r}\, \sigma_{22}\big|_{\theta=0} \right)$$

- *J* integral EDI method is by far the most accurate method
- Interpolation of K from ***u*** is more accurate from σ: 1) higher convergence rate, 2)nonsingular field. Unlike σ it is almost insensitive to surface crack or far field loading
- Except the first contour (*J* integral) or very small *r* the choice of element has little effect

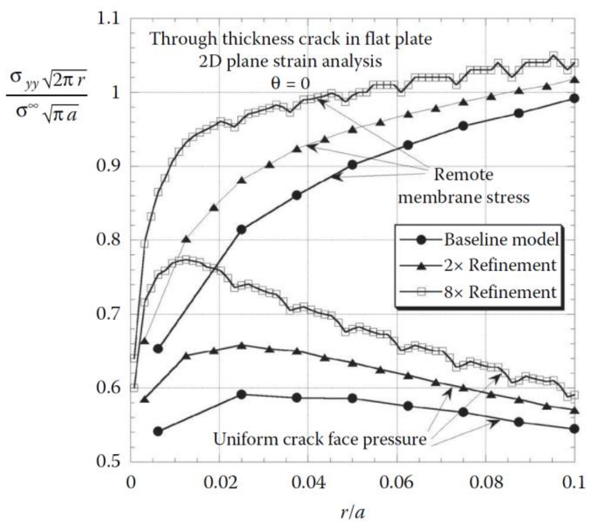
Different elements/methods to compute K

- Effect of adaptivity on local field methods



K from displacement ***u***

$$K_I = \lim_{r \to 0} \left[\frac{E' u_y}{4} \sqrt{\frac{2\pi}{r}} \right] \qquad (\theta = \pi)$$



K from stress σ

$$K_I = \lim_{r \to 0} \left(\sqrt{2\pi r}\, \sigma_{22}\big|_{\theta=0} \right)$$

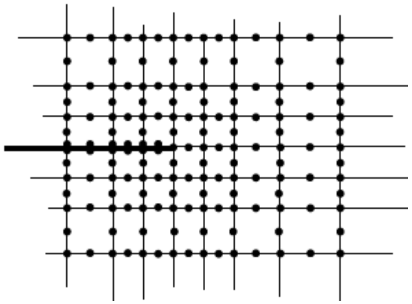
- Even element h-refinement cannot improve K values by much particularly for stress based method

General Recommendations

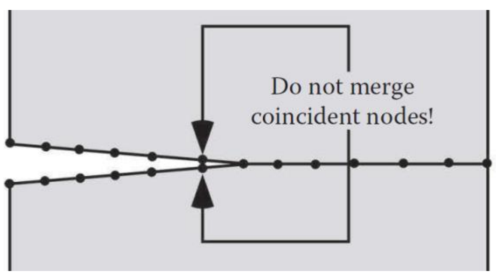
Crack surface meshing

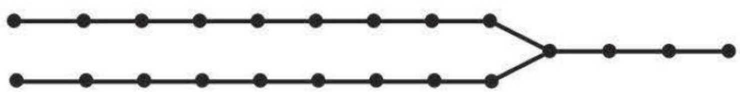
Nodes in general should be duplicated:

- Modern FEM can easily handle duplicate nodes

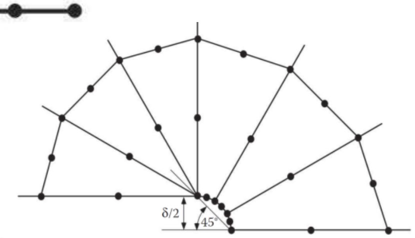


- If not, small initial separation is initially introduced

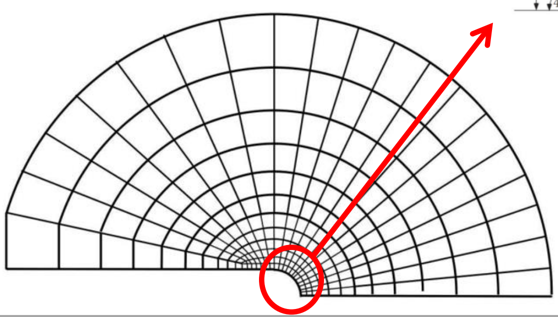




A small but finite gap between crack faces avoids having coincident nodes



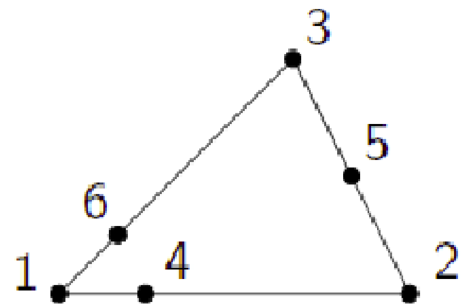
- When large strain analysis is required, initial mesh has finite crack tip radius. The opening should be smaller than 5-10 times smaller than CTOD.



