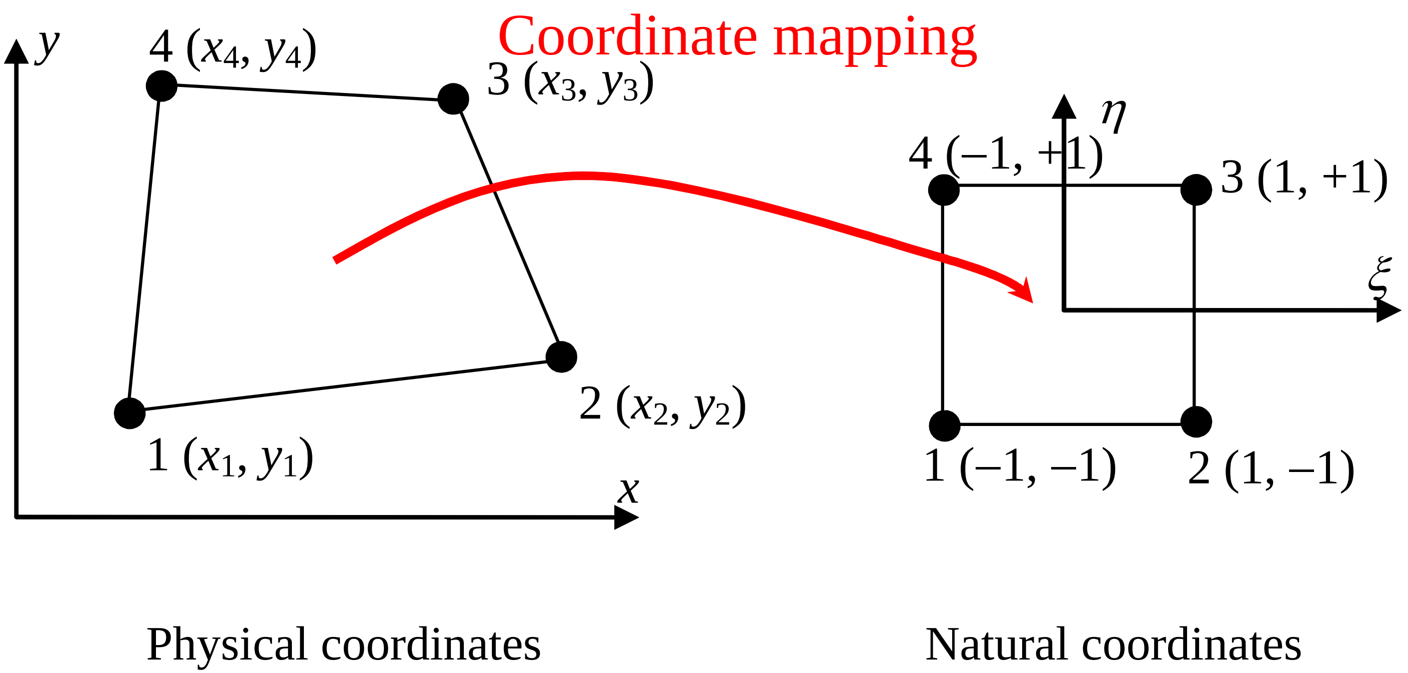
المان مربعی خطی (مختصات طبیعی)

انتقال از دستگاه مختصات (x,y) به دستگاه مختصات طبیعی



$$\mathbf{u}(\xi,\eta) = \mathbf{N}(\xi,\eta)\mathbf{d}$$ (Interpolation of displacements)

$$\mathbf{X}(\xi,\eta) = \mathbf{N}(\xi,\eta)\mathbf{x}_e$$ (Interpolation of coordinates)

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$$\mathbf{X}(\xi,\eta)=\mathbf{N}(\xi,\eta)\mathbf{x}_e$$

where

$$\mathbf{X}=\begin{Bmatrix} x \\ y \end{Bmatrix} \quad , \quad \mathbf{x}_e=\begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix} \begin{matrix} \text{coordinate at node 1} \\ \\ \text{coordinate at node 2} \\ \\ \text{coordinate at node 3} \\ \\ \text{coordinate at node 4} \end{matrix}$$

$$N_1=\tfrac{1}{4}(1-\xi)(1-\eta)$$

$$N_2=\tfrac{1}{4}(1+\xi)(1-\eta)$$

$$N_3=\tfrac{1}{4}(1+\xi)(1+\eta)$$

$$N_4=\tfrac{1}{4}(1-\xi)(1+\eta)$$

$$\begin{cases} x=\sum_{i=1}^{4} N_i(\xi,\eta)x_i \\ y=\sum_{i=1}^{4} N_i(\xi,\eta)y_i \end{cases}$$

