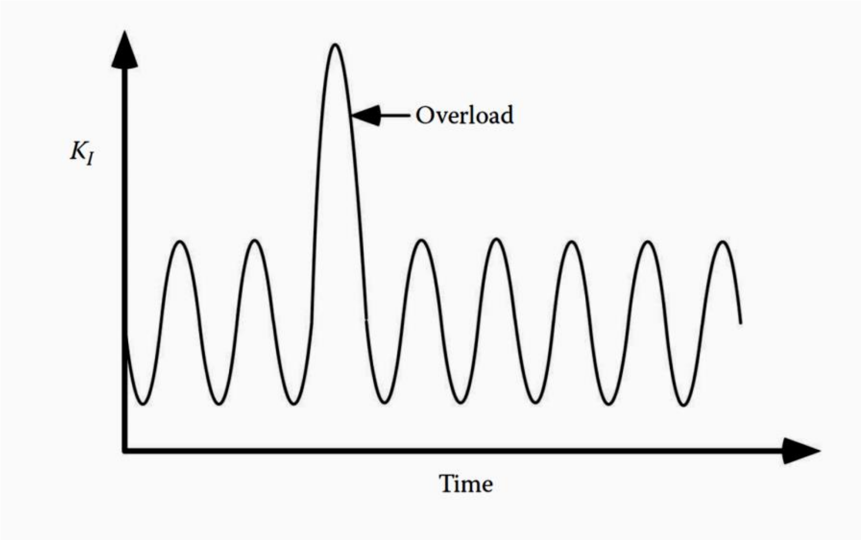
دانشگاه صنعتی اصفهان
دانشکده مکانیک

Fatigue-Crack closure (overload effect)

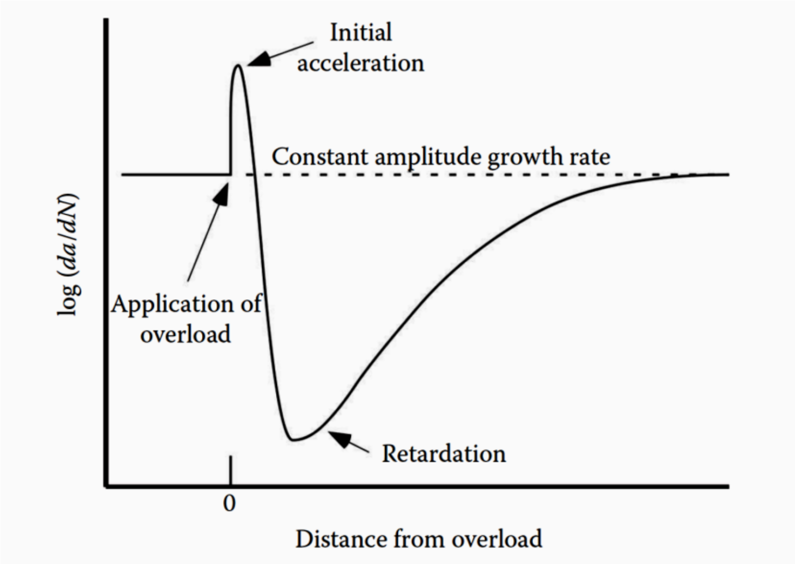
Crack closure

- Overload effect

➢ Constant-amplitude loading is interrupted by a single overload, after which the K amplitude resumes its previous value. Prior to the overload, the plastic zone would have reached a steady-state size, but the overload cycle produces a significantly larger plastic zone.



A single overload during cyclic loading.



Typical crack growth behavior following the application of a single overload.

Crack closure

- Overload effect

➢ Three possible mechanisms have been proposed to explain retardation following an overload:

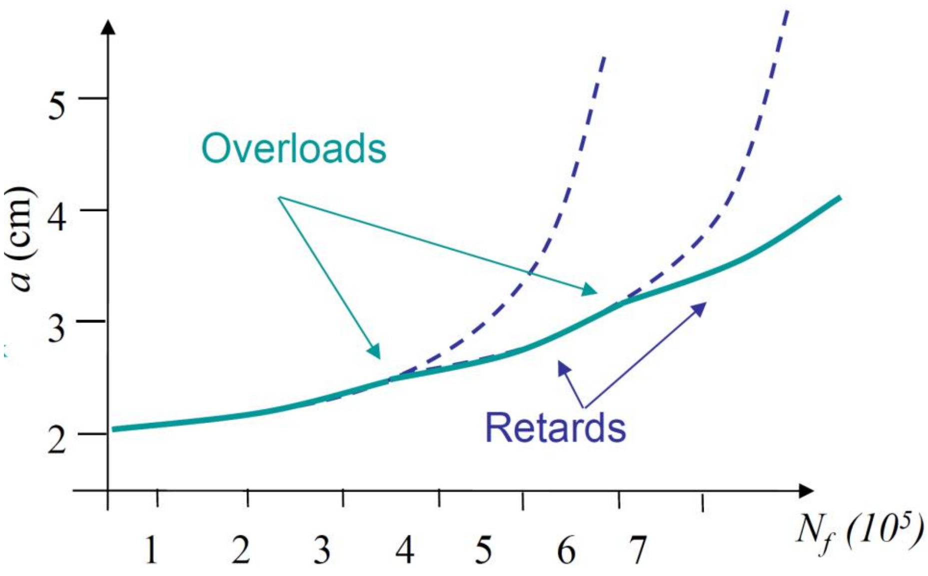
1. The crack blunts following an overload, and crack growth is delayed while the crack tip re-sharpens.
2. The compressive residual stresses in front of the crack tip retard the crack growth rate.
3. As the crack grows into the overload zone, residual stresses behind the crack tip result in plasticity-induced closure.

Crack closure

- Overload effect

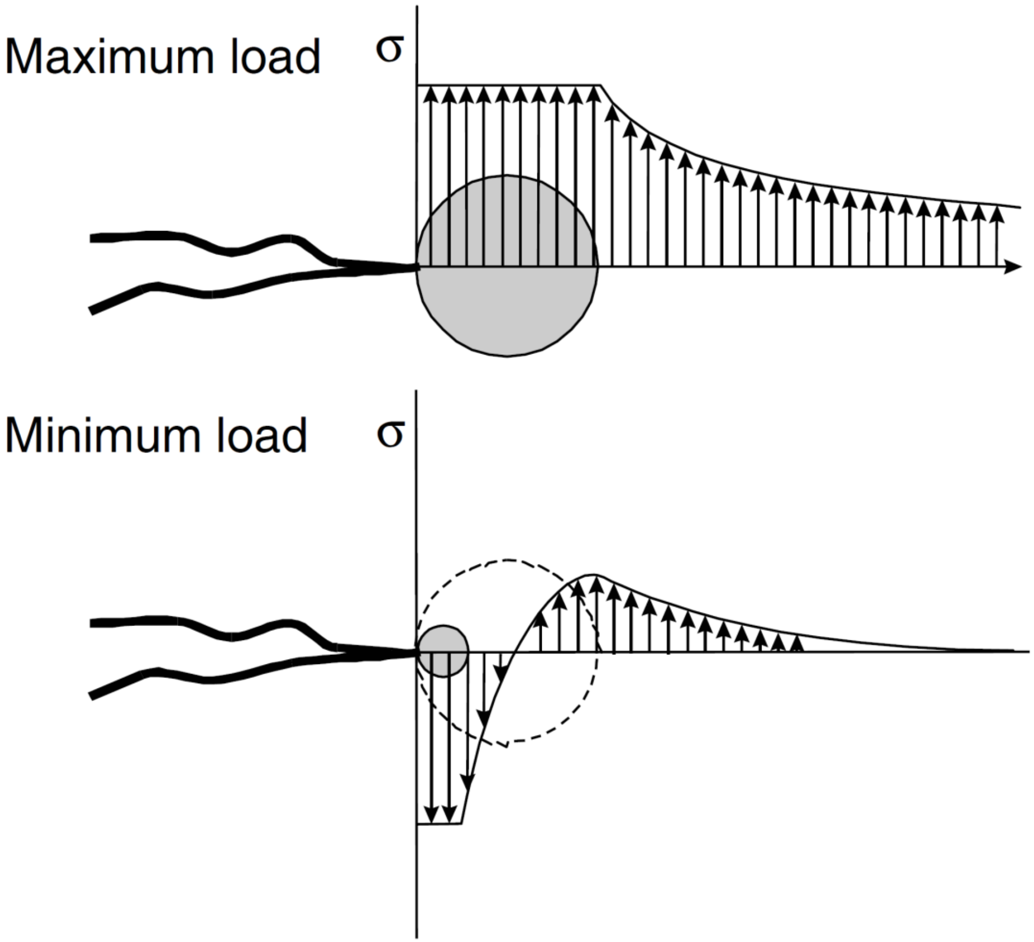
➢ As the plastic wake is temporarily increased, ΔK_{eff} is reduced due to Plasticity Induced Crack Closure ----> there is a retard effect in the crack propagation

