



Physics-informed neural networks for local and nonlocal modeling in continuum damage mechanics

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ABSTRACT

In Continuum Damage Mechanics (CDM), numerical algorithms are widely used to predict material degradation in engineering applications. However, these methods typically require iterative computations at each time step, which creates significant challenges for complex geometries. This study presents a Physics-Informed Neural Network (PINN) model based on the Lemaitre damage model, a well-established framework in CDM, for predicting damage evolution in ductile materials. Training data for the PINN were generated from 2,000 random strain paths, with stress and damage values computed using a numerical Lemaitre model implemented in an Abaqus UMAT. The numerical model was validated with benchmark tests to ensure accuracy. The PINN architecture integrates Gated Recurrent Units (GRUs), enabling it to capture strain path dependency and history effects. Results show that the PINN closely matches the numerical solution in benchmark evaluations. Furthermore, incorporating damage growth equations into the loss function significantly improves predictions, especially for data outside the training region, and enhances physical consistency compared to standard neural networks. The application of PINNs in nonlocal damage mechanics was also investigated. A key challenge in local damage mechanics is the mesh dependency of numerical solutions, which can lead to unrealistic localization effects. While nonlocal damage models mitigate this issue by introducing a characteristic length to smooth damage field, this study demonstrates that the trained PINN, despite being trained on local data, successfully predicts nonlocal damage behavior, highlighting its potential as a data-driven tool for both local and nonlocal damage modeling.

1. Introduction

In computational mechanics, the development and implementation of nonlinear material models capable of describing complex material behavior remain challenging tasks. Constitutive models for inelastic materials are typically derived within a thermodynamic framework that introduces internal variables and evolution laws governing the material response (Lemaitre, 1985). After establishing the theoretical formulation, implementing these models in a computational environment requires significant numerical effort and specialized programming. In finite element analysis, the constitutive equations are typically solved using iterative procedures such as the Newton–Raphson method (De Souza Neto et al., 2008). Because these nonlinear equations must be solved at every integration point and for every iteration of the global equilibrium procedure, the computational cost can become substantial, particularly for complex constitutive models and large-scale simulations.

1.1. NNs in constitutive modeling

In computational material mechanics, a growing trend involves using machine learning techniques, such as neural networks, to mitigate the high computational cost of traditional simulations. This approach focuses on training neural networks with extensive datasets to predict material properties (Mayer et al., 2022; Tipu et al., 2022; Weber et al., 2022) or using them as surrogate models for specific material systems (Bock et al., 2019; Friemann et al., 2023; Fritzen et al., 2019). Among the various neural network architectures employed for this purpose, recurrent neural networks (RNNs) have proven particularly effective for modeling path-dependent material behavior because they naturally capture temporal dependencies in sequential data. Applications of RNN-based models in constitutive mechanics include the work of Mozaffar et al. (2019), who demonstrated the capability of recurrent neural networks to learn history-dependent constitutive responses, and Zhang and Mohr (2020), who developed neural network-based

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Table 1

Some extended PINNs techniques (Lawal et al., 2022).

Reference	Technique
(Jagtap et al., 2020)	Conservative physics-informed neural network (cPINN)
(Jagtap and Karniadakis, 2020)	Extended physics-informed neural networks (XPINNs)
(De Ryck et al., 2024)	PINN error estimates
(Pang et al., 2020)	Nonlocal physics-informed neural networks (nPINNs)
(Yang et al., 2021)	Bayesian physics-informed neural networks (B-PINNs)
(Kharazmi et al., 2021)	Variational physics-informed neural networks (hp-VPINNs)
(Pu et al., 2021)	Improved PINN method
(Schiassi et al., 2021)	Physics-informed neural network theory of functional connections (PINN-TFC)
(Rafiq et al., 2022)	Deep spectral feature aggregation physics-informed neural network (DSFA-PINN)
(Raynaud et al., 2022)	Modal physics-informed neural networks (ModalPINNs)
(Wight and Zhao, 2021)	Adaptive physics informed neural networks
(Haitsiukevich and Ilin, 2023)	PINNs with ensemble models

constitutive models capable of accurately representing complex stress–strain relationships under multiaxial loading conditions. Further developments in this direction include hybrid data–driven frameworks combining machine learning with classical constitutive modeling principles (Friemann et al., 2023; Ray et al., 2024).

Moreover, neural networks have been integrated with phase-field models to overcome limitations of traditional phase-field models, such as the need for fine meshes and small time steps (Koepe et al., 2017). The networks are trained to replicate the response of a finite element–phase-field system, enabling accurate prediction of brittle fracture behavior with significantly reduced computational cost. Building on this, Xie et al. (2023) used backpropagation neural networks with phase-field models to predict the remaining life of structures. The network, trained on strain–lifespan data from the phase-field model, accurately estimates structural lifespan with minimal computational cost (see also Feng et al., 2021).

While purely data–driven neural networks rely entirely on training datasets, recent research has focused on embedding physical principles directly into machine learning models to improve robustness and generalization. These approaches incorporate thermodynamic constraints or physical laws into the learning process, ensuring physically consistent predictions even when training data are limited or noisy. For example, Masi et al. (2021) introduced thermodynamics–based artificial neural networks that embed thermodynamic consistency into the network architecture. Vlassis and Sun (2021) proposed an interpretable deep learning framework for elastoplasticity in which the yield function evolves as a level set controlled by machine learning–based hardening laws. Flaschel et al. (2023) developed an unsupervised framework for discovering material laws for generalized standard materials while enforcing convexity and thermodynamic consistency. Other hybrid approaches combining phenomenological models and machine learning have also been proposed to improve extrapolation capabilities in low–data regimes (Fuhg et al., 2023). Additional work on thermodynamically consistent neural network constitutive models can be found in Wei et al. (2023) and Klein et al. (2022).

A particularly influential branch of this line of work is that of physics–informed neural networks (PINNs) and related variational formulations, which have been proposed as mesh–free alternatives for solving forward and inverse problems governed by partial differential equations (PDEs).

1.2. Physics–informed neural networks

The physics–informed neural networks (PINNs) framework was first introduced by Raissi et al. (2019) as a deep learning methodology for solving forward and inverse problems governed by partial differential

equations. In the classical PINN formulation, the unknown solution fields of a PDE—such as displacement, temperature, or velocity—are represented by a neural network. The network parameters are then trained such that the residuals of the governing PDE, together with the corresponding boundary and initial conditions, are minimized at a set of collocation points within the computational domain. This approach can be interpreted as a strong–form collocation or weighted residual method, where the loss function typically includes contributions from the PDE residual, boundary conditions, and initial conditions.

Beyond solving field equations, PINNs have also been explored for constitutive modeling and inverse identification problems in solid mechanics. For example, Niu et al. (2023) developed a PINN framework for finite–strain elastoplasticity with isotropic hardening, embedding equilibrium equations, boundary conditions, and constitutive relations directly into the loss function. Haghighat et al. (2023) proposed a PINN–based methodology for discovering elastoplastic constitutive models by incorporating inequality constraints associated with plastic admissibility conditions. In earlier work, Haghighat et al. (2021) applied PINNs to inverse analysis and surrogate modeling in solid mechanics, demonstrating improved convergence when combined with Iso-geometric Analysis (IGA). Other recent developments include the Elastoplastic Neural Network (EPNN) proposed by Eghbalian et al. (2023), which captures key features of classical elastoplasticity while improving training efficiency and extrapolation capabilities.

To improve generalization, several extensions of the original PINN framework have been proposed, some of which are summarized in Table 1. For example, Variational Physics-Informed Neural Networks (VPINNs) adopt a weak–form discretization in which the residual is integrated against suitable test functions over the domain, resulting in a neural network analog of classical Galerkin methods (Kharazmi et al., 2021).

1.3. PINNs in damage mechanics

In damage mechanics, local material failure is typically characterized by stiffness degradation, described by various damage variables. Zheng et al. (2022) employed PINNs to simulate crack propagation in quasi-brittle materials by incorporating an energy minimization principle and a four-parameter damage model, ensuring thermodynamic consistency. Their approach effectively captured intricate crack paths under diverse loading conditions, enhancing convergence and addressing ill-conditioned problems. Additionally, the PINNs strategy has been integrated with the Finite Element Method (FEM) for simulating the damaged constitutive behavior of materials. Pantidis and Mobasher (2023) introduced the Integrated Finite Element Neural Network (I-FENN) framework to accelerate nonlinear computational mechanics by embedding a PINN in the finite element stiffness function. I-FENN utilizes different PINNs to predict the nonlocal equivalent strain and its derivative with respect to the local strain, enabling efficient modeling of nonlocal damage behavior in quasi-brittle materials. Rezaei et al. (2024) applied physics-informed neural networks (PINNs) to solve constitutive relations for nonlinear, path-dependent material behavior. The proposed framework enforces thermodynamic constraints and predicts key material state variables, including stress, free energy, and internal variables, under arbitrary loading conditions. The approach eliminates the need for iterative Newton-type solvers typically required in traditional constitutive updates and can be directly implemented as a user-defined material model in finite element simulations. The methodology was validated using the classical von Mises plasticity model with nonlinear hardening as well as local damage models for interface cracking with nonlinear softening, demonstrating its capability to capture complex three-dimensional path-dependent damage behavior.

1.4. Variational energy–based neural networks

In contrast to the residual–based formulations, an alternative and

closely related line of research leverages the variational (energetic) structure of boundary value problems. In this context, the Deep Ritz Method (DRM) and related approaches formulate the training problem in terms of minimizing a discretized energy functional rather than enforcing pointwise PDE residuals. Building on this variational philosophy, Samaniego et al. (2020) introduced the Deep Energy Method (DEM) for computational solid mechanics. In DEM, the unknown mechanical fields—such as displacement or rotation fields—are represented by a neural network, while the loss function is defined as a numerical approximation of the total potential energy of the system. This energy functional is evaluated using numerical quadrature at a set of integration points, typically associated with a background mesh.