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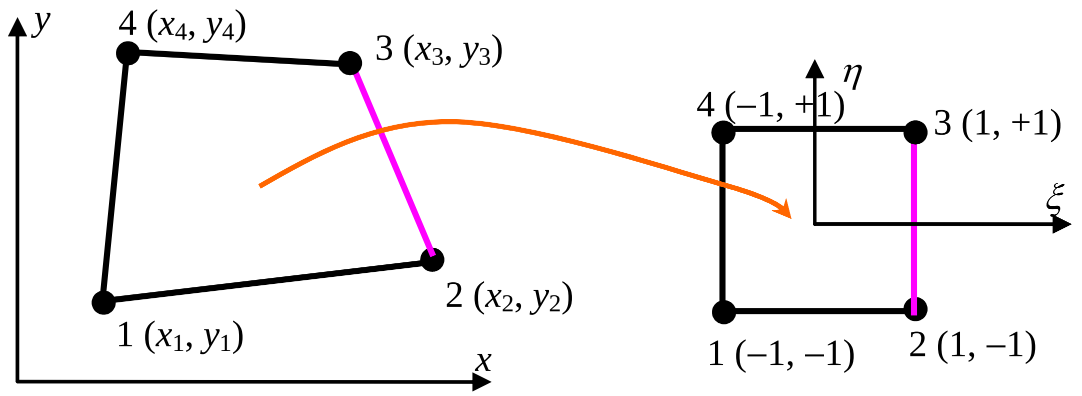
Substitute $\xi = 1$ into $$x = \sum_{i=1}^{4} N_i(\xi,\eta)x_i$$

$$x = \tfrac{1}{2}(1-\eta)x_2 + \tfrac{1}{2}(1+\eta)x_3$$
$$y = \tfrac{1}{2}(1-\eta)y_2 + \tfrac{1}{2}(1+\eta)y_3$$

or

$$x = \tfrac{1}{2}(x_2 + x_3) + \tfrac{1}{2}\eta(x_3 - x_2)$$
$$y = \tfrac{1}{2}(y_2 + y_3) + \tfrac{1}{2}\eta(y_3 - y_2)$$

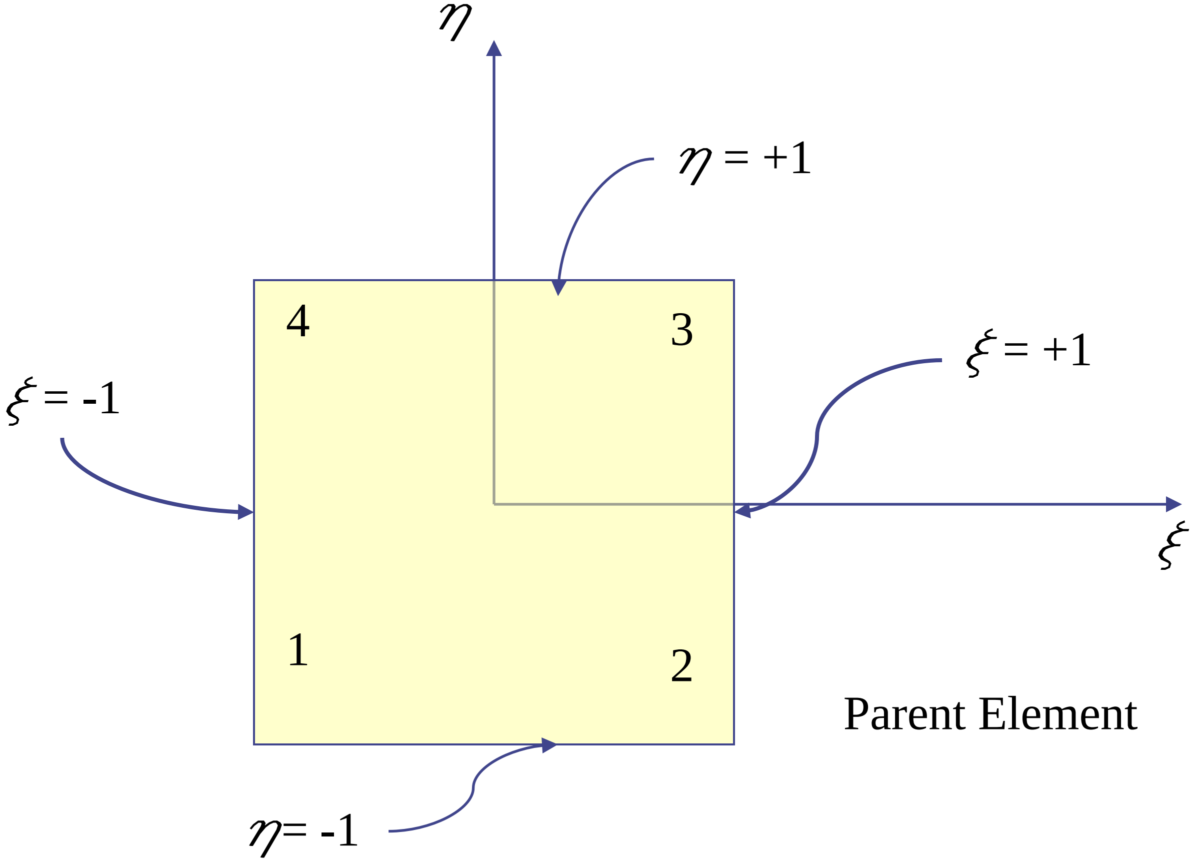
Eliminating η, $$y = \frac{(y_3 - y_2)}{(x_3 - x_2)}\{x - \tfrac{1}{2}(x_2 + x_3)\} + \tfrac{1}{2}(y_2 + y_3)$$