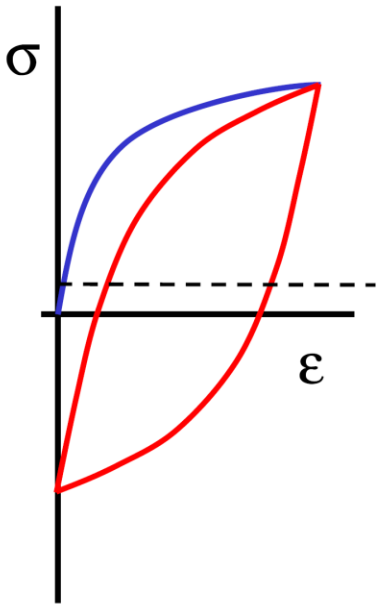
- Infrequent overloads help
- Frequent overloads may help
- Too frequent overloads are damaging, as it actually corresponds to increasing K_{max}



Crack closure

- Overload effect



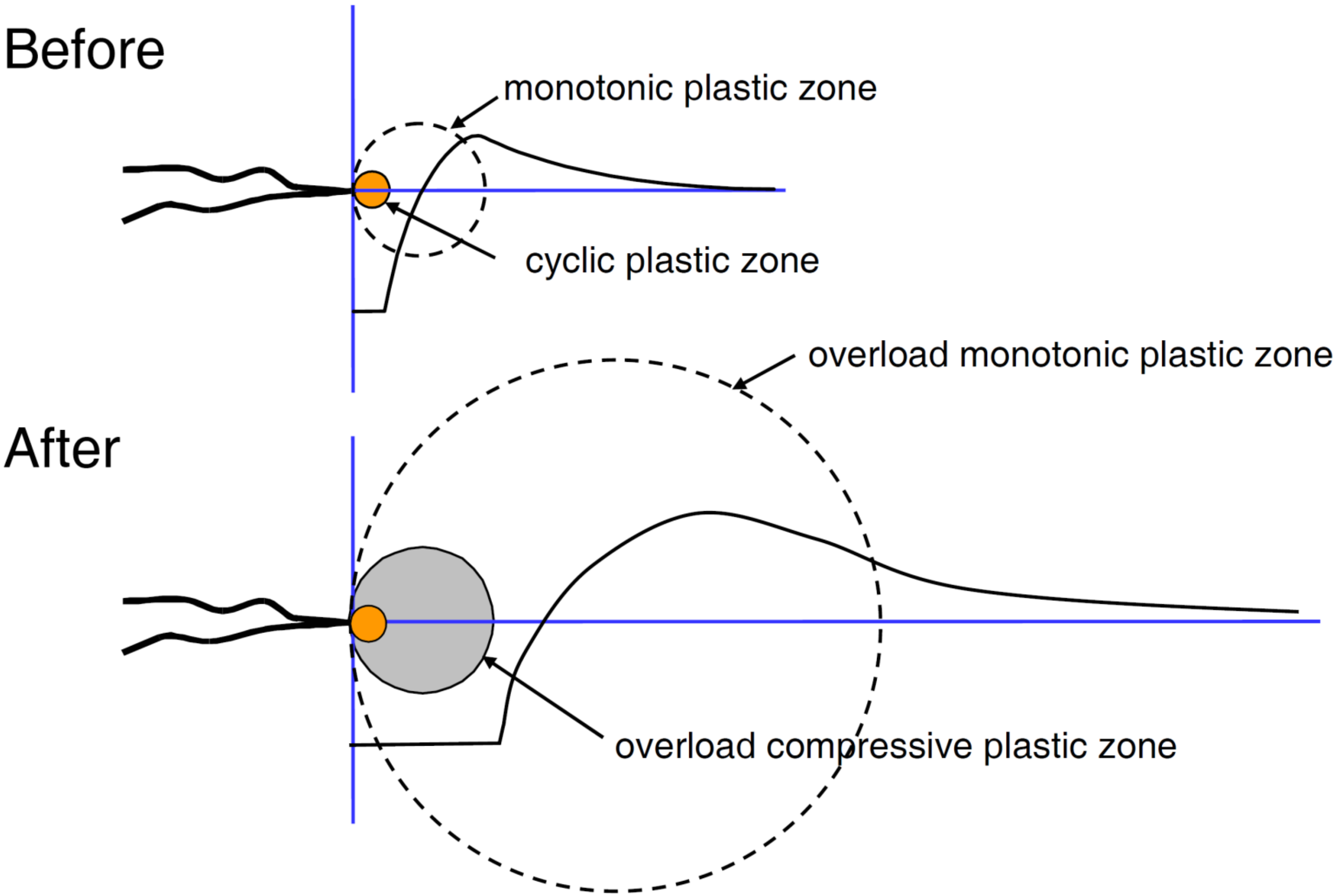


Mean stresses in plastic zone are small

- ➢ A (tensile) overload will introduce (compressive) residual stresses
- ➢ These residual stresses will influence ΔK and thus the rate of crack propagation