More specifically, in DEM the neural network outputs serve as trial functions for the mechanical fields, while the loss function corresponds to the discretized total potential energy consisting of internal strain energy and the potential of external loads. By minimizing this energy with respect to the network parameters, the method yields an approximate equilibrium configuration consistent with the variational principle of minimum potential energy. Samaniego et al. (2020) demonstrated that DEM can be successfully applied to various problems in solid mechanics, including hyperelasticity, Kirchhoff plates, and topology optimization, and clearly highlighted the conceptual differences between DEM and residual-based PINNs.

Compared with residual-based PINNs, these variational energy-based approaches offer several advantages. Because the loss function directly corresponds to the system's total potential energy, the formulation retains a clear thermodynamic interpretation. Natural boundary conditions are automatically satisfied through the variational principle, and the order of derivatives appearing in the loss function is typically lower than in strong-form residual approaches. These features often result in improved numerical stability and training efficiency for problems involving high-order PDEs and fracture phenomena.

Following the work of Samaniego et al. (2020), energy-based neural formulations have been extended to phase-field models of brittle fracture. In this context, Goswami et al. (2020b) developed a class of variational, energy-minimizing neural network approaches in which both the displacement field and the phase-field variable are represented by neural networks, and the equilibrium configuration is obtained by minimizing the total potential energy of the system. Building upon this variational framework, subsequent studies introduced enhancements such as transfer learning (Goswami et al., 2020a) to improve computational efficiency across loading steps, and adaptive local refinement strategies to accurately resolve crack initiation and propagation. In a related development, He et al. (2023) extended the DEM to elastoplasticity by incorporating path dependence and irreversibility. By coupling DEM with the radial return algorithm and a loss function derived from the discrete variational formulation of plasticity, their framework accurately predicts stress-strain responses, including cyclic loading and heterogeneous material behavior, highlighting the potential of energy-based physics-informed neural networks for elastoplastic simulations.

More recently, Zhao and Shao (2024) developed the Discontinuity Embedded Deep Energy Method (DEDEM), an extension of DEM designed for fracture mechanics problems involving strong discontinuities. In DEDEM, cracks and interfaces are represented using discontinuous enrichment functions derived from signed distance functions that are embedded directly into the neural network representation of the solution fields. As in DEM, the total potential energy of the system serves as the loss function, and the equilibrium solution is obtained by minimizing this energy. The method has been shown to provide accurate and computationally efficient solutions for fracture problems involving discontinuities, while retaining the advantages of variational energy-based formulations. In a related contribution, Wang et al. (2026) examined stability and robustness challenges in DEM for both forward and inverse settings and introduced the Energy-Informed Neural Operator (EINO), which combines FEM-informed regularization with a neural-operator

architecture to further improve the reliability of energy-based neural formulations.

Overall, these developments demonstrate that deep energy and variational neural methods provide powerful alternatives to classical residual-based PINNs for solving complex mechanics problems. By exploiting the underlying variational structure of continuum mechanics, these methods offer improved physical consistency, numerical robustness, and interpretability.

1.5. Scope of the present work

The present contribution addresses a conceptually different level of modeling compared to the above-mentioned DEM and energy-based PINN formulations. Rather than using a neural network to directly solve the boundary value problem by minimizing a global variational energy or PDE residual, we employ a recurrent neural network as a data-driven surrogate constitutive model for integral-type nonlocal damage. In this sense, the proposed architecture is closer in spirit to a UMAT-like material subroutine in a finite element code than to a mesh-free solver for PDEs. In the context of finite element analysis, a User Material (UMAT) subroutine is a user defined constitutive routine that enables the implementation of custom material models within a finite element framework. During each load increment and at every integration point, the finite element solver provides the current strain state and relevant history variables to the UMAT, which returns the corresponding stress response and updates the internal variables according to the prescribed constitutive relations. In this way, the UMAT acts as the constitutive operator linking strain histories to stress and internal variable evolution, while the global equilibrium equations and boundary conditions are handled by the finite element solver. Because many advanced material models are implemented through such material interfaces, UMATs provide a flexible mechanism for incorporating complex constitutive behavior into numerical simulations. From this perspective, the recurrent neural network proposed in the present work can be interpreted as a data driven constitutive operator that plays a role conceptually similar to a UMAT, mapping strain histories to the corresponding damage evolution at the material point level.

It is therefore important to emphasize that the present approach does not belong to the class of neural methods that are directly employed as solvers for partial differential equations. Instead, it represents a supervised data-driven constitutive modeling strategy in which the equilibrium equations and boundary conditions are enforced by the reference numerical solver used to generate the training data. The neural network thus acts as an efficient surrogate representation of the nonlocal constitutive law.

In this work, we consider a class of nonlocal damage models in which the damage driving variable at a material point is defined as a weighted spatial average of strain histories over a neighborhood with a prescribed characteristic length. This formulation leads to a nonlocal evolution of damage that regularizes strain localization and yields mesh-objective structural responses. However, the evaluation of the nonlocal averages at each integration point and time step can be computationally intensive. To alleviate this cost, we propose to learn the mapping from local and nonlocal strain histories to the corresponding damage evolution using a gated recurrent neural network (GRU). The network takes as input the strain history at a given integration point together with a discretized nonlocal average of neighboring strain histories, and outputs the damage evolution at that point. Because recurrent neural networks are designed to process sequential data, they are particularly well suited for representing the path-dependent behavior typical of damage and inelastic materials.

The remainder of the paper is organized as follows. Section 2 briefly reviews the nonlocal damage formulation and the corresponding integral averaging operator that motivates the nonlocal features used in the neural network. Section 3 presents the proposed recurrent neural network architecture together with the construction of the training

dataset and the supervised learning procedure. Section 4 provides numerical examples demonstrating the predictive performance of the trained surrogate model for different strain paths. Finally, Section 5 summarizes the main findings of the study and outlines possible directions for future research.

2. Damage constitutive model

Lemaitre's ductile damage model (Lemaitre, 1992) is a CDM approach for describing the progressive deterioration of materials under mechanical loading. It characterizes damage evolution in ductile materials by introducing a damage variable, D , that accounts for void nucleation, growth, and coalescence, resulting in material softening and eventual failure. The damage variable D ranges between 0 and 1, where $D = 0$ represents the undamaged state and $D = 1$ represents complete material failure. In the Lemaitre damage model, the free energy ψ depends on a set of state variables:

$$\psi = \psi(\epsilon^e, R, \mathbf{X}, D) \quad (1)$$

where ϵ^e is the elastic strain tensor and R and D are the scalar internal variables associated, respectively, with isotropic hardening and isotropic damage. The second-order tensor $\mathbf{X}$ represents the internal variable associated with kinematic hardening.

Under the hypothesis of decoupling between elasticity-damage and plastic hardening, the specific free energy is assumed to be given by the sum:

$$\psi = \psi^{ed}(\epsilon^e, D) + \psi^p(R, \mathbf{X}) \quad (2)$$

where ψ^{ed} and ψ^p are, respectively, the elastic-damage and plastic contribution to the free energy. The thermodynamic force conjugate to the damage internal variable is given by:

$$Y = \bar{\rho} \frac{\partial \psi}{\partial D} = -\frac{1}{2} \epsilon^e : \mathbf{D}^e : \epsilon^e \quad (3)$$

where $\bar{\rho}$ denotes mass density and $\mathbf{D}^e$ is the standard isotropic elasticity tensor. Using the elastic stress-strain law gives:

$$Y = \frac{-1}{2E(1-D)^2} \left[(1+\nu) \boldsymbol{\sigma} : \boldsymbol{\sigma} - \nu (tr \boldsymbol{\sigma})^2 \right] \quad (4)$$

where $\boldsymbol{\sigma}$ is Cauchy stress tensor and E and ν are, respectively, the Young's modulus and Poisson's ratio. The plastic contribution $\psi^p(R, \mathbf{X})$ to the free energy is chosen as a sum of an isotropic and a kinematic hardening-related term:

$$\psi^p(R, \mathbf{X}) = \psi^I(R) + \frac{a}{2\bar{\rho}} \mathbf{X} : \mathbf{X} \quad (5)$$

where a is a material constant and the isotropic hardening contribution, $\psi^I(R)$, is an arbitrary function of the single argument R . The thermodynamic force associated with isotropic hardening is:

$$\kappa(R) = \bar{\rho} \frac{\partial \psi^p(R, \mathbf{X})}{\partial R} = \bar{\rho} \frac{\partial \psi^I(R)}{\partial R} \quad (6)$$

the thermodynamic force associated with kinematic hardening, the back-stress tensor, β is:

$$\beta = \bar{\rho} \frac{\partial \psi}{\partial \mathbf{X}} = a \mathbf{X} \quad (7)$$

For the yield function Φ , the following von Mises-type form is adopted:

$$\Phi(\boldsymbol{\sigma}, \kappa, \beta, D) = \frac{\sqrt{3J_2(s - \beta)}}{1 - D} - \sigma_{y0} - \kappa \quad (8)$$

where the material parameter σ_{y0} is the uniaxial yield stress of the virgin material and s is the deviatoric part of the Cauchy stress tensor. The flow

potential Ψ is assumed to be given by:

$$\Psi = \Phi + \frac{b}{2a} \beta : \beta + \frac{r}{(1-D)(1+s)} \left(\frac{-Y}{r} \right)^{s+1} \quad (9)$$

where a , b , r and s are material constants and the damage evolution law is defined as:

$$\dot{D} = -\lambda \frac{\partial \Psi}{\partial Y} = \lambda \frac{1}{1-D} \left(\frac{-Y}{r} \right)^s \quad (10)$$

where λ is the plastic multiplier. Applying the second law of thermodynamics in terms of the Clausius-Duhem inequality leads to the following condition for the damage evolution law:

$$\dot{D} \geq 0. \quad (11)$$

The model uses thermodynamic principles to derive damage and hardening evolution laws, ultimately capturing ductile failure. The presented formulations provide a foundation for developing an improved loss function for the neural network.