Remarks

- Shape functions used for interpolating the coordinates are the same as the shape functions used for interpolation of the displacement field. Therefore, the element is called an *isoparametric element.*
- Note that the shape functions for coordinate interpolation and displacement interpolation do not have to be the same.
- Using the different shape functions for coordinate interpolation and displacement interpolation, respectively, will lead to the development of so-called *subparametric* or *superparametric* elements.

Natural Coordinates ξ-η



استخراج ماتریس سختی المان ایزوپارامتریک

$$x = a_1 + a_2\xi + a_3\eta + a_4\xi\eta$$

$$y = a_5 + a_6\xi + a_7\eta + a_8\xi\eta$$

$$x = \frac{1}{4}\Big((1-\xi)(1-\eta)x_1 + (1+\xi)(1-\eta)x_2 + (1+\xi)(1+\eta)x_3 + (1-\xi)(1+\eta)x_4\Big)$$

$$y = \frac{1}{4}\Big((1-\xi)(1-\eta)y_1 + (1+\xi)(1-\eta)y_2 + (1+\xi)(1+\eta)y_3 + (1-\xi)(1+\eta)y_4\Big)$$

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ x_3 \\ y_3 \\ x_4 \\ y_4 \end{Bmatrix}$$

$$N_1 = \frac{(1-\xi)(1-\eta)}{4}$$

$$N_2 = \frac{(1+\xi)(1-\eta)}{4}$$

$$N_3 = \frac{(1+\xi)(1+\eta)}{4}$$

$$N_4 = \frac{(1-\xi)(1+\eta)}{4}$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \end{Bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x} \end{Bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial (\ )}{\partial x} & 0 \\ 0 & \dfrac{\partial (\ )}{\partial y} \\ \dfrac{\partial (\ )}{\partial y} & \dfrac{\partial (\ )}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$\boldsymbol{\varepsilon} = \mathbf{Bd}$$

Coordinate transformation is unique and invertible.

$$\begin{aligned} x &= x(\xi,\eta) \\ y &= y(\xi,\eta) \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \xi &= \xi(x,y) \\ \eta &= \eta(x,y) \end{aligned}$$

Chain Rule:

$$\frac{\partial f}{\partial \xi} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \xi} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \xi}$$

$$\frac{\partial f}{\partial \eta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \eta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \eta}$$

$$\begin{Bmatrix} \dfrac{\partial f}{\partial \xi} \\ \dfrac{\partial f}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{Bmatrix}$$

Jacobian matrix:

$$\mathbf{J} = \begin{bmatrix} \dfrac{\partial x}{\partial \xi} & \dfrac{\partial y}{\partial \xi} \\ \dfrac{\partial x}{\partial \eta} & \dfrac{\partial y}{\partial \eta} \end{bmatrix}$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$\begin{Bmatrix} \frac{\partial f}{\partial \xi} \\ \frac{\partial f}{\partial \eta} \end{Bmatrix} = [J] \begin{Bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{Bmatrix} \rightarrow \begin{Bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial f}{\partial \xi} \\ \frac{\partial f}{\partial \eta} \end{Bmatrix} \qquad [J] = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

$$\frac{\partial (\;)}{\partial x} = \frac{1}{|J|} \left[\frac{\partial y}{\partial \eta} \frac{\partial (\;)}{\partial \xi} - \frac{\partial y}{\partial \xi} \frac{\partial (\;)}{\partial \eta} \right]$$

$$\frac{\partial (\;)}{\partial y} = \frac{1}{|J|} \left[\frac{\partial x}{\partial \xi} \frac{\partial (\;)}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial (\;)}{\partial \xi} \right]$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{|J|} \begin{bmatrix} \frac{\partial y}{\partial \eta} \frac{\partial (\;)}{\partial \xi} - \frac{\partial y}{\partial \xi} \frac{\partial (\;)}{\partial \eta} & 0 \\ 0 & \frac{\partial x}{\partial \xi} \frac{\partial (\;)}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial (\;)}{\partial \xi} \\ \frac{\partial x}{\partial \xi} \frac{\partial (\;)}{\partial \eta} - \frac{\partial x}{\partial \eta} \frac{\partial (\;)}{\partial \xi} & \frac{\partial y}{\partial \eta} \frac{\partial (\;)}{\partial \xi} - \frac{\partial y}{\partial \xi} \frac{\partial (\;)}{\partial \eta} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$\boldsymbol{\varepsilon} = \mathbf{DNd}$$

