

shows the acceleration polygon, from which the required acceleration \bar{A}_C is determined to be

$$\bar{A}_C = 12.2 \text{ in./sec}^2 \quad (\text{as directed})$$

In the example above, it is to be noted that only two calculations were necessary to complete the analysis after the scales were determined. These were to determine V_B and A_B^T , the velocity and acceleration magnitudes of the first link. If this link were to rotate with a constant angular velocity, the calculation of A_B^T would not have been necessary, since the value of this acceleration would be zero. Thus, for a complete graphical analysis, the maximum number of calculations necessary is two.

12.2 EQUIVALENT LINKAGE METHOD

Determining the acceleration of points on many *higher-paired mechanisms*, such as those with rolling and sliding contacts, can become rather involved if a point-to-point analysis is attempted. This is because of the need to know the curvature of the path traced by a point on one link relative to the other and to apply the Coriolis law. If no easily recognized path is found, the analysis can be difficult.

To simplify this problem, the use of equivalent linkages has been found most effective. In application, an equivalent linkage replaces a higher-paired contact with appropriate lower pairs that will produce the correct values of velocities and accelerations for the instantaneous phase under consideration. An *equivalent linkage* may then be defined as one that produces identical motion as the part being analyzed for a given position or phase.

Figures 12.6 and 12.7 show several mechanisms with their equivalent linkages depicted as line diagrams on the right. Note that the rolling and sliding surfaces have been replaced by pin joints as part of a more simplified four-bar linkage. Note also that in each case, the floating link of the equivalent linkage is drawn along the common normal of the two contacting surfaces and connects the centers of curvature of the surfaces.

Although an equivalent linkage is generally valid only for a given instant or phase and does not ordinarily apply to a complete cycle, there are some instances in which the equivalent linkages of some higher-paired mechanisms will duplicate the input/output motion of those mechanisms throughout their motion cycle. Some examples are shown in Figures 12.6 and 12.7.