3. Neural network modeling

In this section, a PINN is developed to learn the nonlinear constitutive behavior governed by Lemaitre's ductile damage model, capturing the relationship between input strain paths and their corresponding stress and damage responses. To achieve this, a suitable loss function is defined, and a comprehensive dataset is generated using random strain paths as inputs and stress-damage outputs as targets.

PINNs leverage machine learning techniques to solve scientific problems by constraining the model to adhere to established physical laws (Chen et al., 2020; Kissas et al., 2020; Raissi et al., 2020). This is typically achieved by incorporating residuals of the governing differential equations into the loss function as additional terms during training (Monaco and Apletti, 2023). The key to understanding why PINNs enforce physical constraints lies in neural network training. In standard supervised learning, weights and biases are adjusted to minimize the loss function. In PINNs, governing physical equations are embedded directly into this loss. During backpropagation, the network updates parameters to drive the entire loss toward zero, which, in turn, drives the physics term toward zero. This is equivalent to satisfying the governing equations, meaning the network's predictions naturally respect physical constraints.

In the present framework, the neural network is not employed to directly solve the structural boundary value problem. In contrast to many physics informed neural network approaches in computational mechanics, where the neural network approximates the displacement field and the strain field is obtained through spatial differentiation, the present study adopts a different strategy. The strain paths at the Gauss integration points are assumed to be known for the considered problem and are directly provided as inputs to the neural network. Consequently, the PINN does not predict structural quantities such as displacements or forces, nor does it solve the equilibrium equations of the system. Instead, it acts as a surrogate constitutive model that maps the strain history at each integration point to the corresponding damage evolution.

3.1. Loss function

The choice of a loss function for a neural network may vary depending on the specific application. For regression problems, a common approach is to use functions such as mean squared error (MSE) or mean root mean squared error (RMSE). In this work, RMSE, as defined in Eq. (12), is used because it penalizes larger errors more heavily and provides a loss value in the same unit as the predicted variable, making it sensitive to significant deviations and easy to interpret:

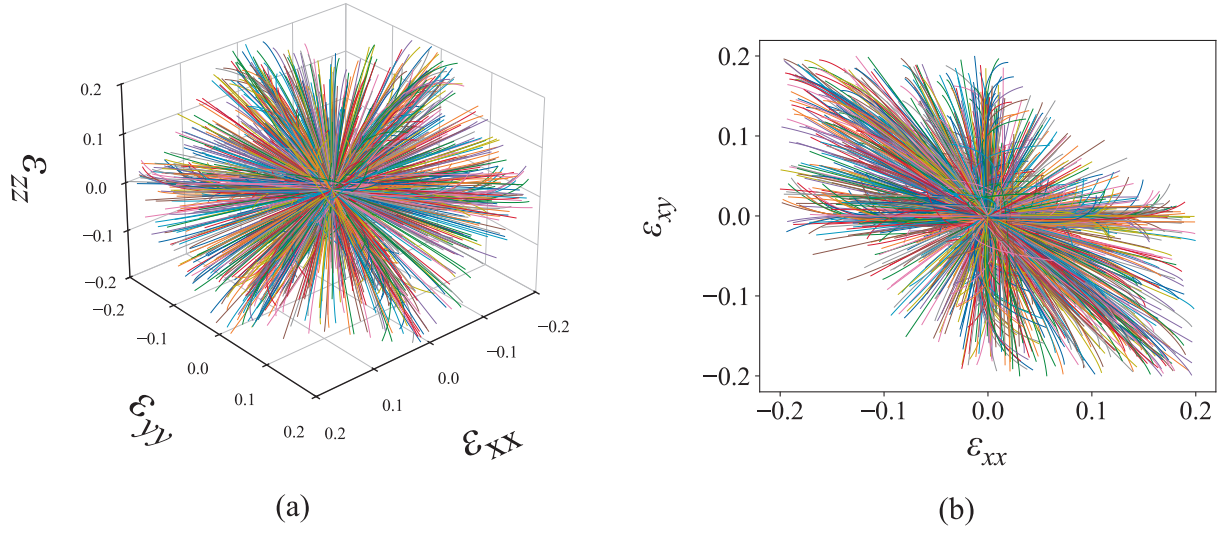


Fig. 1. 2,000 randomly generated strain paths satisfying Eq. (19), shown in (a) the three-dimensional normal strain space and (b) two arbitrary shear components.

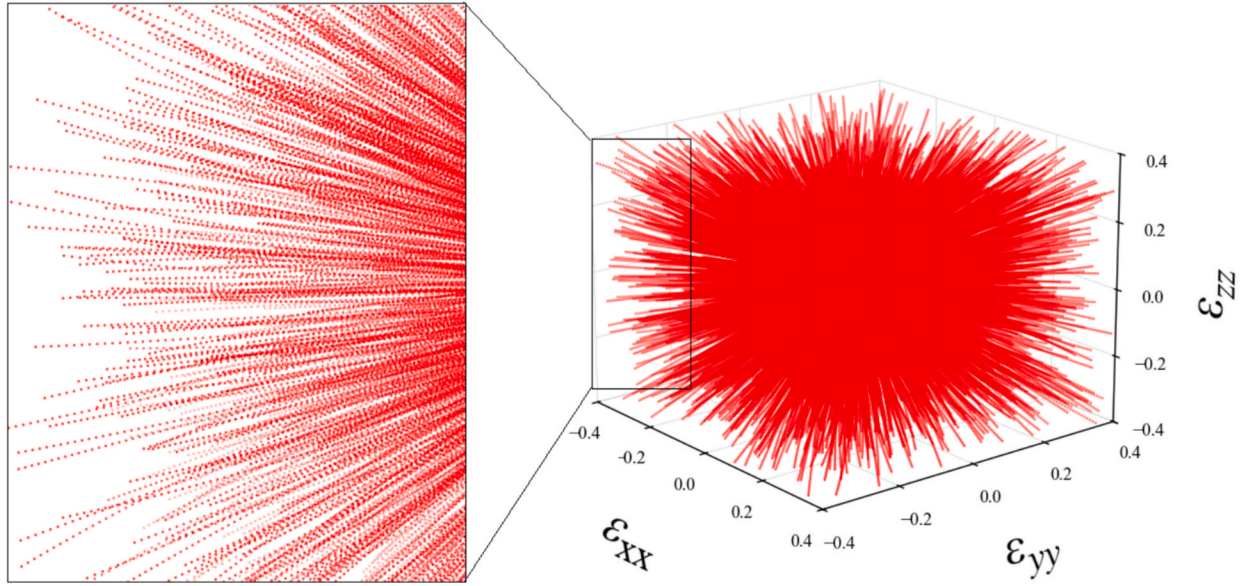


Fig. 2. Distribution of 400,000 collocation points in the normal strain space, generated from 4,000 strain paths with 100 increments each. A similar distribution appears in the shear strain space.

Table 2

Material properties of cylindrical notched bar problem (De Souza Neto et al., 2008).

E	210 GPa
ν	0.3
Hardening curve	$\sigma_y(R) = 620 + 3300(1 - \exp(-0.4 R))$ MPa
s	1.0
r	3.5 MPa

$$\mathcal{L}_{RMSE} = \sqrt{\frac{1}{N} \sum_i^N (U_{NN}(x_i; \theta) - U_{true}(x_i))^2} \quad (12)$$

where U_{NN} is the neural network prediction, U_{true} is the desired output, x_i is the input data, θ is the network parameter, and N is the number of samples. Using the constitutive equation introduced for the Lemaitre damage model in Eq. (10), it is desirable that Eq. (13) be satisfied:

$$\dot{D} - \dot{\lambda} \frac{1}{1-D} \left(\frac{-Y}{r} \right)^s = 0 \quad (13)$$

Therefore, the neural network loss function is enhanced by incorporating the damage-based loss term $\mathcal{L}_D$, to ensure that the thermodynamic constraints on damage growth are satisfied in the network's predictions:

$$\mathcal{L}_D = D_{n+1} - D_n - \frac{d\lambda}{1-D_{n+1}} \left(\frac{-Y(d\lambda)_{n+1}}{r} \right)^s \quad (14)$$

where $d\lambda$ is the incremental plastic multiplier, and the subscripts n and $n+1$ denote the known and the desired increments in the strain path, respectively. In addition, the yield condition and the stress update rule for the plastic-damage response, based on the Return-Mapping algorithm proposed by de Souza Neto et al. (2008), were also introduced as additional loss terms, as expressed in the following equations respectively:

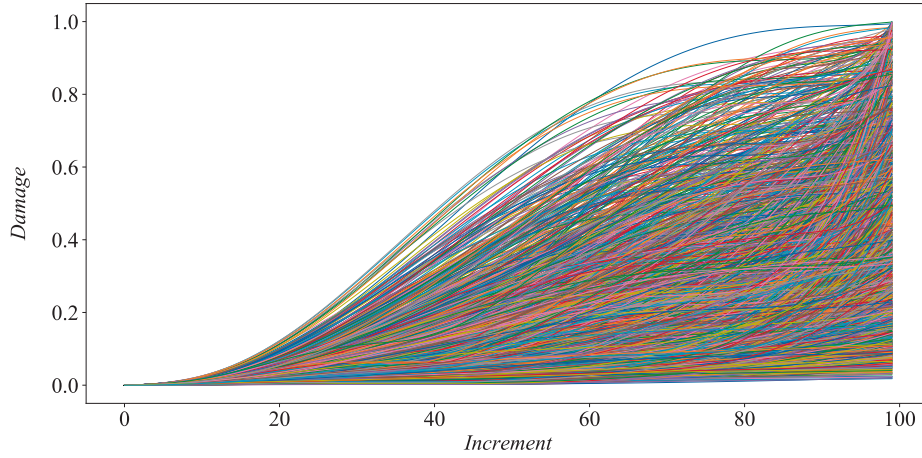


Fig. 3. The resulting damage paths corresponding to the 2,000 randomly generated strain paths.