$$\mathbf{D} = \frac{1}{|J|}\begin{bmatrix} \frac{\partial y}{\partial \eta}\frac{\partial (\ )}{\partial \xi} - \frac{\partial y}{\partial \xi}\frac{\partial (\ )}{\partial \eta} & 0 \\ 0 & \frac{\partial x}{\partial \xi}\frac{\partial (\ )}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial (\ )}{\partial \xi} \\ \frac{\partial x}{\partial \xi}\frac{\partial (\ )}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial (\ )}{\partial \xi} & \frac{\partial y}{\partial \eta}\frac{\partial (\ )}{\partial \xi} - \frac{\partial y}{\partial \xi}\frac{\partial (\ )}{\partial \eta} \end{bmatrix}$$

$$\underset{(3\times 8)}{\mathbf{B}} = \underset{(3\times 2)}{\mathbf{D}} \ \underset{(2\times 8)}{\mathbf{N}}$$

$$x = \frac{1}{4}\begin{pmatrix} (1-\xi)(1-\eta)x_1 + (1+\xi)(1-\eta)x_2 \\ +(1+\xi)(1+\eta)x_3 + (1-\xi)(1+\eta)x_4 \end{pmatrix}$$

$$\frac{\partial x}{\partial \xi} = \frac{1}{4}\left(-(1-\eta)x_1 + (1-\eta)x_2 + (1+\eta)x_3 - (1+\eta)x_4\right)$$

$$\frac{\partial x}{\partial \eta} = \frac{1}{4}\left(-(1-\xi)x_1 - (1+\xi)x_2 + (1+\xi)x_3 + (1-\xi)x_4\right)$$

$$y = \frac{1}{4}\begin{pmatrix} (1-\xi)(1-\eta)y_1 + (1+\xi)(1-\eta)y_2 \\ +(1+\xi)(1+\eta)y_3 + (1-\xi)(1+\eta)y_4 \end{pmatrix}$$

$$\frac{\partial y}{\partial \xi} = \frac{1}{4}\left(-(1-\eta)y_1 + (1-\eta)y_2 + (1+\eta)y_3 - (1+\eta)y_4\right)$$

$$\frac{\partial y}{\partial \eta} = \frac{1}{4}\left(-(1-\xi)y_1 - (1+\xi)y_2 + (1+\xi)y_3 + (1-\xi)y_4\right)$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$|\mathbf{J}| = \frac{1}{8}\{X_c\}^T \begin{bmatrix} 0 & 1-\eta & \eta-\xi & \xi-1 \\ \eta-1 & 0 & \xi+1 & -\xi-t \\ \xi-\eta & -\xi-1 & 0 & \eta+1 \\ 1-\xi & \xi+\eta & -\eta-1 & 0 \end{bmatrix}\{Y_c\}, \quad \{X_c\} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} \qquad \{Y_c\} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix}$$

$$[B(\xi,\eta)] = \frac{1}{|\mathbf{J}|}\left[B_1 \;\vdots\; B_2 \;\vdots\; B_3 \;\vdots\; B_4\right]$$

$$[B_i] = \begin{bmatrix} a(N_{i,\xi}) - b(N_{i,\eta}) & 0 \\ 0 & c(N_{i,\eta}) - d(N_{i,\xi}) \\ c(N_{i,\eta}) - d(N_{i,\xi}) & a(N_{i,\xi}) - b(N_{i,\eta}) \end{bmatrix}$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$N_{1,\xi}=\frac{\partial N_1}{\partial \xi}=\frac{-1(1-\eta)}{4}=\frac{(\eta-1)}{4} \qquad N_{1,\eta}=\frac{\partial N_1}{\partial \eta}=\frac{(1-\xi)(-1)}{4}=\frac{(\xi-1)}{4}$$

$$N_{2,\xi}=\frac{\partial N_2}{\partial \xi}=\frac{(1)(1-\eta)}{4}=\frac{(1-\eta)}{4} \qquad N_{2,\eta}=\frac{\partial N_2}{\partial \eta}=\frac{(1+\xi)(-1)}{4}=\frac{-(\xi+1)}{4}$$

$$N_{3,\xi}=\frac{\partial N_3}{\partial \xi}=\frac{(1)(1+\eta)}{4}=\frac{(1+\eta)}{4} \qquad N_{3,\eta}=\frac{\partial N_3}{\partial \eta}=\frac{(1+\xi)(1)}{4}=\frac{(\xi+1)}{4}$$

