دانشگاه صنعتی اصفهان

دانشکده مکانیک

Experimental Stress Analysis for Determination of Fracture Toughness

Experimental Stress Analysis Methods

- The different methods used for experimental determination of stress intensity factors belong to a broader domain called *experimental stress analysis*. Here, we briefly review the usage of electrical resistance strain gages and the method of photoelasticity for determination of stress intensity factors.

Experimental Stress Analysis Methods

- electrical resistance strain gages
- photoelasticity method
- Ultrasonic Methods
- Moire Method
- Brittle Coating
- Digital Image Correlation . . .

References:

Modern Experimental Stress Analysis , By: James F. Doyle

Experimental Stress Analysis For Materials And Structures: Stress Analysis ...
By Alessandro Freddi, Giorgio Olmi, Luca Cristofolini

- The stress intensity factor can be determined experimentally by placing one or more strain gages near the crack tip. However, to avoid severe strain gradients, the gages should not be placed at very near field. On the other hand, as we move further from the crack tip, we need more terms to be able to express the field parameters correctly. The three-term representation of the strain field is:

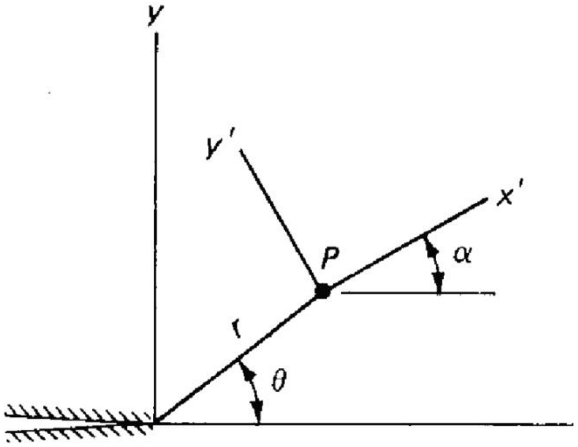
$$E\varepsilon_{xx} = A_o r^{-\frac{1}{2}}\cos(\frac{\theta}{2})\left[(1-\nu)-(1+\nu)\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})\right]+2B_0+A_1 r^{\frac{1}{2}}\cos(\frac{\theta}{2})\left[(1-\nu)-(1+\nu)\sin^2(\frac{\theta}{2})\right]$$

$$E\varepsilon_{yy} = A_o r^{-\frac{1}{2}}\cos(\frac{\theta}{2})\left[(1-\nu)-(1+\nu)\sin(\frac{\theta}{2})\sin(\frac{3\theta}{2})\right]-2\nu B_0+A_1 r^{\frac{1}{2}}\cos(\frac{\theta}{2})\left[(1-\nu)-(1+\nu)\sin^2(\frac{\theta}{2})\right]$$

$$2\mu\gamma_{xy} = A_o r^{-\frac{1}{2}}\left[\sin\theta\cos(\frac{3\theta}{2})\right]-A_1 r^{\frac{1}{2}}\left[\sin\theta\cos(\frac{\theta}{2})\right]$$

where A_0 , B_0 , and A_1 are unknown coefficients which depend on loading and the geometry of the specimen. For instance we have: $A_o = \frac{K_I}{\sqrt{2\pi}}$

- In general, we need three strain gages to be able to determine the above three unknowns. However, it can be shown that it is possible to use only one gage oriented at angle and positioned along the P_x axis:



- Accordingly, the stress intensity factor can be determined from:

$$2\mu E\,\varepsilon_{x'x'} = \frac{K_I}{\sqrt{2\pi r}}\left[k\,\cos\left(\frac{\theta}{2}\right) - \frac{1}{2}\sin\theta\sin\left(\frac{3\theta}{2}\right)\cos 2\alpha + \frac{1}{2}\sin\theta\cos\left(\frac{3\theta}{2}\right)\sin 2\alpha\right]$$

where: $k = \dfrac{1-\nu}{1+\nu}$

- The choice of the angles and depend on the Poisson's ratio and can be determined from the following table:

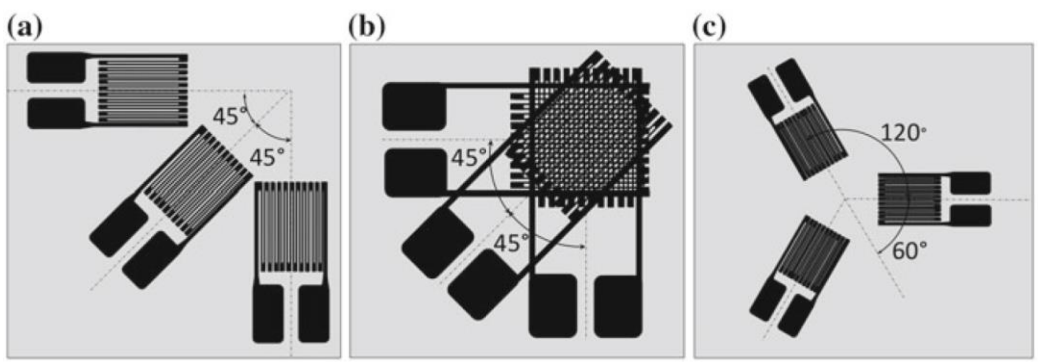
Strain Gage Method

- For the choice $\alpha = \vartheta = 60^o$, the required expression is very simple:

$$K_I = E\sqrt{\frac{8\pi}{3} r\varepsilon_{x'x'}}$$

Angles α and θ as a function of Poisson's ratio ν

ν	θ, deg	α, deg
0.250	73.74	63.43
0.300	65.16	61.29
0.333	60.00	60.00
0.400	50.76	57.69
0.500	38.97	54.74

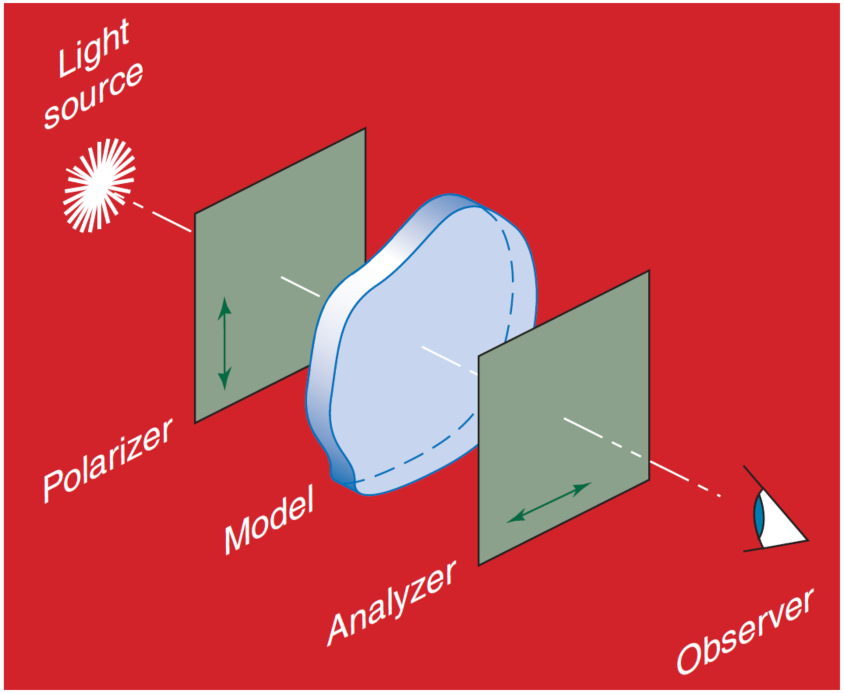


Schematic representation of three-gage Rosettes with different layout of the strain gages at 45 and 60/120° with separated or stacked grids.