Table 3
Network parameters.

Component	Value
training epochs	7000
Number of samples	2000
Number collocation points	400,000
Mini-batch size	32
k_1 (weight factor in $\mathcal{L}_{PINN}$)	1
k_2 (weight factor in $\mathcal{L}_{PINN}$)	10
k_3 (weight factor in $\mathcal{L}_{PINN}$)	500
k_4 (weight factor in $\mathcal{L}_{PINN}$)	2
Initial learning rate	0.0008
Learning rate decay factor	0.99
Learning rate decay period	20
β_1 (in ADAM optimizer)	0.9
β_2 (in ADAM optimizer)	0.999
ϵ (in ADAM optimizer)	10^{-8}
Test data percentage	10
Validation data percentage	10

$$\mathcal{L}_Y = \frac{\sigma_{eq}^{n+1}}{1 - D_{n+1}} - \sigma_y(R_{n+1}) \quad (15)$$

$$\mathcal{L}_C = \sigma_{ij}^{n+1} - \frac{1 - D_{n+1}}{1 - D_n} \sigma_{ij}^n + (1 - D_{n+1}) C_{ijkl} d\epsilon_{kl} - 3Gd\lambda \frac{S_{ij}^{n+1}}{\sigma_{eq}^{n+1}} \quad (16)$$

where C_{ijkl} is the elasticity tensor, G is the shear modulus, S_{ij} is the deviatoric part of the Cauchy stress tensor, and σ_{eq} is the von Mises equivalent stress. Therefore, the final physics-informed neural network

loss function is defined as:

$$\mathcal{L}_{PINN} = k_1 \cdot \mathcal{L}_{RMSE} + k_2 \cdot \mathcal{L}_Y + k_3 \cdot \mathcal{L}_D + k_4 \cdot \mathcal{L}_C \quad (17)$$

where k_i are the weighting factors. It should be noted that incorporating additional penalty terms into the loss function, while enabling the enforcement of physical constraints, also introduces its own challenges. This includes the need to balance the weighting factors, which directly affects the trade-off between satisfying the governing equations and minimizing data mismatch (Sau and Yin, 2024).

The loss function is constructed to measure the discrepancy between the predicted damage evolution and the reference responses obtained from the underlying constitutive model. The weighting of the loss components was selected to balance their relative contributions and to ensure stable convergence during training. While adaptive weighting strategies may be employed in more general settings, fixed loss weights were found to be sufficient for the supervised constitutive learning task considered in this study and consistently resulted in stable training behavior.

In the present work, the weighting factors in Eq. (17) were determined through preliminary numerical tests. A set of finite element solutions was examined to estimate the order of error associated with each

Table 4
Final values of the individual loss terms.

Loss term	$\mathcal{L}_{RMSE}$	$\mathcal{L}_Y$	$\mathcal{L}_D$	$\mathcal{L}_C$	$\mathcal{L}_{Validation}$
Final value	0.0219	0.0184	0.0004	0.0961	0.0018

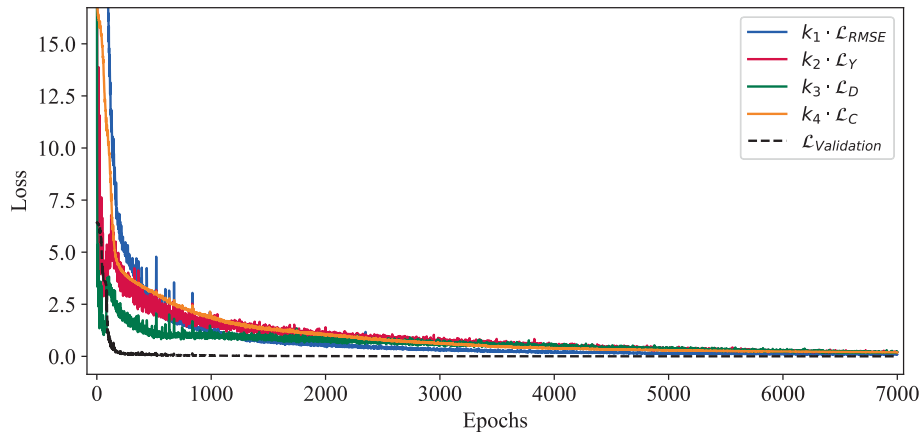
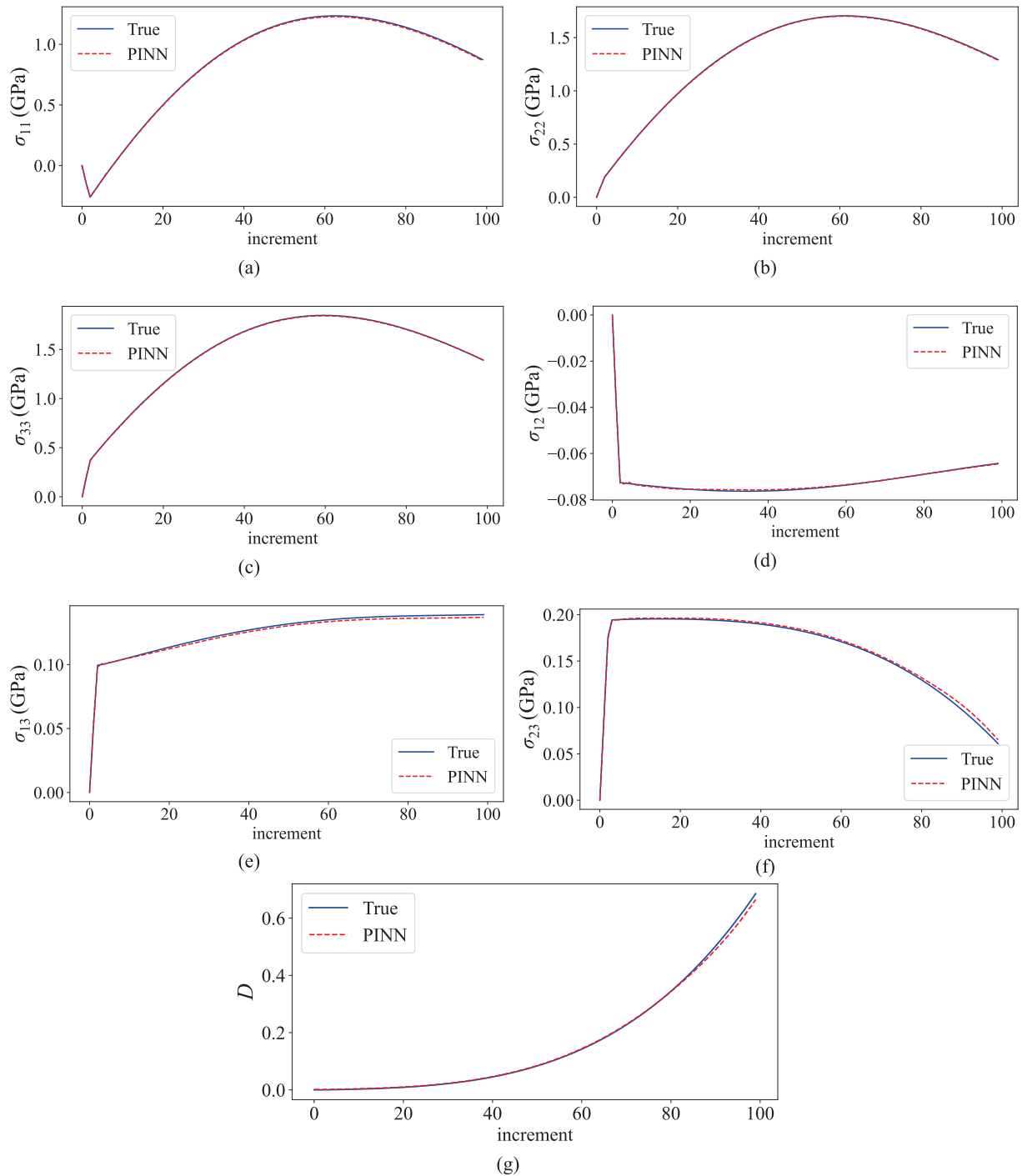


Fig. 4. Loss values during the training process.

Table 5

Prediction errors of the default NN and PINN for damage and stress components on 200 test samples.

NN	RMSE σ_{11} (GPa)	RMSE σ_{22} (GPa)	RMSE σ_{33} (GPa)	RMSE σ_{12} (GPa)	RMSE σ_{13} (GPa)	RMSE σ_{23} (GPa)	RMSE D
Default NN(inside trained domain)	0.0078	0.0076	0.0078	0.0008	0.0011	0.0007	0.0024
Default NN(outside trained domain)	0.0232	0.0222	0.0228	0.0047	0.0047	0.0045	0.0097
PINN	0.0136	0.0136	0.0139	0.0010	0.0011	0.0010	0.0043

**Fig. 5.** Comparison of stress components and damage predicted by PINN and FEM for some randomly selected test data: (a–f) stress components, (g) damage parameter.