$$N_{4,\xi}=\frac{\partial N_4}{\partial \xi}=\frac{(-1)(1+\eta)}{4}=\frac{-(1+\eta)}{4} \qquad N_{4,\eta}=\frac{\partial N_4}{\partial \eta}=\frac{(1-\xi)(1)}{4}=\frac{(1-\xi)}{4}$$

$$a=1/4\left[y_1(\xi-1)+y_2(-\xi-1)+y_3(\xi+1)+y_4(1-\xi)\right]$$

$$b=1/4\left[y_1(\eta-1)+y_2(1-\eta)+y_3(\eta+1)+y_4(-1-\eta)\right]$$

$$c=1/4\left[x_1(\eta-1)+x_2(1-\eta)+x_3(\eta+1)+x_4(-1-\eta)\right]$$

$$d=1/4\left[x_1(\xi-1)+x_2(-\xi-1)+x_3(\xi+1)+x_4(1-\xi)\right]$$

استخراج ماتریس سختی المان ایزوپارامتریک

$$\mathbf{k} = \iint_A \mathbf{B}^T \mathbf{E} \mathbf{B}\, t\, dxdy$$

$$\iint_A f(x,y)dxdy = \iint_A f(\xi,\eta)\left|\mathbf{J}\right| d\xi d\eta$$

$$\mathbf{k} = \int_{-1}^{1}\int_{-1}^{1} \mathbf{B}^T \mathbf{E} \mathbf{B}\, t\, \left|\mathbf{J}\right| d\xi d\eta$$

$$\boldsymbol{f}_b = \int_{-1}^{1}\int_{-1}^{1} \mathbf{N}^T \mathbf{X}_b\, t\left|\mathbf{J}\right|\, d\xi d\eta$$

$$\boldsymbol{f}_s = \int_L \mathbf{N}^T T\;\; t\left|\mathbf{J}\right|\; dL$$

Gauss integration

- For evaluation of integrals in **k** (in practice)

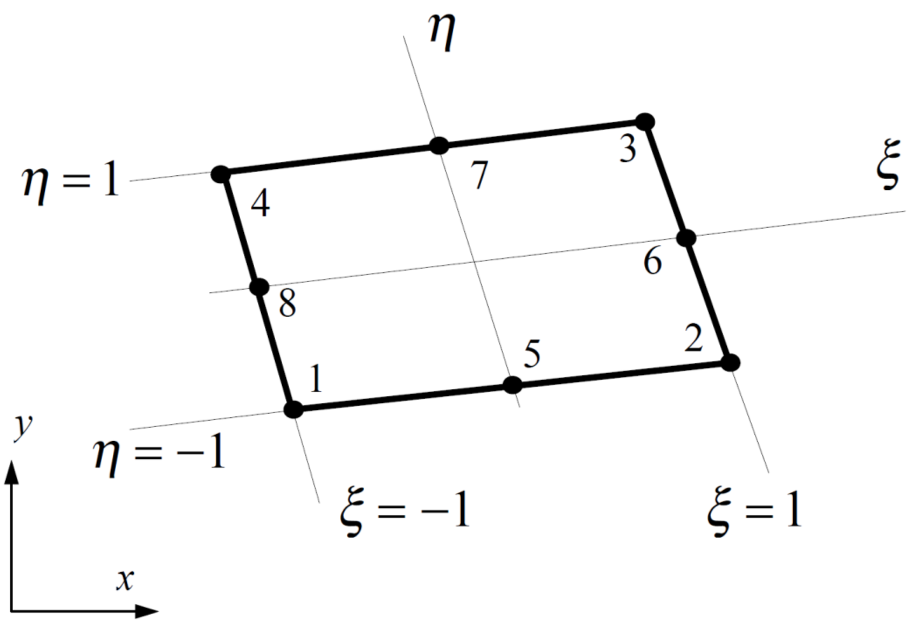
In 1 direction: $$I = \int_{-1}^{+1} f(\xi)\mathrm{d}\xi = \sum_{j=1}^{m} w_j f(\xi_j)$$

m gauss points gives exact solution of polynomial integrand of $n = 2m - 1$

In 2 directions: $$I = \int_{-1}^{+1}\int_{-1}^{+1} f(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta = \sum_{i=1}^{n_x}\sum_{j=1}^{n_y} w_i w_j f(\xi_i,\eta_j)$$

- This is the most widely used element for 2-D problems due to its high accuracy in analysis and flexibility in modeling.