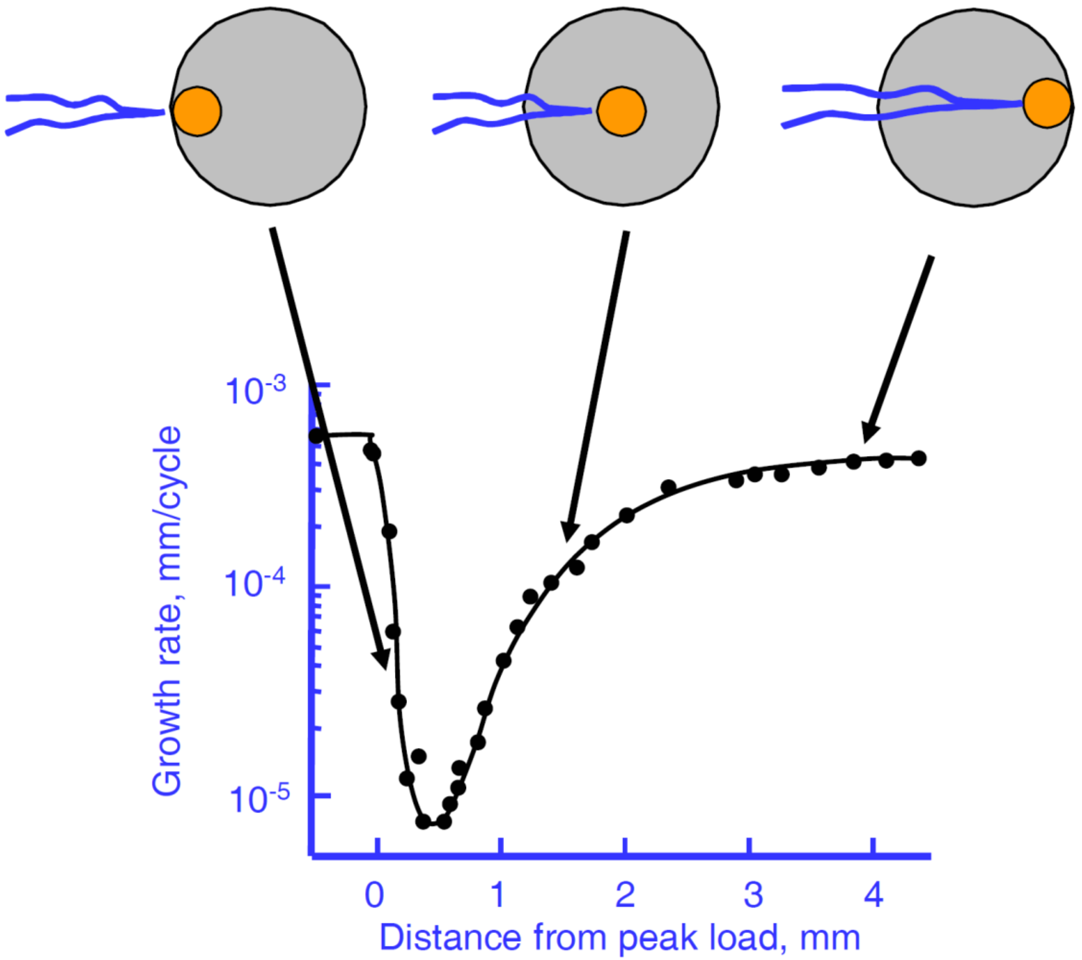
Stress Fields After Overload

- Overload effect



Crack Growth Retardation

- Overload effect

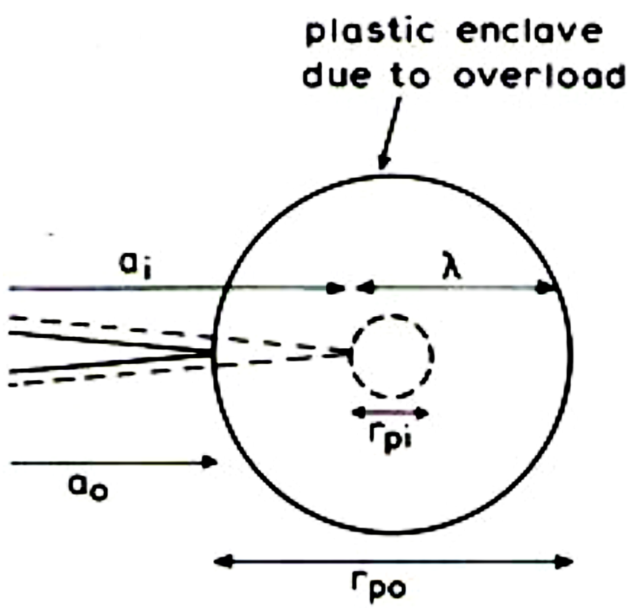


Retardation Models

- The **Wheeler model** (Wheeler 1972) is a simple model that calculates the fatigue crack growth retardation following a single tensile overload by introducing a retardation parameter, Φ_R, which prescribes a reduced growth rate for fatigue cracks advancing through the expanded plastic zone produced by the overload.

- The retardation parameter Φ_R can be multiplied to any constant amplitude fatigue model. Therefore, applying the Wheeler model to the Paris law the following equation is obtain:

$$\left(\frac{da}{dN}\right)_R = \Phi_R \frac{da}{dN} = \Phi_R [C \Delta K^m]$$



C and m are materials parameters in the Paris equation.

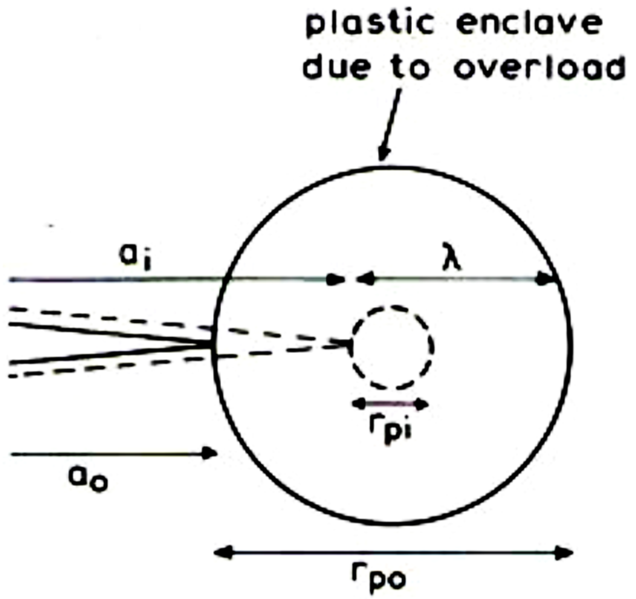
The Wheeler model

- The retardation parameter, Φ_R, is defined as (Wheeler 1972):

$$\Phi_R = \left(\frac{r_{pi}}{\lambda}\right)^m = \left(\frac{r_{pi}}{a_0 + r_{p0} - a_i}\right)^m$$

$$\text{If} \quad a_i + r_{pi} \geq a_0 + r_{p0} \qquad \Phi_R = 1$$