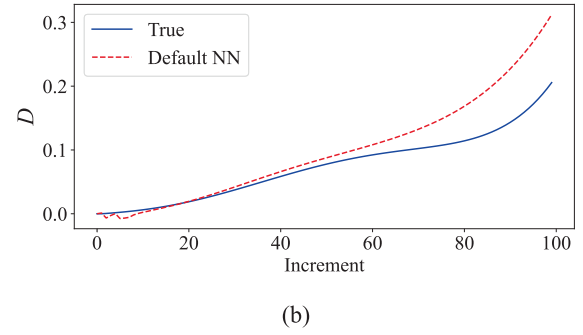
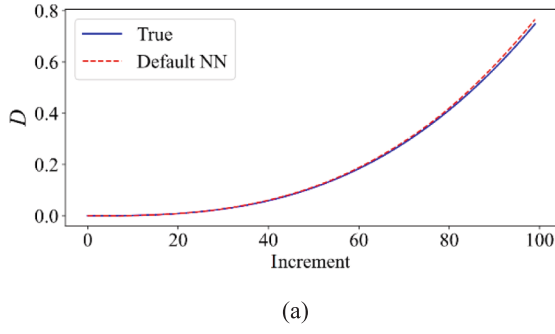


Fig. 6. Damage values predicted by the neural network with the default loss function; the network cannot perform acceptably outside its training domain. Strain paths: (a) within the training domain, (b) outside the training domain.

of Eqs. (14)–(16), and the values of k_2 , k_3 , and k_4 were selected accordingly to balance the contributions of the respective loss components and prevent any single term from dominating the training. Moderate variations in these weights were found to have negligible impact on the reported results.

3.2. Data generation

3.2.1. Generation of random strain paths

Developing a network capable of learning the correct path dependency of damage and plasticity requires generating representative strain paths. A total of 2,000 random points in strain space were selected, with each point connected to the origin using a quadratic path. Each path was then divided into 100 equal increments, resulting in strain paths of the following form:

$$\mathbf{x}^i = \begin{bmatrix} \epsilon_{11}^{t=0}, \epsilon_{11}^{t=1}, \epsilon_{11}^{t=2}, \dots, \epsilon_{11}^{t=100} \\ \epsilon_{22}^{t=0}, \epsilon_{22}^{t=1}, \epsilon_{22}^{t=2}, \dots, \epsilon_{22}^{t=100} \\ \epsilon_{33}^{t=0}, \epsilon_{33}^{t=1}, \epsilon_{33}^{t=2}, \dots, \epsilon_{33}^{t=100} \\ \epsilon_{12}^{t=0}, \epsilon_{12}^{t=1}, \epsilon_{12}^{t=2}, \dots, \epsilon_{12}^{t=100} \\ \epsilon_{13}^{t=0}, \epsilon_{13}^{t=1}, \epsilon_{13}^{t=2}, \dots, \epsilon_{13}^{t=100} \\ \epsilon_{23}^{t=0}, \epsilon_{23}^{t=1}, \epsilon_{23}^{t=2}, \dots, \epsilon_{23}^{t=100} \end{bmatrix}_{6 \times 101} \quad (18)$$

In practice, loading paths may be complex, non-uniform, or discontinuous. To assess the proposed model's capability, quadratic strain paths were employed, enabling (1) the representation of loading steps using higher-order functions rather than simple linear relations, and (2) the more accurate simulation of non-uniform loading paths. The quadratic function used to generate strain paths produces second-order curves in a three-dimensional space—whether in the space of normal strains or shear strains. The projection of these curves onto the coordinate planes reveals how each strain component evolves, allowing for a variety of behaviors beyond strictly quadratic trends. This flexibility enables generation of diverse strain paths, ensuring that the network is not constrained to purely quadratic patterns. However, this simplified assumption was chosen to explore the overall concept and is currently applicable only to simple geometries. A more rigorous approach would involve running numerous finite element simulations to generate a dedicated dataset, though this would demand significantly higher computational resources and time. To expand the operational range of the neural network, strain paths needed to cover a significant portion of the strain space. While this is feasible, it proportionally increases the training time. Therefore, based on the characteristics of the problems to be analyzed, the permissible strain path space was restricted by applying the optional constraints given in Eq. (19), as shown in Fig. 1.

$$-0.2 < \epsilon_{ij} < 0.2 \quad (19)$$

3.2.2. Collocation points

In PINNs, collocation points are specific locations within the computational domain where the neural network's predictions are evaluated and constrained to satisfy governing physical laws. These points are used to compute the physics-based component of the loss function, guiding the network to learn solutions that respect the underlying equations. In this work, 400,000 collocation points were generated using random sampling within the admissible strain space (Fig. 2). To ensure broader coverage and improve the robustness of the learned solution, the admissible domain for collocation points was defined as:

$$-0.4 < \epsilon_{ij} < 0.4 \quad (20)$$

which is twice the range used for the training strain paths. This extended domain allows the neural network to enforce the physical constraints over a wider strain range than it has seen during data-driven training, thereby enhancing generalization and stability.

3.2.3. Calculation of stress and damage for the generated strain paths

A Return-Mapping algorithm for implementing Lemaitre's ductile damage model was introduced by De Souza Neto et al. (2008). In the absence of kinematic hardening, they proposed a more computationally efficient single-equation method. This algorithm was employed to compute stress and damage for each generated strain path, based on the material properties listed in Table 2, which correspond to the cylindrical notched bar problem in Table 12.1 in (De Souza Neto et al., 2008). The implementation was validated, and the results showed full agreement with those reported in the reference. The resulting damage paths corresponding to the 2,000 strain paths shown in Fig. 1 are presented in Fig. 3.

3.3. ANN architecture

The history-dependent nature of stress makes recurrent neural networks (RNNs) a suitable choice. However, due to challenges such as vanishing and exploding gradients in RNNs, more advanced architectures like Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) have been adopted. The Gated Recurrent Unit (GRU) is similar to the Long Short-Term Memory (LSTM) network (Hochreiter and Schmidhuber, 1997), but has a simpler gating mechanism and higher computational efficiency (Tsantekidis et al., 2022; Wang et al., 2022). It uses only two gates, namely the reset and update gates, and does not employ a protected cell state; instead, the gates are applied directly to the hidden state. This design makes the GRU more computationally efficient than the LSTM while still enabling the network to learn long-term dependencies without suffering excessively from the gradient vanishing problem (Bengio et al., 1994; Ghosh et al., 2023). A GRU architecture with three layers, each containing 600 hidden units, is used in this study, since deeper configurations showed no notable

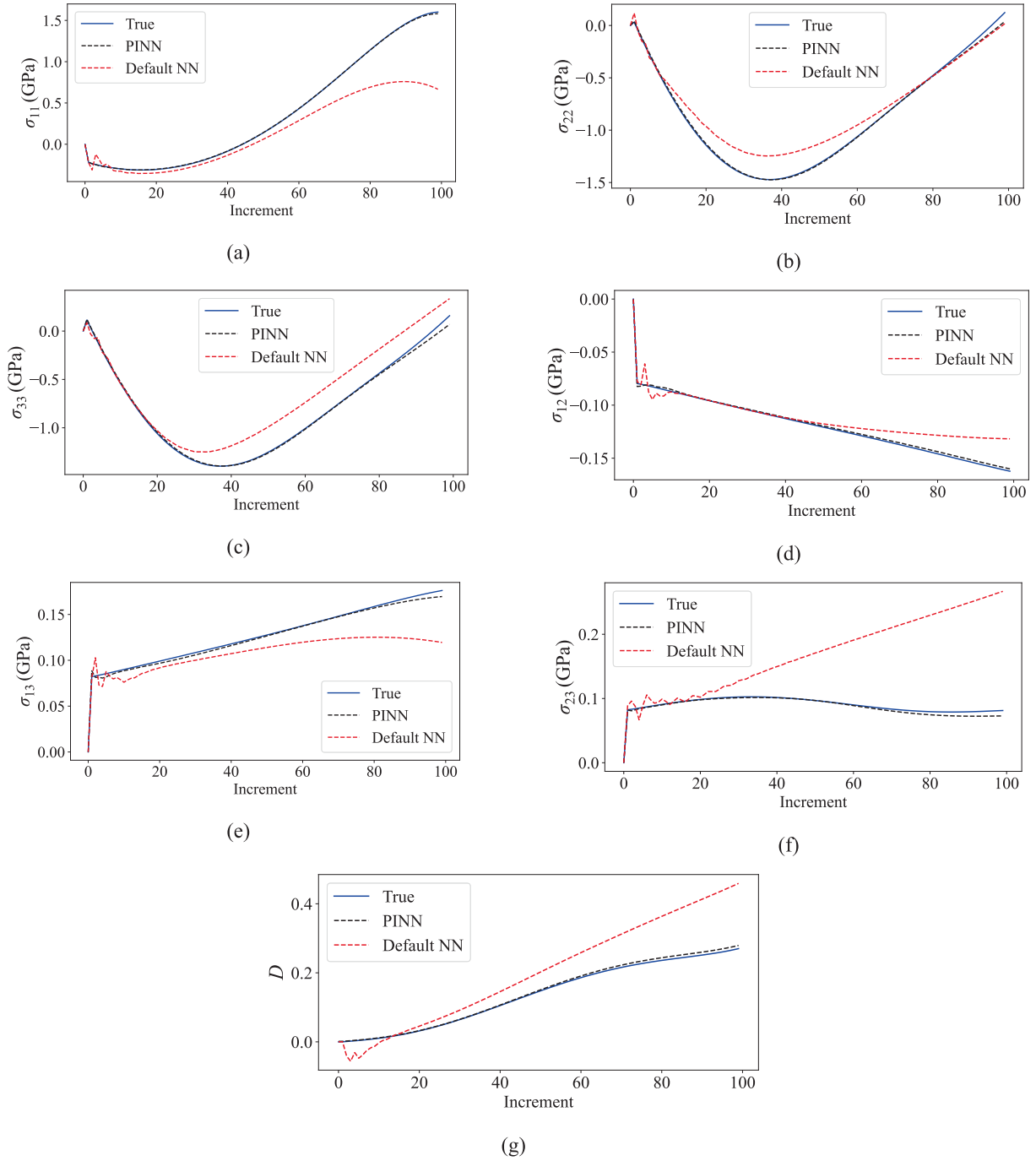


Fig. 7. Comparison of PINN and default NN in predicting damage and stress components outside the training domain for randomly selected test cases (a–g).