General Recommendations

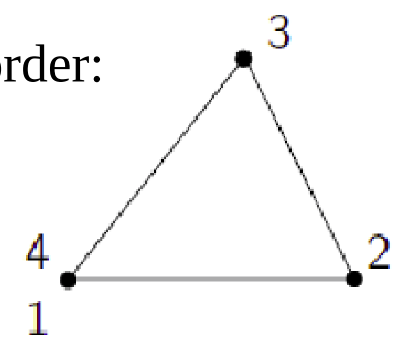
- Quarter point vs mid point elements / Collapsed elements around the crack tip

- For LEFM singular elements triangle quarter elements are better than normal tri/quad, and quarter point quad elements (collapsed or not)

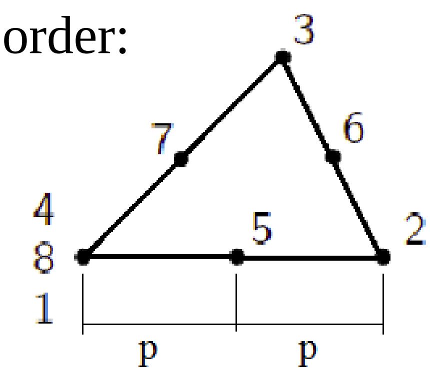


- For elastic perfectly plastic material collapsed quad elements (1^{st} / 2^{nd} order) are recommended

1^{st} order:



2^{nd} order:



- Use of crack tip singular elements are more important for local field interpolation methods (***u*** and σ). EDI J integral method is less sensitive to accuracy of the solution except the 1^{st} contour is used.

General Recommendations

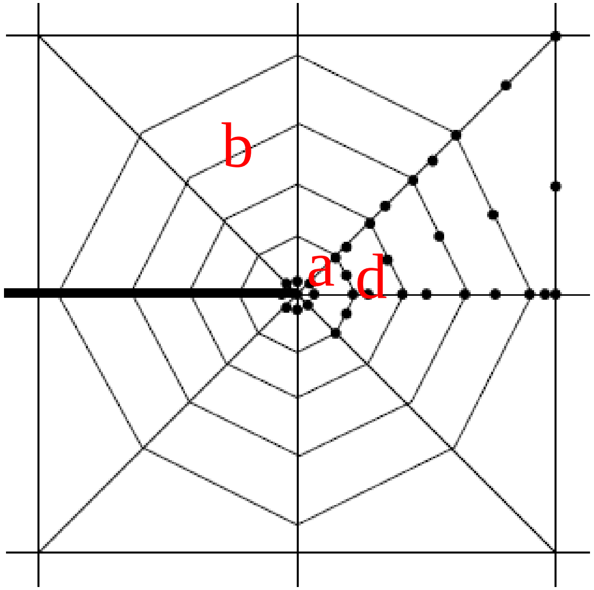
- Shape of the mesh around a crack tip

a) Around the crack tip triangular singular elements are recommended (little effect for EDI J integral method)

b) Use quad elements (2nd order or higher) around the first contour

c) Element size: Enough number of elements should be used in region of interest: r_s, r_p, large strain zone, etc.

d) Use of transition elements away from the crack tip although increases the accuracy has little effect

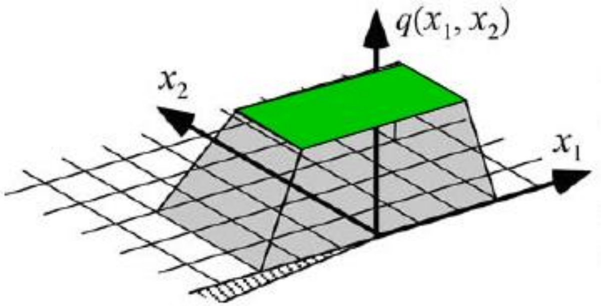


Spider-web mesh (rozet)

General Recommendations

- Method for computing K

  - Energy methods such as J integral and G virtual crack extension (virtual stiffness derivative) are more reliable
    - J integral EDI is the most accurate and versatile method
      - Least sensitive method to accuracy of FEM solution at CT particularly if plateau q is used



  - K based on local fields is the least accurate and most sensitive to CT solution accuracy.
    - Particularly stress based method is not recommended.
    - Singular/ quarter point elements are recommended for these methods especially when *K* is obtained at very small *r*.