where a_{p0} is the crack length when the overload is applied, a_i is the crack length at each load cycle i, r_{p0} is the size of the plastic zone produced by the overload at a_{p0}, r_{pi} is the size of plastic zone produced by the post-overload constant amplitude loading at current crack length a_i and N is an adjustable experiment-based shaping exponent, which depends on the type of material, geometry and overload magnitude.



The Wheeler model

- **Example**: A wide plate with a central crack is subjected to a constant amplitude fatigue load with $\Delta\sigma$ = 60 MPa and R = 0. At a crack length $2a$ = 30 mm an overload occurs with a stress of 120 MPa. How high is the crack growth rate i) immediately after the overload, ii) after 0.1 mm, iii) after 0.15 mm, iv) after 0.3 mm of crack growth (at each tip)?

  Assume plane stress conditions and, for convenience, consider K to remain constant during the indicated crack growth intervals.

- Given: Wheeler exponent m = 1.5, yield strength σ_{ys} = 420 Mpa,

  Paris relation: $da/dN = 1.5\times10^{-8}(\Delta K)^4$ mm/cycle with ΔK in $MPa.m^{1/2}$

- **solution**: $\sigma_{min} = 0, \quad \sigma_{max} = 60\ MPa, \quad \sigma_0 = 2\Delta\sigma = 120\ MPa.$