improvement in performance. The ADAM optimizer is employed for loss function optimization, with parameters listed in Table 3. Hyperparameter tuning was performed using random search, guided by similar works such as Frieman et al. (2023). The network's input consists of time series of length 101, with seven features representing the independent components of the strain tensor and the incremental plastic multiplier $d\lambda$. The outputs include the six independent components of the stress tensor and one damage variable, D .

The neural network employed in the present framework is designed to approximate the constitutive mapping between the strain history and the corresponding damage evolution at each integration point. Since the strain paths are known a priori and are directly provided as inputs to the network, the training process follows a supervised learning strategy

rather than a classical physics-informed formulation based on partial differential equation residuals. Consequently, the network does not require the direct enforcement of essential boundary conditions. Its role is conceptually similar to the constitutive update performed at Gauss points in a UMAT subroutine.

4. Results

This section, which is divided into five parts, begins by evaluating the overall performance of the physics-informed neural network in predicting stress and damage paths for unseen strain paths, using the test dataset. The second part compares the PINN with a conventional neural network to assess how incorporating physical constraints into the loss

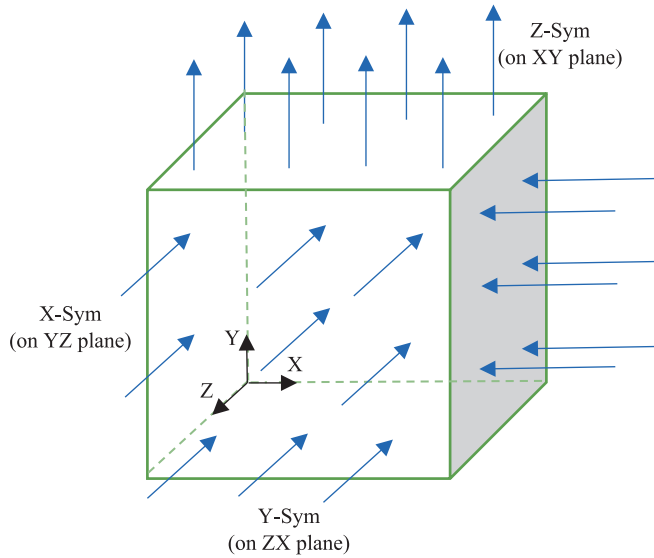


Fig. 8. A cubic C3D20 element with a side length of one millimeter, with random displacement boundary conditions applied to three perpendicular faces and symmetry boundary conditions on the other faces.

function influences the results. Next, a single element is simulated in Abaqus under boundary and loading conditions that ensure its strain stays within the network's training range, allowing for a direct comparison between Abaqus and the PINN predictions. This is followed by an evaluation of the network's performance on a benchmark problem. Lastly, the network's ability to generalize is examined in the context of predicting nonlocal damage behavior.

During the training process, the loss function terms introduced in Eq. (17) were monitored to evaluate the network's convergence. As illustrated in Fig. 4, the steady decrease in each term indicates that the network effectively captured the underlying patterns in the data, and also confirms that the gradients were appropriately balanced during backpropagation, ensuring that no single term dominated the training. By the end of training, the loss values converged to those listed in Table 4, confirming successful learning and optimization.

4.1. PINN performance on the test dataset

In the first step, the performance of the PINN was evaluated on the test dataset. The assessment was conducted on 200 test samples, with the RMSE reported in Table 5 to quantify prediction accuracy. To evaluate these errors, the performance of the PINN in predicting damage and stress paths was visualized for randomly selected samples from the test set in Fig. 5. As observed, the reported errors are within an acceptable range, indicating that the PINN accurately predicts the

damage and stress paths on the test dataset.

4.2. PINN comparison with the default NN

To analyze the impact of the physics-informed constraint on neural network predictions, training was also conducted using the default RMSE loss function, while keeping all conditions and settings identical to those in Table 3. The results from both cases were then compared. The neural network trained with the default RMSE loss function performs accurately within the domain of the training dataset; however, its performance degrades significantly when predicting outside the training domain, as illustrated in Fig. 6(a) and (b), respectively. The corresponding RMSE values are also listed in Table 5.

On the other hand, it was observed that the PINN, by incorporating the physics-informed constraint into the loss function, showed significant improvements in stress and damage predictions, indicating that the PINN can also generalize beyond its training domain. The impact of this physics-informed loss is illustrated in Fig. 7 for randomly selected samples from the test set. The corresponding RMSE values are also listed in Table 5, confirming the advantage of the PINN approach over the default NN when applied to data beyond the training domain.

In Fig. 7(g), the damage predicted by the neural network with the default loss function shows a negative slope and even negative values in the initial increments, which are physically unrealistic and violate the thermodynamic damage growth relation Eq. (11). In contrast, the damage predicted by the PINN satisfies the thermodynamic constraint throughout all increments, ensuring thermodynamically consistent predictions.

4.3. Testing the PINN on practical strain scenarios

The random strain paths used for training the neural network were generated by approximating typical paths with a quadratic function. Therefore, it was necessary to verify the network's performance when exposed to more conventional strain paths. To further evaluate our PINN, a cubic C3D20 element with a side length of one millimeter and material properties from Table 2 was modeled in Abaqus software, as illustrated in Fig. 8.

Displacement boundary conditions with random values (columns 2–4 in Table 6) were applied to three mutually perpendicular faces of the cube, creating ten different boundary condition scenarios, which are listed in Table 6. The remaining faces were assigned symmetry boundary conditions. The applied displacements were chosen to ensure that the resulting strains remained within the constraints outlined in Eq. (20). Next, the strain paths experienced by the element in these ten cases were used as input for the PINN, which then predicted the damage and stress components. The PINN error compared to FEM for these ten cases is presented in Table 6.

In Fig. 9(a), the damage predicted by the PINN is compared with FEM for ten different boundary condition scenarios listed in Table 6. The

Table 6

Ten different boundary condition scenarios for the C3D20 element and the PINN error in predicting stress and damage values.

State	Face displacement in -X direction	Face displacement in Y direction	Face displacement in -Z direction	RMSE σ_{11} (GPa)	RMSE σ_{22} (GPa)	RMSE σ_{33} (GPa)	RMSE σ_{12} (GPa)	RMSE σ_{13} (GPa)	RMSE σ_{23} (GPa)	RMSE D
1	0.189	0.247	0.038	0.0067	0.0066	0.0061	0.0006	0.0011	0.0009	0.0010
2	0.011	0.241	0.210	0.0228	0.0191	0.0218	0.0016	0.0016	0.0017	0.0072
3	0.198	0.274	0.002	0.0326	0.0410	0.0368	0.0018	0.0011	0.0015	0.0155
4	0.039	0.199	0.112	0.0361	0.0451	0.0408	0.0015	0.0011	0.0011	0.0193
5	0.122	0.160	0.052	0.0101	0.0076	0.0111	0.0009	0.0013	0.0014	0.0058
6	0.233	0.280	0.038	0.0509	0.0405	0.0457	0.0008	0.0022	0.0015	0.0153
7	0.068	0.181	0.144	0.0363	0.0375	0.0430	0.0006	0.0010	0.0018	0.0222
8	0.041	0.028	0.017	0.0185	0.0178	0.0276	0.0056	0.0018	0.0041	0.0022
9	0.032	0.131	0.084	0.0029	0.0015	0.0025	0.0014	0.0004	0.0007	0.0007
10	0.102	0.292	0.176	0.0330	0.0253	0.0320	0.0015	0.0034	0.0015	0.0134