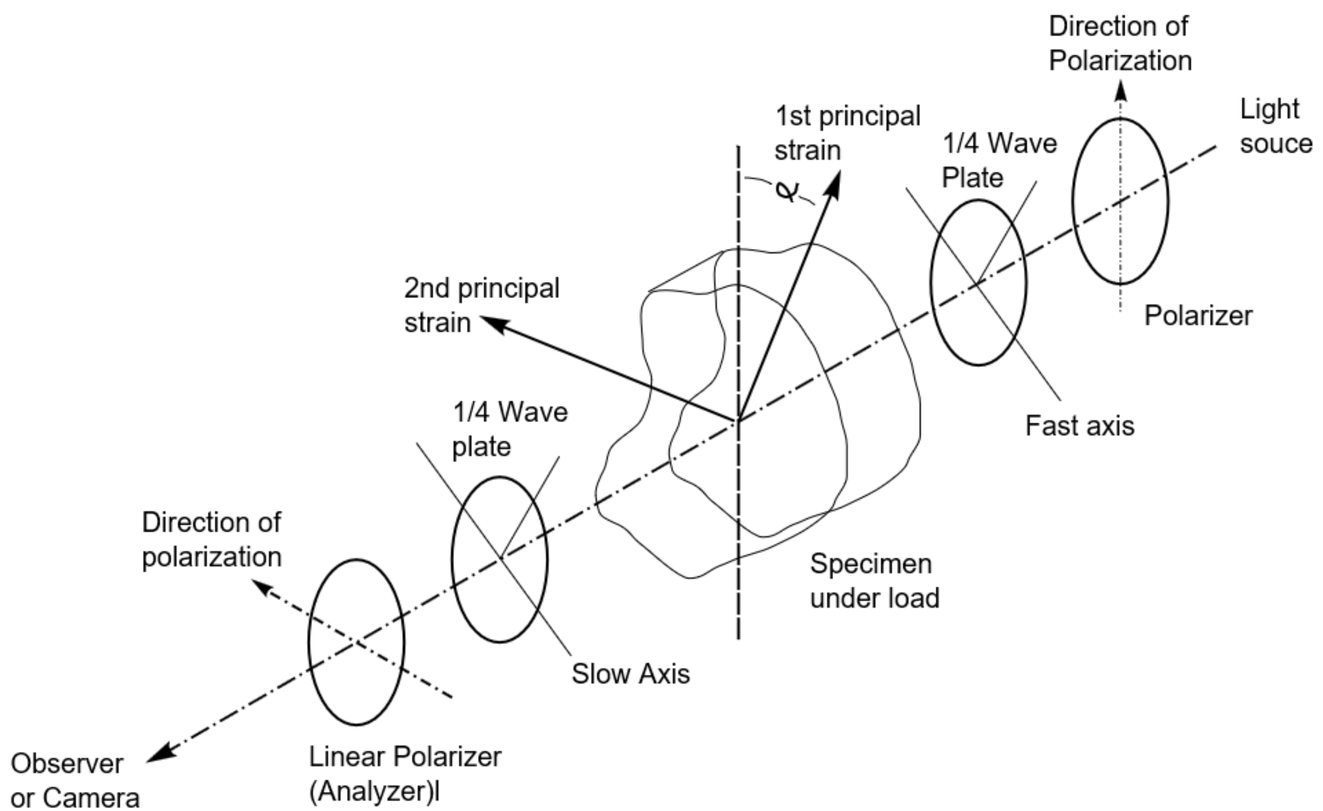
Photoelasticity Method

- Photoelasticity is a whole-field stress analysis technique based on an optical-mechanical property, possessed by many transparent polymers. A loaded photoelastic specimen, combined with other optical elements, and illuminated with an ordinary light source exhibits fringe patterns that are related to the difference between the principal stresses in a plane normal to the light propagation direction.

- A polariscope is needed for viewing the fringes induced by the stresses. Two types of pattern can be obtained: isochromatics and isoclinics. The former is related to the principal-stress differences and the latter to the principal stress directions.