Quadratic Quadrilateral Element (Q8)

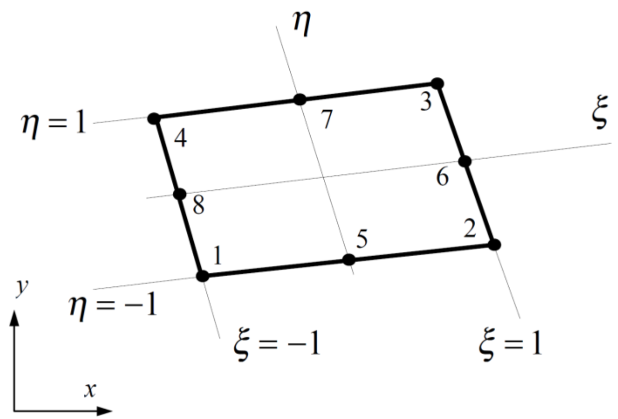


- In the natural coordinate system (ξ, η), the eight shape functions are,

$$N_i = F_i(\xi,\eta)G_i(\xi,\eta)$$

$F_i(\xi,\eta)$ Give a value of zero along the sides of the element that the given node does not contact

$G_i(\xi,\eta)$ Select such that when multiply by F_i, it will produce A value of unity at node *i* and a value of *zero* at other neighboring nodes.



Example: Consider N_3

$$F_3(\xi,\eta) = (1+\xi)(1+\eta)$$

$$G_3(\xi,\eta) = c_1 + c_2\xi + c_3\eta$$

$$G_3(1,0) = 0; \quad G_3(0,1) = 0; \quad N_3(1,1) = F_3(1,1)G_3(1,1) = 1$$

المان مربعی درجه دو

$$G_3(1,0)=0 \Rightarrow c_1+c_2=0$$

$$G_3(0,1)=0 \Rightarrow c_1+c_3=0$$

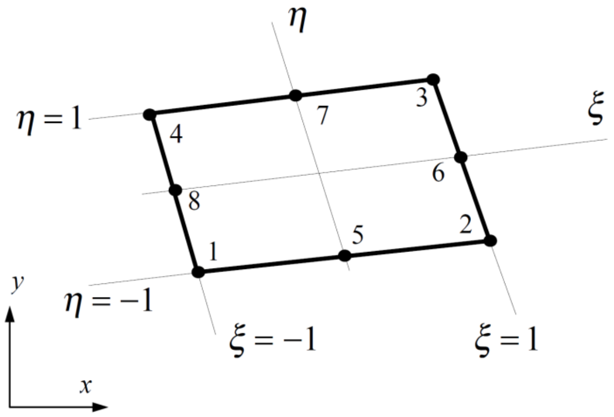
$$N_3(1,1)=1 \Rightarrow 4(c_1+c_2+c_3)=1$$

$$\therefore c_1=-1/4$$

$$c_2=1/4$$

$$c_3=1/4$$

$$N_3=\frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1)$$



المان مربعی درجه دو

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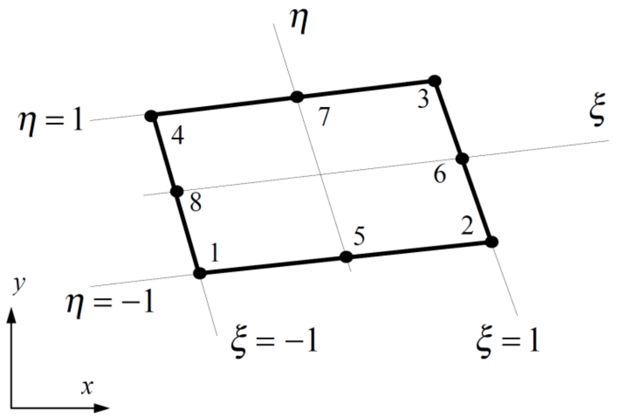

$$N_1 = \frac{1}{4}(1-\xi)(\eta-1)(\xi+\eta+1)$$

$$N_2 = \frac{1}{4}(1+\xi)(\eta-1)(\eta-\xi+1)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1)$$

$$N_4 = \frac{1}{4}(\xi-1)(\eta+1)(\xi-\eta+1)$$

$$N_5 = \frac{1}{2}(1-\eta)(1-\xi^2)$$

$$N_6 = \frac{1}{2}(1+\xi)(1-\eta^2)$$

$$N_7 = \frac{1}{2}(1+\eta)(1-\xi^2)$$

$$N_8 = \frac{1}{2}(1-\xi)(1-\eta^2)$$

المان مربعی درجه دو

- at any point inside the element: $\sum_{i=1}^{8} N_i = 1$

- The displacement field is given by:

$$u = \sum_{i=1}^{8} N_i u_i , \qquad v = \sum_{i=1}^{8} N_i v_i$$

- which are quadratic functions over the element. Strains and stresses over a quadratic quadrilateral element are linear functions, which are better representations.

Notes:

- Q4 and T3 are usually used together in a mesh with linear elements.
- Q8 and T6 are usually applied in a mesh composed of quadratic elements.
- Quadratic elements are preferred for stress analysis, because of their high accuracy and the flexibility in modeling complex geometry, such as curved boundaries.