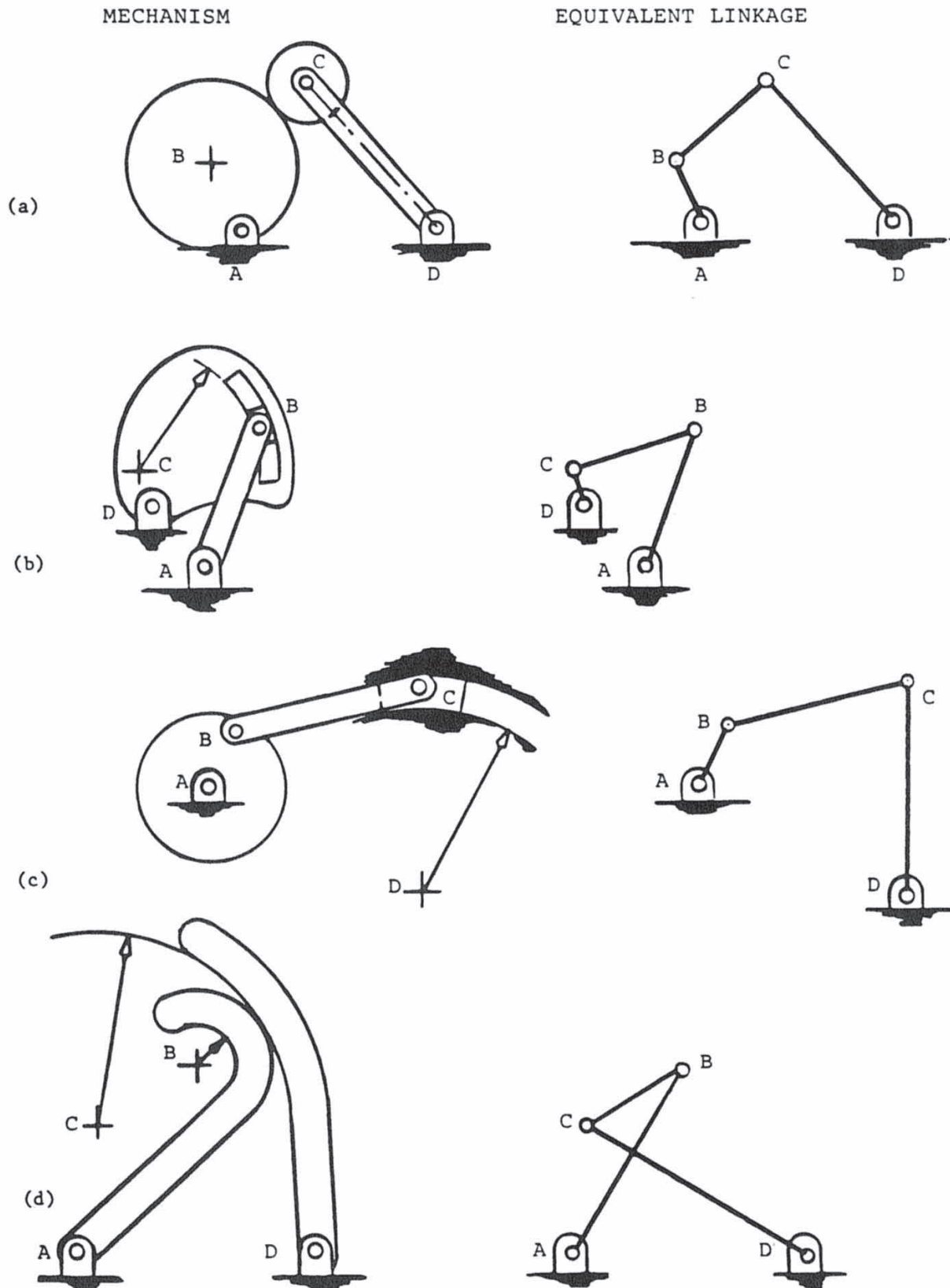


Figure 12.6 Kinematically equivalent four-bar linkages.

