تکلیف سری هفتم — موعد تحویل ۲۷ آذرماه

1- The plate structure shown in Figure 1 is loaded and deforms in the plane of the figure. The applied load at D and the supports at I and N extend over a fairly narrow area. Give a list of what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

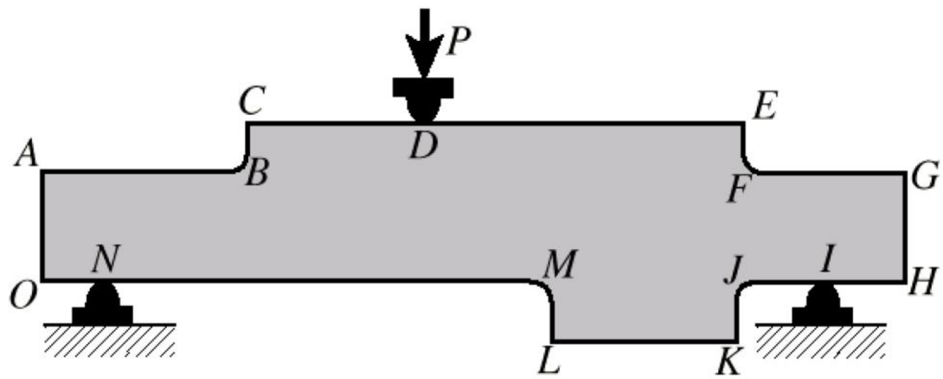
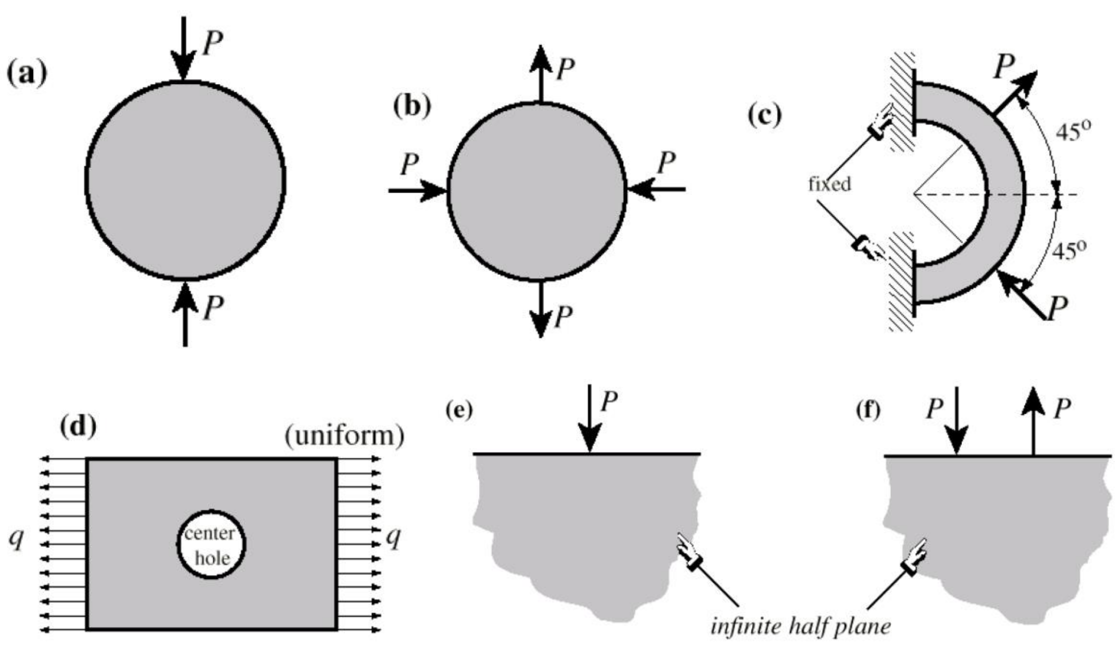