The stress in an element is determined by the following relation,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \mathbf{E} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \mathbf{EBd}$$

where **B** is the strain-nodal displacement matrix and **d** is the nodal displacement vector which is known for each element once the global FE equation has been solved.

Stresses can be evaluated at any point inside the element (such as the center) or at the nodes. Contour plots are usually used in FEA software packages (during post-process) for users to visually inspect the stress results.

The von Mises stress is the *effective* or *equivalent* stress for 2-D and 3-D stress analysis. For a ductile material, the stress level is considered to be safe, if $\sigma_e \le \sigma_Y$

where σ_e is the von Mises stress and σ_Y the yield stress of the material. This is a generalization of the 1-D (experimental) result to 2-D and 3-D situations.

The von Mises stress is defined by

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

in which σ_1, σ_2 and σ_3 are the three principle stresses at the considered point in a structure.

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in which σ_1, σ_2 and σ_3 are the three principle stresses at the considered point in a structure.

For 2-D problems, the two principle stresses in the plane are determined by

$$\sigma_1{}^P = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma_2{}^P = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Thus, we can also express the von Mises stress in terms of the stress components in the *xy* coordinate system. For plane stress conditions, we have,

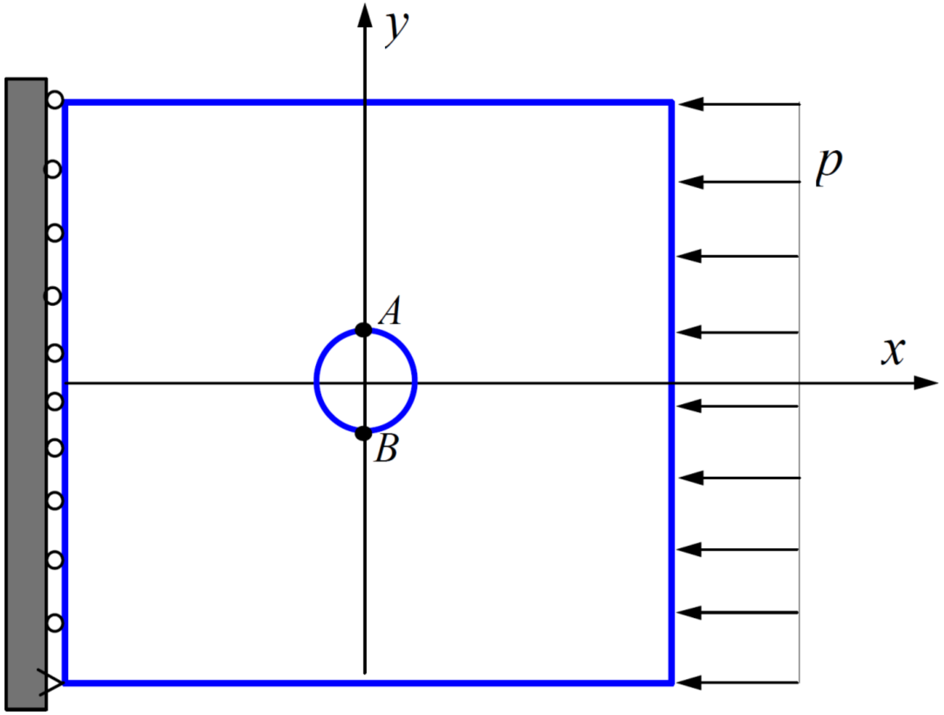
$$\sigma_e = \sqrt{(\sigma_x + \sigma_y)^2 - 3(\sigma_x \sigma_y - \tau_{xy}^2)}$$

***Averaged Stresses*:**

Stresses are usually averaged at nodes in FEA software packages to provide more accurate stress values. This option should be turned off at nodes between two materials or other geometry discontinuity locations where stress discontinuity does exist.

محاسبه تنش در یک مسئله تنش صفحه‌ای

مثال: یک صفحه مربعی با سوراخ دایره‌ای که تحت تنش فشاری قرار گرفته است.



The dimension of the plate is *10* in. *x 10* in., thickness is *0.1* in. and radius of the hole is *1* in. Assume $E = 10 \times 10^6$ psi, $\nu = 0.3$ and $p = 100$ *psi*. Find the maximum stress in the plate.

مثال: یک صفحه مربعی با سوراخ دایره‌ای که تحت تنش فشاری قرار گرفته است.

From the knowledge of stress concentrations, we should expect the maximum stresses occur at points A and B on the edge of the hole. Value of this stress should be around 3p (= 300 psi) which is the exact solution for an infinitely large plate with a hole.