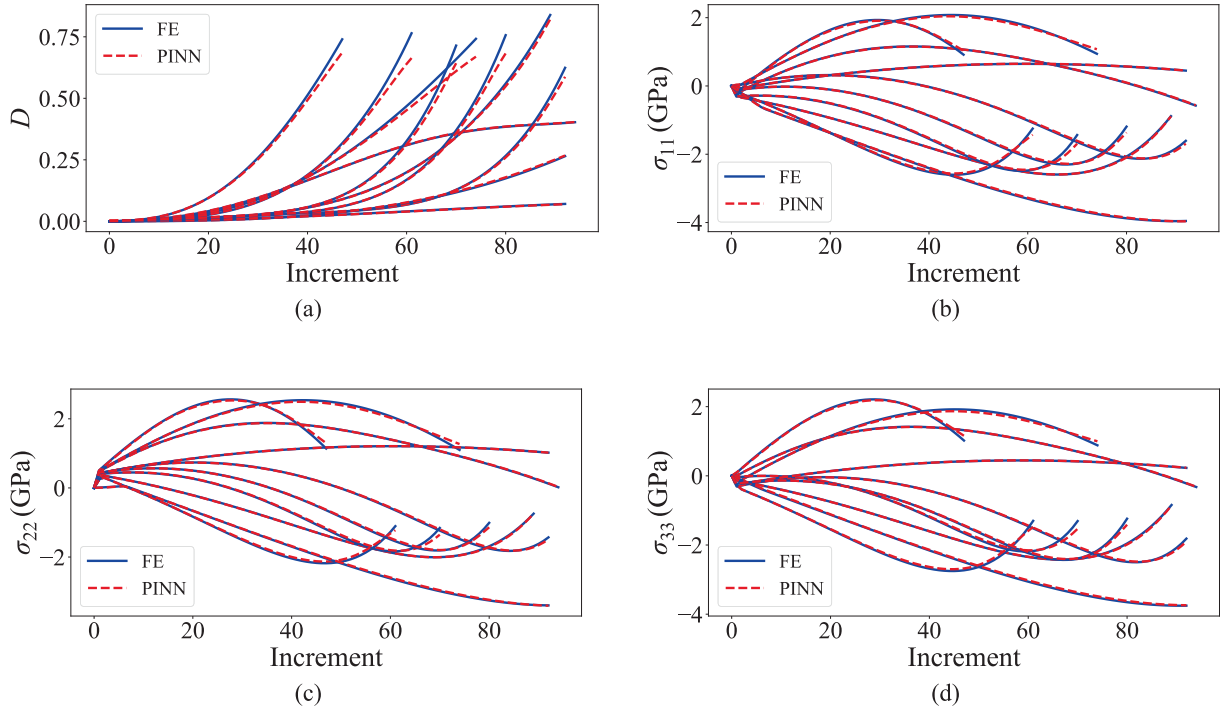


Fig. 9. PINN compared with FEM in ten different boundary condition scenarios as listed in Table 6: (a) damage variable, (b, c, d) stress component σ_{11} , σ_{22} , and σ_{33} .

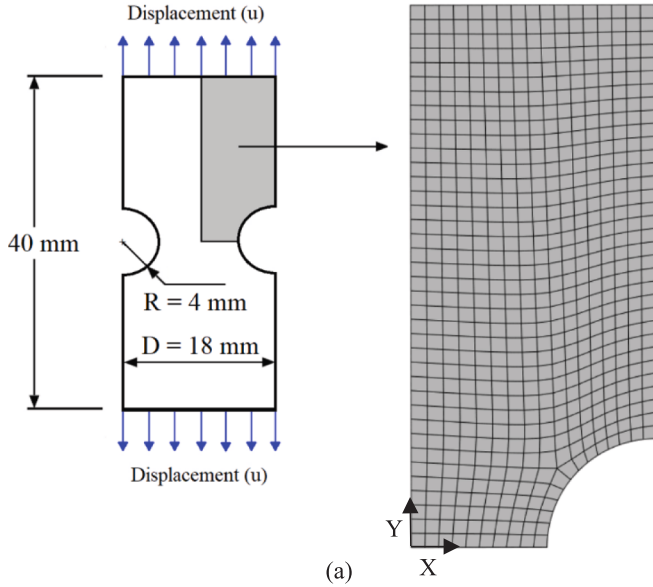


Fig. 10. Cylindrical notched bar subjected to monotonic axial loading (De Souza Neto et al., 2008) with material properties of Table 2: Geometry, boundary conditions, and FE mesh.

Table 7

Aluminum properties of cylindrical notched bar problem (de Almeida César de Sá et al., 2010).

E	69 GPa
ν	0.3
Hardening curve	$\sigma_y(R) = 589(10^{-4} + R)^{0.216}$ MPa
s	1.0
r	1.25 MPa
Characteristic length	0.6325 mm

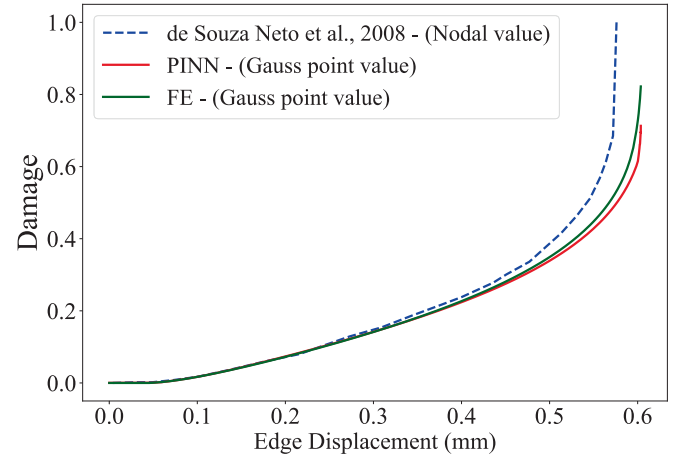


Fig. 11. Damage evolution in the central element of the cylindrical notched bar: comparison between PINN, FE, and (De Souza Neto et al., 2008).

results show that the PINN performs well in predicting damage within the range of training strains, even in practical cases. Similarly, Fig. 9(b–d) present this comparison for the normal stress components, further validating the accuracy of the PINN in predicting stress and damage responses under various loading conditions.

4.4. PINN applied to a benchmark problem

In the next stage of the PINN evaluation, damage prediction was carried out for a benchmark problem using the material properties listed in Table 2. The selected problem consists of a cylindrical notched bar subjected to monotonic axial loading (de Souza Neto et al., 2008), as illustrated in Fig. 10. As no load reversal occurs in this case, the simplified version of Lemaitre's ductile damage model provides an appropriate framework for the analysis. Taking advantage of symmetry, the analysis was restricted to one-quarter of the specimen, with all

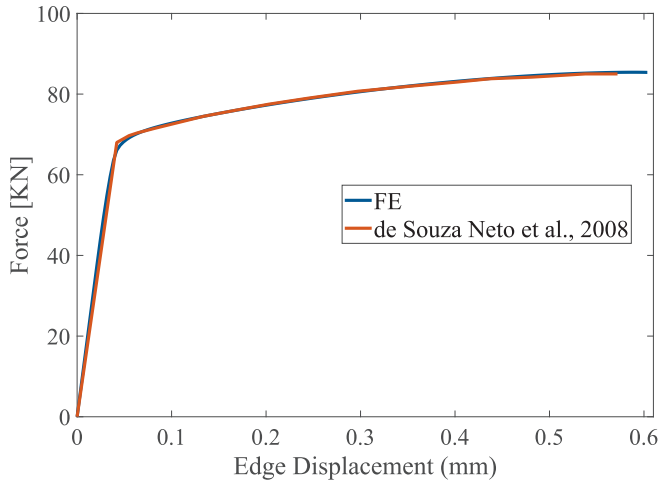


Fig. 12. Reaction force versus edge displacement of the cylindrical notched bar: comparison between FE, and (De Souza Neto et al., 2008).

meshes and contour results mirrored for clarity. The strain paths at each integration point of every element were provided as inputs to the PINN to predict the corresponding damage evolution. The predictions were compared with FE results for various edge displacements, as

summarized in Table 7. The FE analyses were performed in Abaqus using 612 CAX8R elements (8-node biquadratic axisymmetric quadrilateral with reduced integration). The reaction force and the damage evolution of the central element are also presented in Figs. 11 and 12, respectively.

The results demonstrate excellent agreement, with the predicted damage evolution closely matching the FE results, thereby confirming the robustness and reliability of the proposed approach. De Souza Neto et al. (2008) showed that, in the early stages of loading, maximum damage occurs near the root of the notch. As the specimen is progressively stretched, the region of maximum damage gradually shifts toward the center and eventually localizes in that region. A similar trend was observed in the predictions of the PINN, as shown in Fig. 13. To provide a clearer illustration, Fig. 14 depicts the damage evolution predicted by the PINN along the specimen's X-axis for various edge displacement values.

It is also worth noting that, as shown in Fig. 15, when the number of collocation points used for training is insufficient, the predicted damage values deviate significantly: the contour shape no longer matches the finite element solution, and negative, physically invalid damage values appear.