The Wheeler model

- solution:

the size of the plastic zone for plane stress:

$$r_p = \frac{1}{6\pi}\left(\frac{K}{\sigma_{ys}}\right)^2 = \frac{1}{6\pi}\left(\frac{1.12\sigma\sqrt{\pi a}}{\sigma_{ys}}\right)^2 = 0.209a\left(\frac{\sigma}{\sigma_{ys}}\right)^2$$

before the overload:

$$r_{pi} = 0.209\times 15\left(\frac{60}{420}\right)^2 = 0.064\ mm$$

the size of the plastic zone produced by the overload:

$$r_{po} = 0.209\times 15\left(\frac{120}{420}\right)^2 = 0.256\ mm$$

retardation parameter :

$$\Phi_R = \left(\frac{r_{pi}}{a_0 + r_{p0} - a_i}\right)^m = \left(\frac{0.064}{15+0.256-15}\right)^{1.5} = 0.125$$

the range of the stress intensity factor :

$$\Delta K = 1.12\Delta\sigma\sqrt{\pi a} = 1.12\times 60\times\sqrt{\pi\times 0.015} = 14.588\quad MPa\sqrt{mm}$$

the ordinary crack growth rate:

$$\frac{da}{dN} = 1.5\times 10^{-8}(\Delta K)^4 = 1.5\times 10^{-8}(14.588)^4 = 0.679\times 10^{-3}\ \frac{m}{cyc} = 0.679\ \frac{mm}{cyc}$$

Retardation Models

- solution:

i) the crack growth rate at the moment of overload:

$$\left(\frac{da}{dN}\right)_{retadation} = \Phi_R \left(\frac{da}{dN}\right)_{ordinary} = 0.125 \times 0.679 = 0.0848 \quad mm/cyc$$

ii) the crack growth rate after 0.1mm crack propagated: $a_i = 15 + 0.1 = 15.1 \quad mm$

$$\Delta K = 1.12 \times 60 \times \sqrt{\pi \times 0.0151} = 14.637 \quad MPa\sqrt{m}$$

$$\left(\frac{da}{dN}\right)_{ordinary} = 1.5 \times 10^{-8} (14.637)^4 = 0.688 \quad \frac{mm}{\sec}$$

$$r_{pi} = 0.209a\left(\frac{\sigma}{\sigma_{ys}}\right)^2 = 0.209 \times 15.1\left(\frac{60}{420}\right)^2 = 0.0644 \quad mm$$

$$\Phi_R = \left(\frac{r_{pi}}{a_0 + r_{p0} - a_i}\right)^m = \left(\frac{0.0644}{15 + 0.256 - 15.1}\right)^{1.5} = \left(\frac{0.0644}{0.156}\right)^{1.5} = 0.256$$

$$\left(\frac{da}{dN}\right)_{retadation} = \Phi_R \left(\frac{da}{dN}\right)_{ordinary} = 0.256 \times 0.688 = 0.1825 \quad mm/cyc$$

The Wheeler model

- solution:

iii) the crack growth rate after 0.15mm crack propagated:

$$a_i = 15 + 0.15 = 15.15 \quad mm$$

$$\Delta K = 1.12 \times 60 \times \sqrt{\pi \times 0.01515} = 14.660 \quad MPa\sqrt{m}$$

$$\left(\frac{da}{dN}\right)_{ordinary} = 1.5 \times 10^{-8} (14.66)^4 = 0.693 \quad \frac{mm}{\sec}$$

$$r_{pi} = 0.209a\left(\frac{\sigma}{\sigma_{ys}}\right)^2 = 0.209 \times 15.15\left(\frac{60}{420}\right)^2 = 0.0646 \quad mm$$

$$\Phi_R = \left(\frac{r_{pi}}{a_0 + r_{p0} - a_i}\right)^m = \left(\frac{0.0646}{15 + 0.256 - 15.15}\right)^{1.5} = \left(\frac{0.0646}{0.106}\right)^{1.5} = 0.476$$

$$\left(\frac{da}{dN}\right)_{retadation} = \Phi_R \left(\frac{da}{dN}\right)_{ordinary} = 0.476 \times 0.693 = 0.33 \quad mm/cyc$$

The Wheeler model

- solution:

iv) the crack growth rate after 0.3mm crack propagated:

$$a_i = 15 + 0.3 = 15.3 \quad mm$$

$$\Delta K = 1.12 \times 60 \times \sqrt{\pi \times 0.0153} = 14.733 \quad MPa\sqrt{m}$$

$$\lambda = a_0 + r_{p0} - a_i = 15 + 0.256 - 15.3 = -0.044 \ \ mm < 0$$

the crack tip has gone out from the plastic region created by the overload: $\Phi_R = 1$

$$\frac{da}{dN} = 1.5 \times 10^{-8} (14.733)^4 = 0.7067 \quad \frac{mm}{cyc}$$

Examples for Fatigue

- Example 1:

A large plate contains a crack of length $2a_0$ and is subjected to a constant-amplitude tensile cyclic stress normal to the crack which varies between 100 MPa and 200 MPa. The following data were obtained: for $2a_0 = 2$ mm it was found that N = 20,000 cycles were required to grow the crack to $2a_f = 2.2$ mm, while for 2a = 20 mm it was found that N = 1000 cycles were required to grow the crack to $2a_f = 22$ mm. The critical stress intensity factor is $K_c = 60\ MPa\sqrt{m}$. Determine the constants in the Paris and Formam equations.