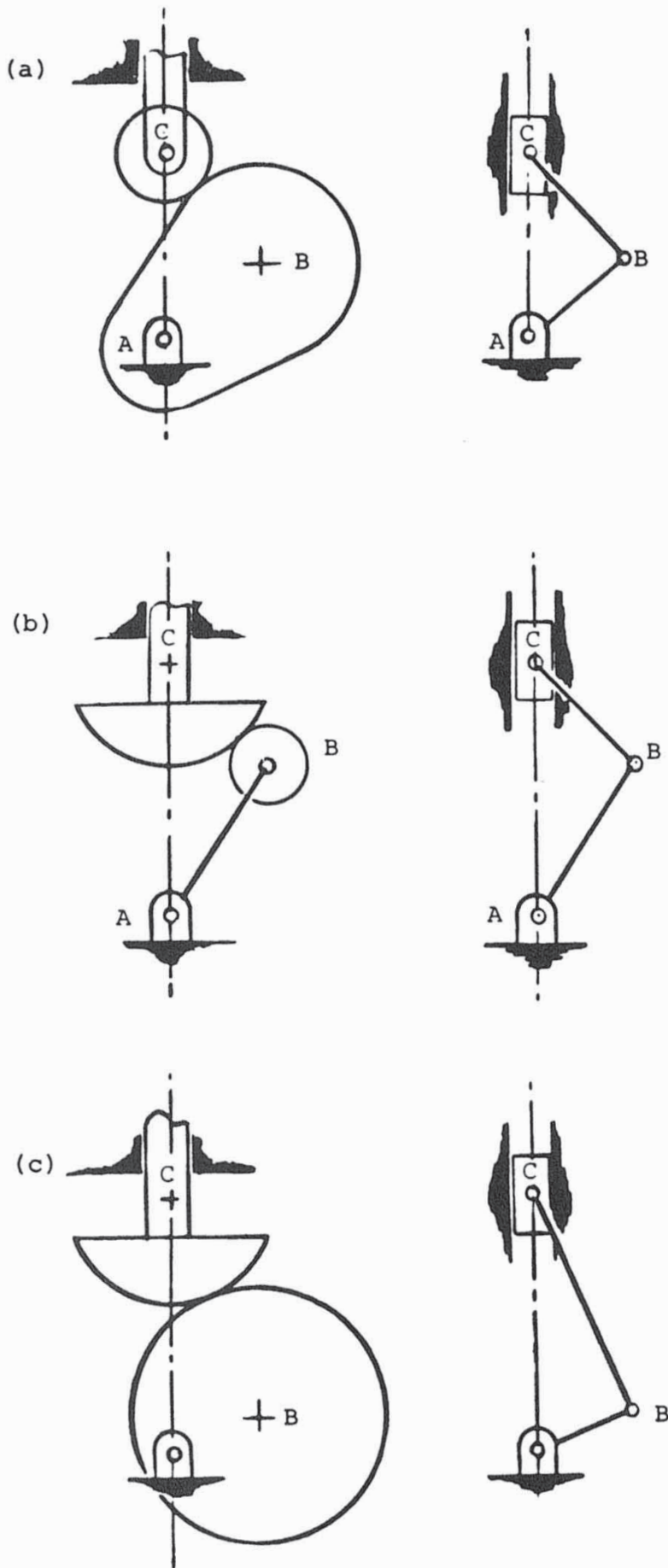


Figure 12.7 Kinematically equivalent slider-crank linkages.

EXAMPLE 12.2

Consider the cam mechanism shown in Figure 12.8(a). The cam (2) rotates counterclockwise at a constant angular velocity of 2 rad/sec. Find the acceleration of follower (4) using the equivalent linkage method.

SOLUTION

The equivalent mechanism for the cam mechanism given is the simple slider-crank ABC shown in Figure 12.8(b), for which the velocity and acceleration diagrams are readily obtained, as shown in Figures 12.8(c) and (d). Applying the velocity polygon construction procedure, we obtain

$$V_B = 0.9(2) = 1.8 \text{ in./sec}$$

$$V_C = 1.45 \text{ in./sec} \quad (\text{from the velocity polygon})$$

$$V_{C/B} = 0.6 \text{ in./sec} \quad (\text{from the velocity polygon})$$

Applying the acceleration construction procedure, we obtain

$$A_B^N = 0.9(2)^2 = 3.6 \text{ in./sec}^2$$

$$A_B^T = 0$$

$$A_{C/B}^N = \frac{(0.6)^2}{2.0} = 0.18 \text{ in./sec}^2$$

$$A_{C/B}^T = 3.8 \text{ in./sec}^2 \quad (\text{from the acceleration polygon})$$

$$\bar{A}_C = 2.8 \text{ in./sec}^2 \quad (\text{directed as shown})$$

Note that the resulting acceleration of point C (\bar{A}_C) is exactly the same as that obtained for point P4 (the same point) in Section 9.7, using an alternative method.

12.3 CENTER OF CURVATURE

The ability to locate the center of curvature of a path generated by a coupler point in plane motion is very important in the analysis or synthesis of a mechanism. With this ability, one is able not only to