Photoelasticity Method

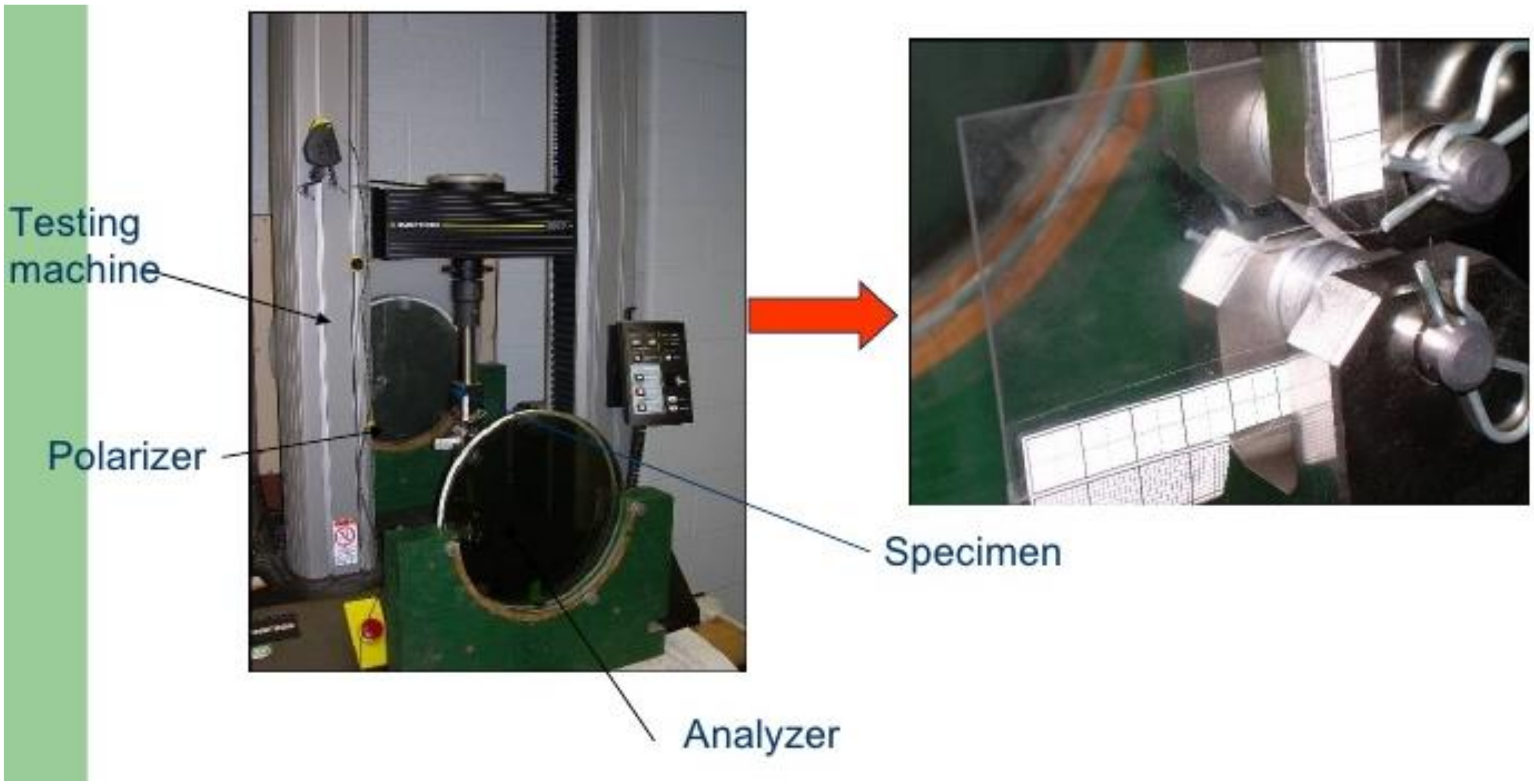


Transmission Circular Polariscope

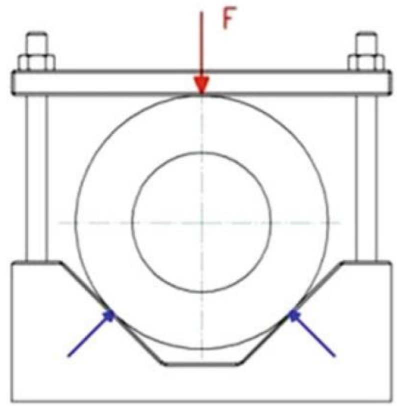
The same device functions as a plane polariscope when quarter wave plates are taken aside or rotated so their axes parallel to polarization axes

Photoelasticity Method

Experimental Set-up



Photoelasticity Method



Isochromatics on a black and bright background of an annular disc loaded at three points