- solution:

The stress intensity factor range ΔK is calculated as $\Delta K = \Delta\sigma\sqrt{\pi a_0}$

For the crack of initial length $2a_o = 2$ mm: $\Delta K = 100\sqrt{\pi \times 1 \times 0.001} = 5.60\ \ MPa\sqrt{m}$

For the crack of initial length $2a_o = 20$ mm: $\Delta K = 100\sqrt{\pi \times 10 \times 0.001} = 17.72\ \ MPa\sqrt{m}$

Examples for Fatigue

- solution:

The crack growth rate da/dN is calculated for the crack of initial length $2a_o$ = 2 mm as:

$$\frac{da}{dN} = \frac{(1.1 - 1.0) \times 0.001\, m}{20000 \;\; cycle} = 5 \times 10^{-9} \quad m / cycle$$

and for the crack of initial length $2a_o$ = 20 mm as:

$$\frac{da}{dN} = \frac{(11 - 10) \times 0.001\, m}{1000 \;\; cycle} = 10^{-6} \quad m / cycle$$

(a) Paris equation:

$$\log \frac{da}{dN} = \log C + m \; \log \Delta K$$

$$\log(5 \times 10^{-9}) = \log C + m \; \log 5.6$$
$$\log(10^{-6}) = \log C + m \; \log 17.72$$

$$C = 1.82 \times 10^{-12} \quad M\ N^{-4.6} m^{7.9} / cycle$$
$$m = 4.6$$

Examples for Fatigue

- solution:

(b) Forman equation :

$$\log\left[[(1-R)K_c-\Delta K]\frac{da}{dN}\right]=\log C+n\log\Delta K$$

$$R=K_{\min}/K_{\max}=\sigma_{\min}/\sigma_{\max}=100/200=0.5$$

$$\log(0.5\times 60-5.6)\times 5\times 10^{-9})=\log C+n\log 5.6$$

$$\log(0.5\times 60-17.72)\times 10^{-6})=\log C+n\log 17.72$$

$$C=1.229\ MN^{-3.3}m^{5.95}/cycle$$

$$n=4.006$$

Examples for Fatigue

- **Example 2**:

A large thick plate contains a crack of length $2a_0=10$ mm and is subjected to a constant-amplitude tensile cyclic stress normal to the crack of which $\sigma_{min} = 100$ MPa and $\sigma_{max} = 200$ MPa. The critical SIF is $K_{Ic} = 60\ MPa\sqrt{m}$. Fatigue is governed by the following equation:

$$\frac{da}{dN} = 0.42\times10^{-11}(\Delta K)^3 \quad m/cycle$$

(↑ ΔK in $MPa\sqrt{m}$)

i) Plot the crack growth curve (a versus N) up to the point of fracture.

ii) If a lifetime of 10^6 cycles is required, discuss the option that the designer has for an improved lifetime.

- **solution**:

at final crack length: $200\sqrt{\pi a_f} = 60 \;\Rightarrow\; a_f = 28.65 \ mm$

Paris' equation:

$$\frac{da}{dN} = 0.42\times10^{-11}\left(\Delta\sigma\sqrt{\pi a}\right)^3$$

Examples for Fatigue

- solution:

$$\frac{da}{dN} = 0.42\times10^{-11}\left(\Delta\sigma\sqrt{\pi a}\right)^3$$

$$N = \frac{1}{0.42\times10^{-11}\times(100\sqrt{\pi})^3}\int_{a_0}^{a_f} a^{-3/2}da$$

$$N = \frac{2}{0.42\times10^{-11}\times(100\sqrt{\pi})^3}\left(\frac{1}{\sqrt{a_0}}-\frac{1}{\sqrt{a_f}}\right)$$

$$N = 7.04\times10^5 \ cycle$$

The curve of crack growth, a, versus number of cycles, N, up to the crack length a_c is calculated by Paris' equation with m = 3, C = 0.42 x 10^{-11} $MN^{-3}m^{5.5}$/cycle and $\Delta K_0 = 100\ (\pi x0.005)^{0.5} = 12.53$ MPa√m. Thus, for the crack to grow from its initial length a_o = 5 mm to a length a = 7 mm the number of cycles *N* required is given by:

Examples for Fatigue

- solution:

$$N = \frac{2\times(5\times10^{-3})}{1\times(0.42\times10^{-11})(12.53)^3}\left[1-\left(\frac{5}{7}\right)^{0.5}\right] = 187,412 \quad cycles$$

In a similar manner the number of cycles required for a crack of length $a_o = 7$ mm to grow to a length a=9 mm is given by :

$$N = \frac{2\times(7\times10^{-3})}{1\times(0.42\times10^{-11})\left[100\sqrt{\pi\times7\times10^{-3}}\right]^3}\left[1-\left(\frac{7}{9}\right)^{0.5}\right] = 120,696 \quad cycles$$

The above procedure is repeated for crack growth of steps 2 mm. Results are shown in the table.

Examples for Fatigue

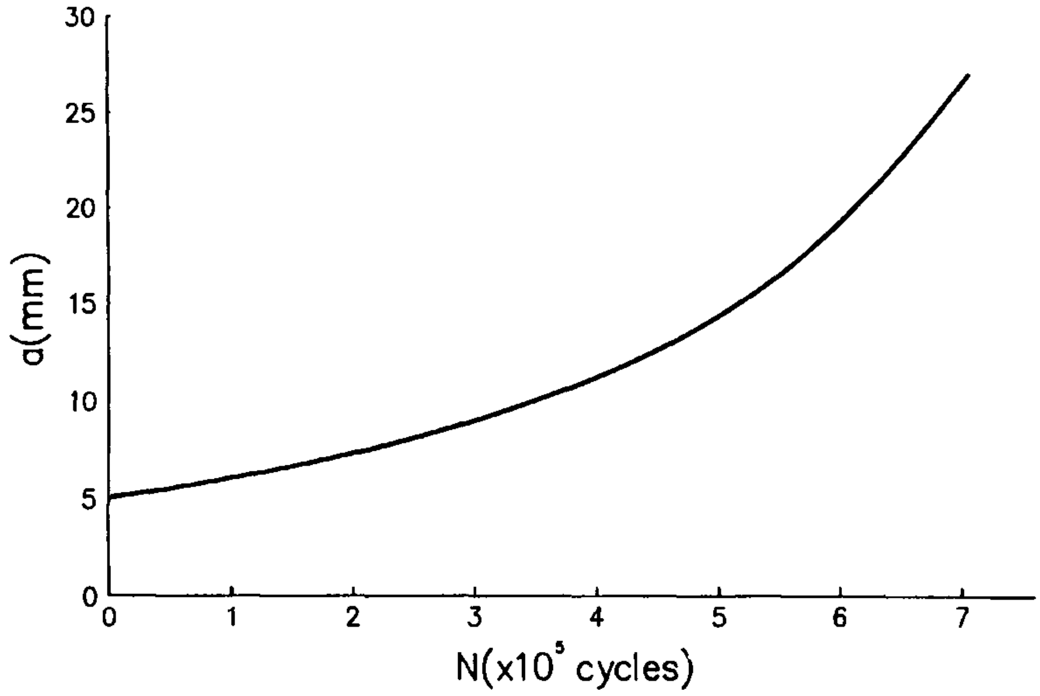
- solution:

a_o(mm)	a(mm)	ΔK_0(MPa√m)	ΔN(cycles)	N(cycles)
5	7	12.53	187,412	187,412
7	9	14.83	120,696	308,108
9	11	16.81	86,056	394,164
11	13	18.59	65,339	459,503
13	15	20.21	51,791	511,294
15	17	21.71	42,358	553,652
17	19	23.11	35,480	589,132
19	21	24.43	30,282	619,414
21	23	25.69	26,241	645,655
23	25	26.88	23,027	668,682
25	27	28.02	20,417	689,099
27	28.65	29.12	15,209	704,308

Examples for Fatigue

- solution:

Crack length versus number of cycles:



Examples for Fatigue

- solution:

If a lifetime of 10^6 cycles is required for the plate the designer may make the following changes:

(a) Employ a different metal with higher K_{Ic}, so as to increase the critical crack length a_c at instability.
(b) Reduce the maximum value of the applied stress $\Delta\sigma_{max}$.
(c) Reduce the stress range $\Delta\sigma$.
(d) Improve the inspection so as to reduce the assumed initial crack length. If for example the initial crack length was reduced from $2a_o$ = 10 mm to $2a_o$ = 6 mm,

the lifetime of the plate would be increased to 1,056,097 cycles, which is more than the required number of 10^6 cycles.