Fig. 1 The plate structure

2- Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in Figure 2a-f. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

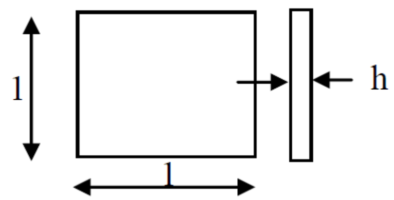


3- Consider the square, isotropic, elastic body of thickness h shown in Figure below. Suppose that the displacement are approximated by:

$$u(x,y) = (1-x)yu_1 + x(1-y)u_2$$

$$v(x,y) = 0$$

Assuming that the body is in the state of the plane stress. Derive 2 by 2 stiffness matrix of the unit square.



4- Construct the weak form and, whenever possible, quadratic functional.

a) ***The Timoshenko (shear-deformable) beam theory:***

$$\left.\begin{aligned} -\frac{d}{dx}\left[GKA\left(\frac{dw}{dx}+\Psi\right)\right] &= f \\ -\frac{d}{dx}\left(EI\frac{d\Psi}{dx}\right) + GKA\left(\frac{dw}{dx}+\Psi\right) &= 0 \end{aligned}\right\} \quad \text{for } 0<x<L$$

$$w(0) = w(L) = 0, \quad \left(EI\frac{d\Psi}{dx}\right)\bigg|_{x=0} = \left(EI\frac{d\Psi}{dx}\right)\bigg|_{x=L} = 0$$

where G, K, A, E, I, and f are functions of x.

b) ***The Euler–Bernoulli–von Kármán nonlinear theory of beams:***

$$-\frac{d}{dx}\left\{a\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]\right\} = q \quad \text{for } 0<x<L$$

$$\frac{d^2}{dx^2}\left(b\frac{d^2w}{dx^2}\right) - \frac{d}{dx}\left\{a\frac{dw}{dx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]\right\} = f$$

$$u = w = 0 \quad \text{at } x = 0, L; \quad \left(\frac{dw}{dx}\right)\bigg|_{x=0} = 0; \quad \left(b\frac{d^2w}{dx^2}\right)\bigg|_{x=L} = M_0$$

where a, b, q, and f are functions of x, and M_0 is a constant. Here u denotes the axial displacement and w the transverse deflection of the beam.

5- Compute the coefficient matrix and the right-hand side of the N-parameter Rayleigh–Ritz approximation of the equation

$$-\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right]=0 \quad \text{for } 0<x<1$$

$$u(0)=0, \qquad u(1)=1$$

Use algebraic polynomials for the approximation functions. Specialize your result for $N=2$ and compute the Ritz coefficients.

Answer: $c_1=\frac{55}{131}$ and $c_2=-\frac{20}{131}$.

6- Solve the Poisson equation governing heat conduction in a square region (see Example 2.6):

$$-k\nabla^2 T=q_0$$

$$T=0 \quad \text{on sides } x=1 \text{ and } y=1$$

$$\frac{\partial T}{\partial n}=0 \text{ (insulated)} \quad \text{on sides } x=0 \text{ and } y=0$$

using a one-parameter Rayleigh–Ritz approximation of the form

$$T_1(x,y)=c_1(1-x^2)(1-y^2)$$

Answer: $c_1=\dfrac{5q_0}{16k}$.

Find a one-parameter approximate solution of the nonlinear equation

$$-2u\frac{d^2u}{dx^2}+\left(\frac{du}{dx}\right)^2=4 \quad \text{for } 0<x<1$$

subject to the boundary conditions $u(0)=1$ and $u(1)=0$, and compare it with the exact solution $u_0=1-x^2$. Use (*a*) the Galerkin method, (*b*) the least-squares method, and (*c*) the Petrov–Galerkin method with weight function $w=1$.

Answer: (*a*) $(c_1)_1=1$, $(c_1)_2=-2$.