Photoelasticity Method

- The sensitivity of a photoelastic material is characterized by its fringe constant f_σ . This constant relates the value N associated with a given fringe to the thickness h of the specimen in the light-propagation direction and the difference between the principal stresses in the plane normal to the light-propagation direction as follows:

$$\sigma_1 - \sigma_2 = \frac{Nf_\sigma}{h} \qquad (*)$$

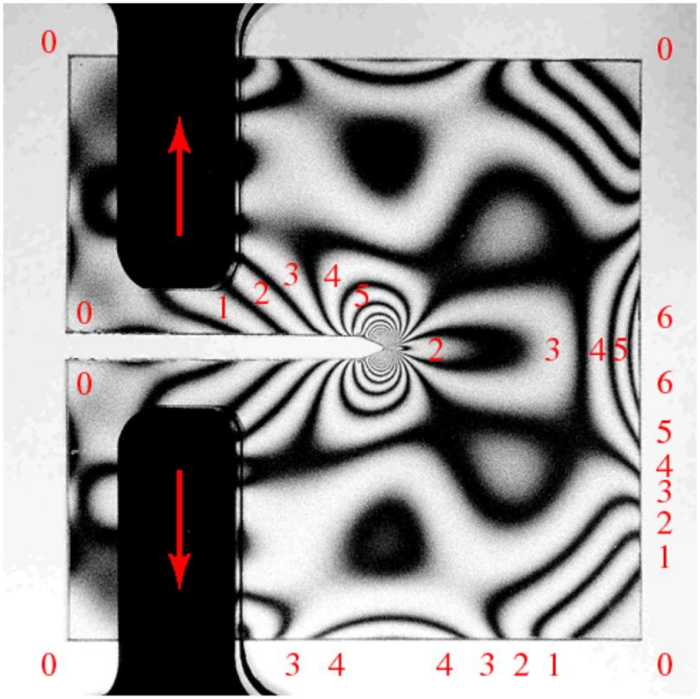
- In practice, the principal stresses are obtained from the stresses defined:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \\ \sigma_{22} \end{Bmatrix} = \frac{K_I}{(2\pi r)^{1/2}} \cos(\theta/2) \begin{Bmatrix} 1 - \sin(\theta/2)\sin(3\theta/2) \\ \sin(\theta/2)\cos(3\theta/2) \\ 1 + \sin(\theta/2)\sin(3\theta/2) \end{Bmatrix}$$

and combined with Eq. (*) to give:

$$\sigma_1 - \sigma_2 = 2\sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}} = \frac{K_I}{\sqrt{2\pi r}}\left|\sin\theta\right| \quad \Rightarrow \quad K_I = \frac{\sqrt{2\pi r} f_\sigma N}{h}$$

- Example: let us calculate the value of the stress intensity factor for the specimen:

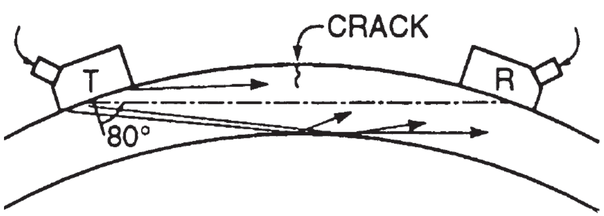


- Suppose that the fringe designated with number 5 is located at the distance 0.23 in. from the crack tip, the material fringe value is 43, and the specimen thickness is 0.213 in. The magnitude of the stress intensity factor can be calculated as:

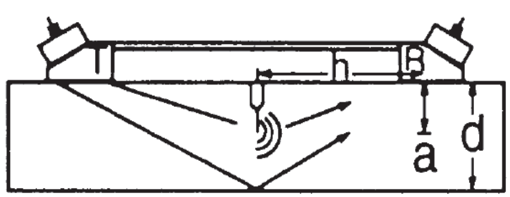
$$K_I = \frac{\sqrt{2\pi r}\, f_\sigma N}{h} = \frac{\sqrt{2\pi(0.23)}(43)(5)}{0.213} = 1.2 \ \text{ksi}\sqrt{\text{in}}$$