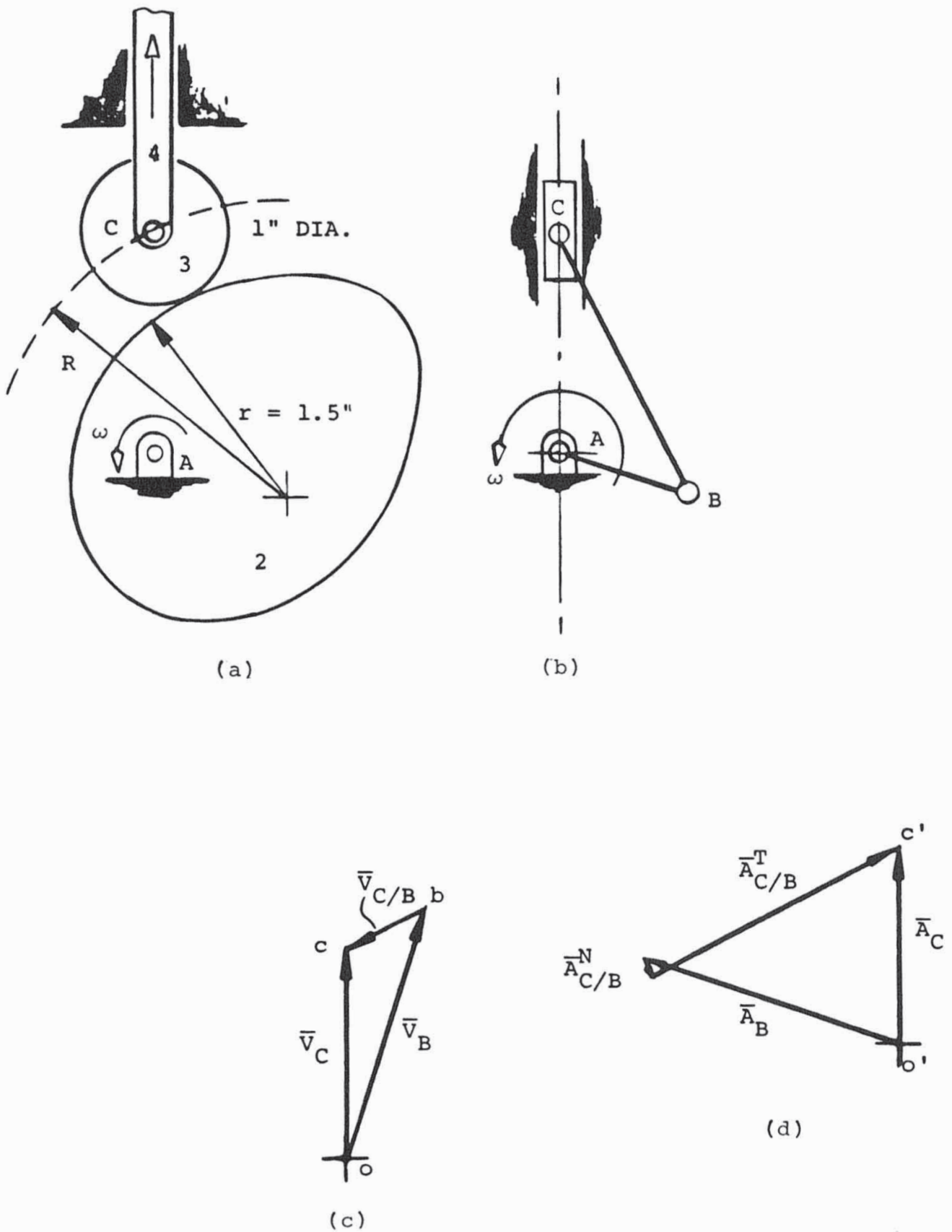


Figure 12.8 Acceleration analysis of a cam-follower mechanism: (a) Cam-follower mechanism; (b) equivalent linkage; (c) velocity polygon; (d) acceleration polygon.

determine the instantaneous velocity and acceleration of that point, but also to locate other key parameters related to the path geometry, such as the inflexion point, which is a useful indicator of the existence of straight-line motion for some finite distance during the mechanism cycle. Also, in rolling contact mechanisms such as cams, planetary gears, and roller bearings, one can, by locating the centers of curvature of the surfaces at the contact points, easily find the equivalent linkages that will simplify the analyses of those mechanisms.

Graphical Procedures

The following graphical methods will illustrate some of the various approaches that are available to determine the centers of curvature of paths described by points in a mechanism. We choose for this illustration a typical planetary gear mechanism where gear 2 rolls on gear 1, which is fixed. The centers of gears 1 and 2 are A_o and A , respectively, and the point of contact between the gears is defined as P . Let it be required to determine the center of curvature B_o of any point B on the revolving gear.

HARTMANN'S METHOD (Figure 12.9)

1. Assume gear 1 is held constant and find $\bar{V}_A = PA\omega_2 (\perp PA)$.
2. Lay out pole velocity \bar{V}_P by drawing a vector parallel to \bar{V}_A to meet a line of proportionality connecting \bar{V}_A to A_o .
3. Determine \bar{V}_B from proportionality as follows:
 - a. Rotate point B about P to point B' on the common normal.
 - b. From B' , draw vector $\bar{V}_{B'}$ parallel to \bar{V}_A and terminating at a line of proportionality, drawn from P through the head of \bar{V}_A .
 - c. Rotate vector $\bar{V}_{B'}$ back to point B .
4. Determine V'_P by dropping a perpendicular from the head of \bar{V}_P to meet a line drawn from P parallel to \bar{V}_P .
5. Draw a line of proportionality connecting heads of vectors \bar{V}_B and \bar{V}'_P and intersecting ray PB at a point B_o . This point will define the required center of curvature.