Elem. Type	*No. Elem.*	*DOF*	*Max. σ (psi)*
T6	966	4056	310.1
Q4	493	1082	286.0
Q8	493	3150	327.1
...	...	...	...
Q8	2727	16,826	322.3

محاسبه تنش در یک مسئله تنش صفحه‌ای

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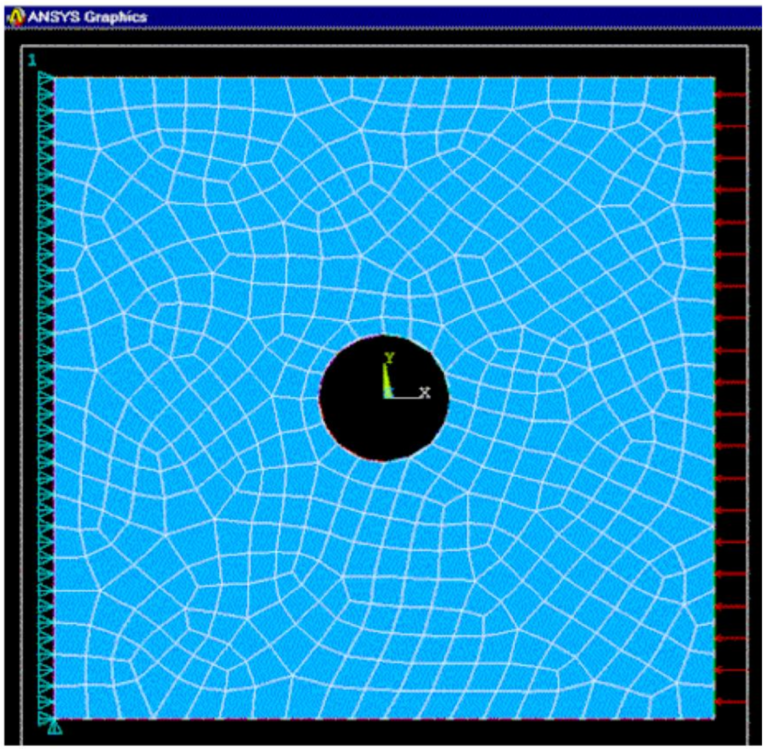
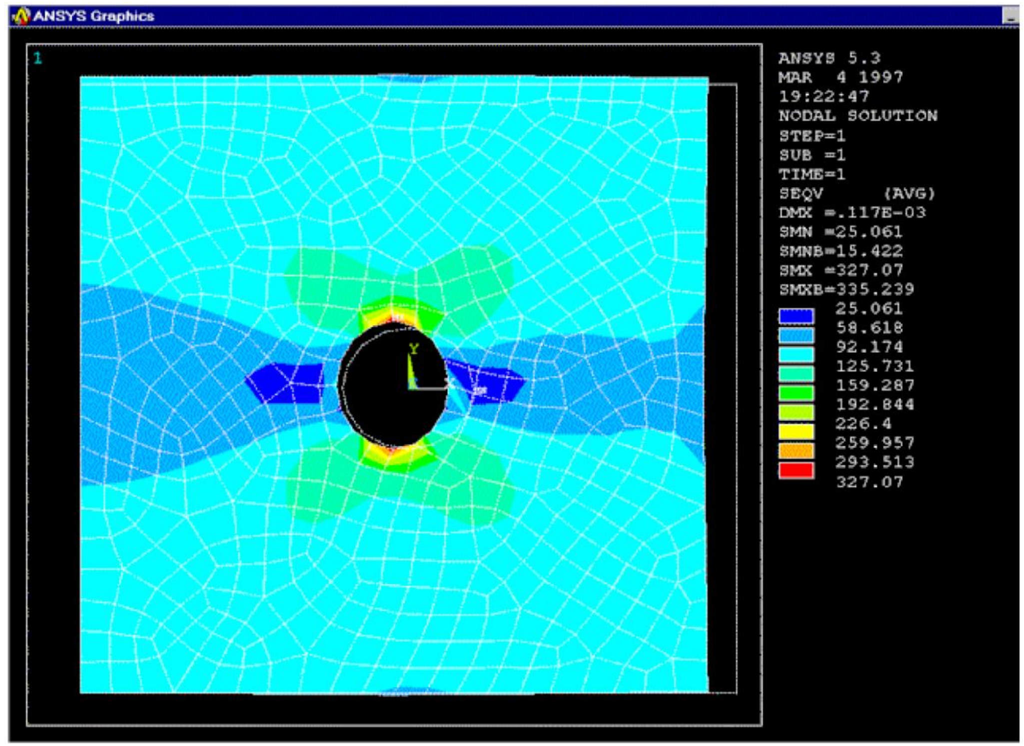


FEA Mesh (Q8, 493 elements)



FEA Stress Plot (Q8, 493 elements)

Discussions:

- Check the deformed shape of the plate
- Check convergence (use a finer mesh, if possible)
- Less elements (~ 100) should be enough to achieve the same accuracy with a better or “smarter” mesh