Crack Length Determination By Ultrasonic Methods

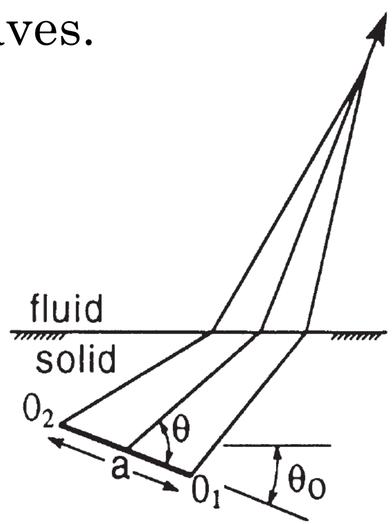
- Accurate calculation of the stress intensity factor on a given component under load relies on an accurate size determination of the flaws present in the component.
- The ultrasonic methods developed fall into three general categories;
  - determination of crack length from signal amplitude measurements
  - determination of crack length from time-of-flight measurements
  - determination of crack length using diffracted waves.



Signal Amplitude



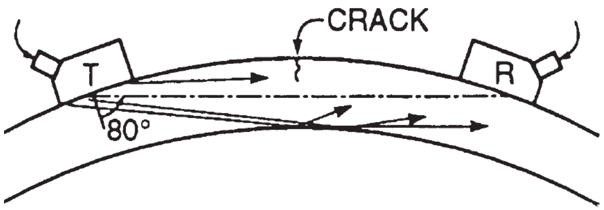
Time of flight



Diffracted wave modulation

Crack Length Determination By Ultrasonic Methods

❖ Crack Length From Signal Amplitude

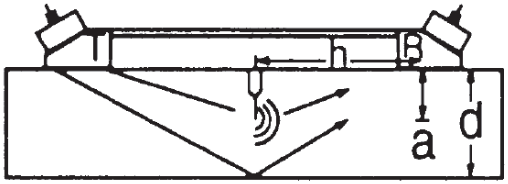


➢ This technique developed by Lumb and coworkers, they used a compression wave to monitor the growth of a through-the-thickness crack. A calibration curve of the ultrasonic signal amplitude versus crack depth was developed using milled slots and destructive measurements of part-through fatigue cracks to determine the depth of penetration of inservice cracks. They claim measurement of growth increments accurate to +/-0.025 mm and larger amounts of growth accurate to +/-0.25 mm.

➢ Particular problems concerning this technique were determined to be the stability of the coupling and temperature induced drift due to attenuation and velocity in the wedges. Additionally, crack closure contributes to inaccuracy of the crack depth measurement since contact between the crack faces allows additional energy to be transmitted that would be blocked if the crack were completely open.

Crack Length Determination By Ultrasonic Methods

❖ Crack Length From Time-of-Flight



- In this method determines the crack length by the time of flight of the wave diffracted by the crack tip and the wave reflected from the back surface. The crack length a is calculated according to

$$a = \sqrt{(C_1 t \, / \, 2)^2 - h^2}$$

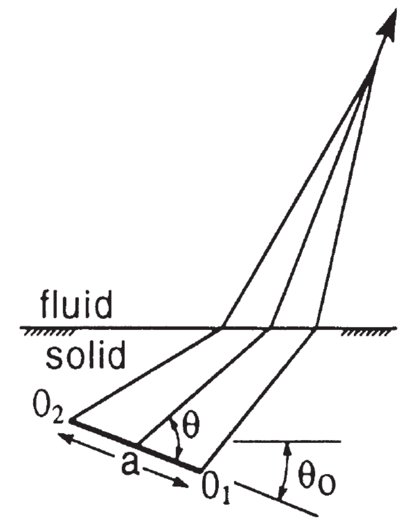
where C_l is the acoustic velocity of the wave and *t* is the time of flight.

It is recognized that any inaccuracies in the measurements of the separation distance of the receiver from the crack or the acoustic velocity would be reflected in the calculated crack length.

Crack Length Determination By Ultrasonic Methods

❖ Crack Length From Diffracted Waves

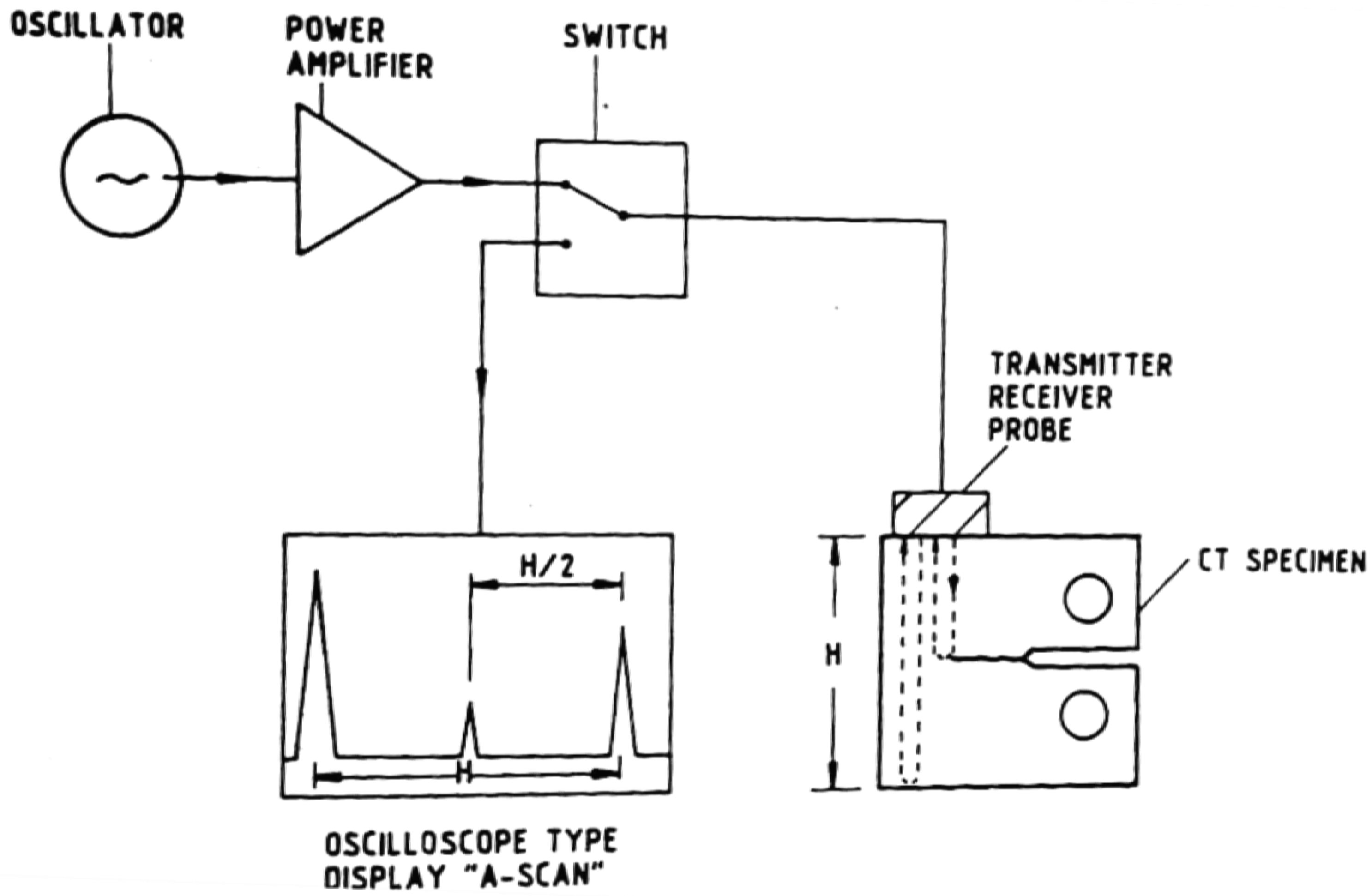
- Achenbach and coworkers have utilized the elastodynamic ray theory] to predict the scattering field for the situation. In this configuration, the crack is completely subsurface and the first arriving wave at the receiving transducer is due to the interference of the longitudinal rays diffracted at the crack tips. This wave exhibits a modulation in the frequency domain with period, *p*,



$$p = \frac{\pi}{a}[\cos\theta - \sin\theta_0]$$

- Since both a and ϑ_0 are unknown, two measurements at different angles are necessary to quantify the crack parameters but with inclusion of the appropriate attenuation values, agreement between their model and the experiment was almost perfect.

Crack Length Determination By Ultrasonic Methods



Crack Length Determination By Ultrasonic Methods