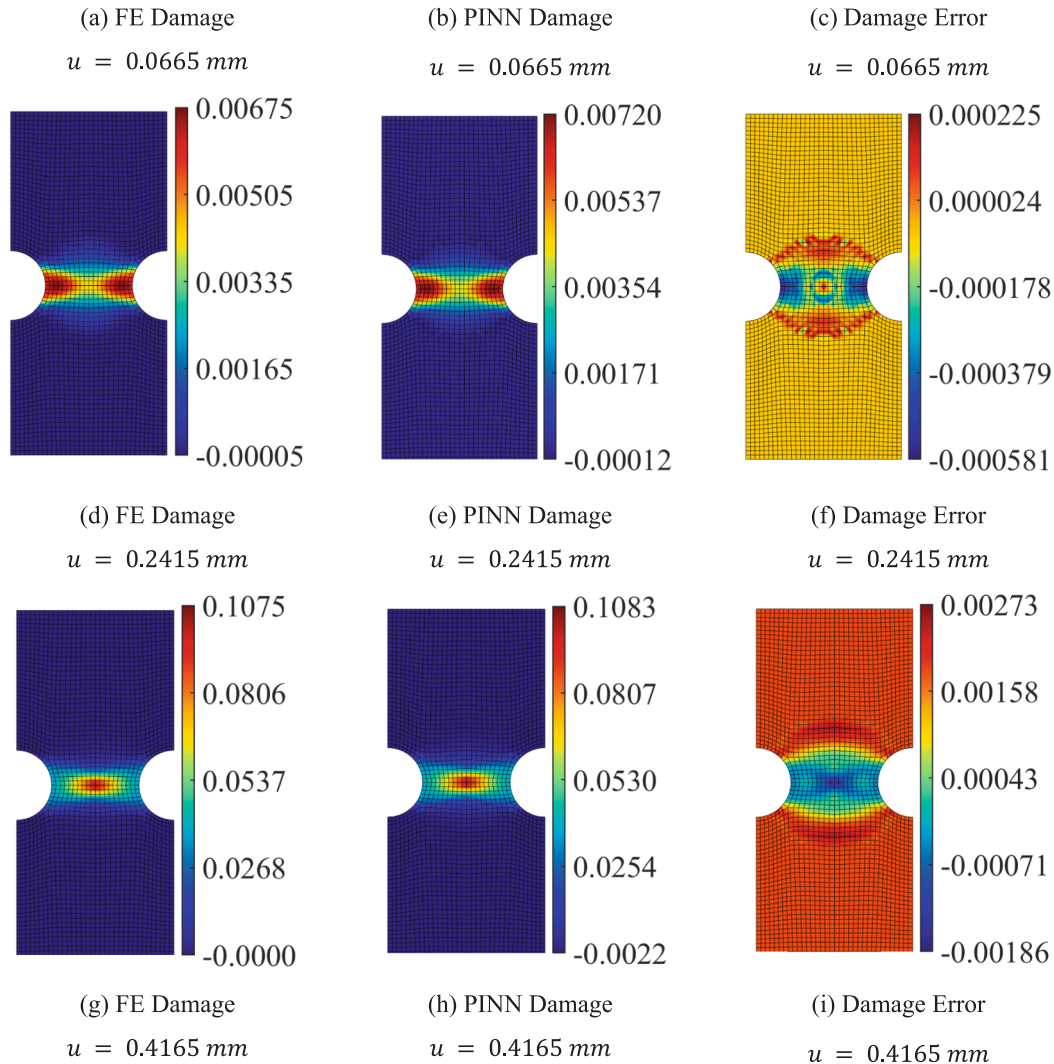


Fig. 13. Damage evolution of the cylindrical notched bar: comparison between PINN, FE (u = Edge Displacement).

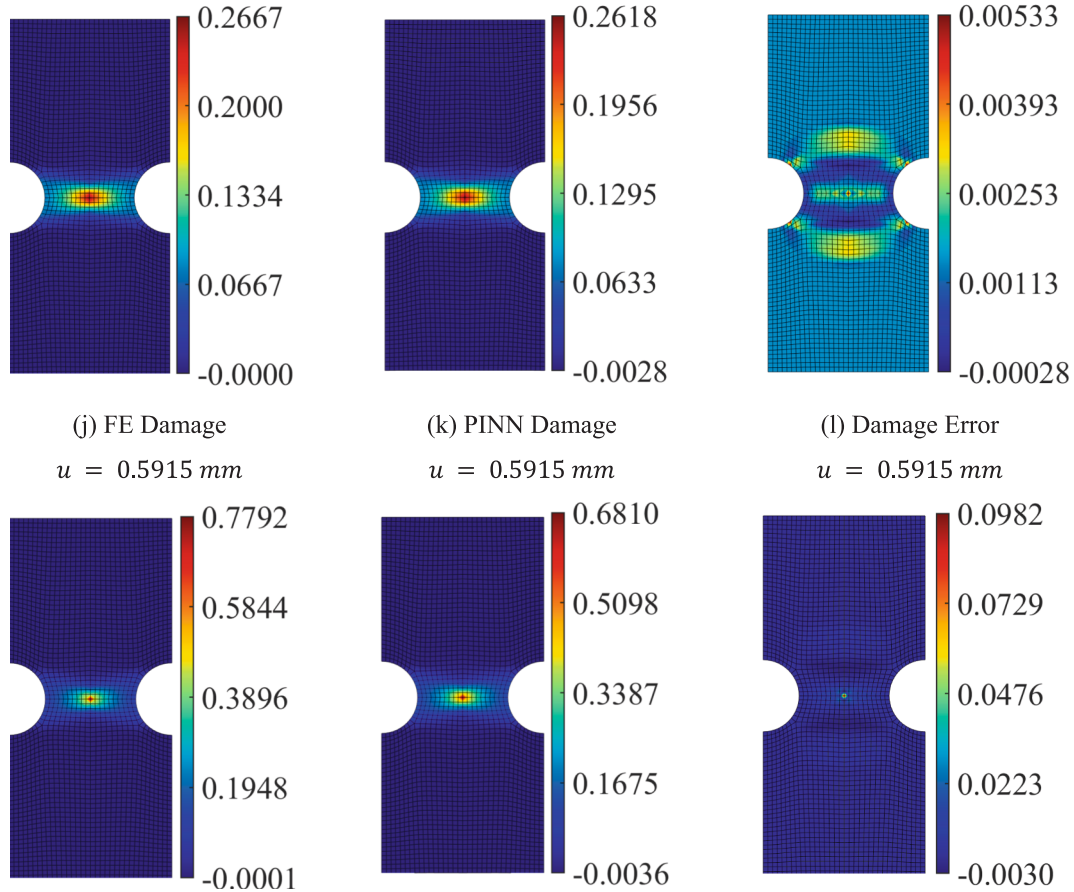
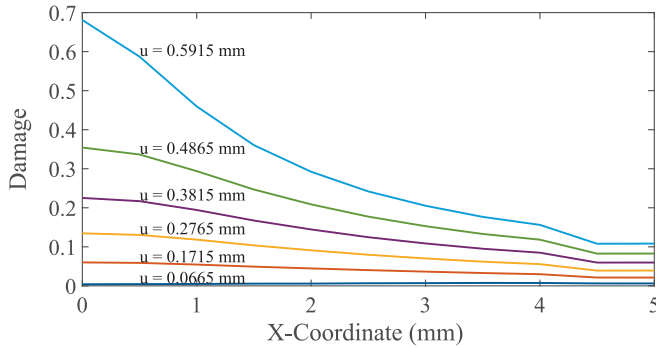


Fig. 13. (continued).

Fig. 14. Damage evolution predicted by the PINN along the X-axis in different edge displacement, u values.

5. Further evaluation of the PINN – nonlocal damage

5.1. Nonlocal damage modeling

In classical continuum damage mechanics, the damage variable at a point depends solely on the local strain or stress state, often resulting in mesh dependency and nonphysical strain localization. Nonlocal damage models overcome these issues by incorporating microstructural effects and interactions between neighboring material points through spatially weighted averages of internal variables over a characteristic length scale. This approach captures phenomena such as void nucleation and coalescence, eventually leading to macroscopic failure. By considering a neighborhood of points, nonlocal formulations mitigate spurious localization and mesh sensitivity, leading to more physically realistic

simulations. For example, in Lemaitre's ductile damage model, nonlocal averaging of the damage evolution equation improves the representation of stress redistribution and energy dissipation during plastic deformation.

5.1.1. Nonlocal damage approaches

There are two main classes of nonlocal damage models: integral-type and gradient-enhanced approaches. The integral-type and gradient-enhanced nonlocal damage models differ fundamentally in their formulation and computational behavior. Integral models introduce nonlocality explicitly by averaging a local damage variable over a neighborhood defined by a kernel function and a characteristic length, directly linking neighboring points. In contrast, gradient models implicitly incorporate nonlocality through a partial differential equation (PDE) that governs damage diffusion, coupling a damage gradient term with the mechanical equilibrium (Fonseca Gonçalves et al., 2025).

5.1.2. Characteristic length and its role

The characteristic length ℓ_r is a material-specific parameter that defines the spatial influence range for nonlocal averaging. It is linked to microstructural features such as void spacing or grain size and determines the radius within which neighboring points influence the nonlocal variables at a given material point. This length scale allows the model to capture microscale interactions while preserving accurate macroscale behavior. It plays a central role in integral-type nonlocal formulations by regularizing the solution, smoothing highly localized fields (e.g., damage or plastic strain), and mitigating pathological mesh sensitivity. For instance, in Gaussian or quartic polynomial weight functions, ℓ_r controls the rate at which the weighting factor decays with distance, ensuring physically meaningful predictions of damage evolution and failure. However, the determination of this parameter for real

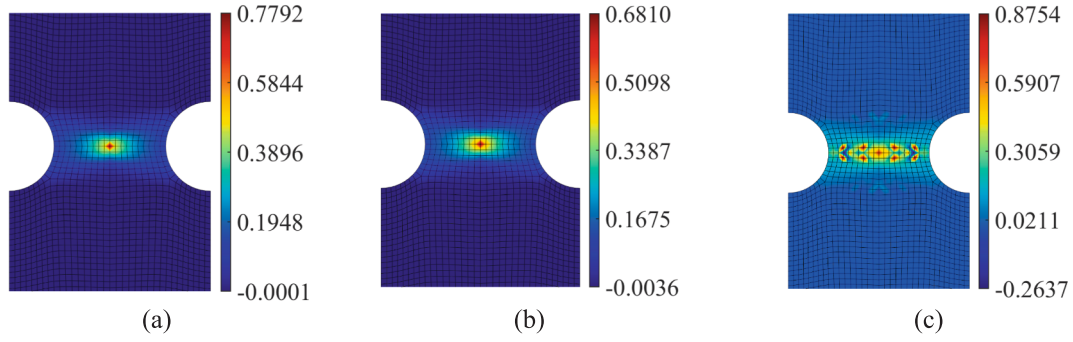


Fig. 15. Effect of the number of collocation points on neural network damage predictions for the cylindrical notched bar problem (de Souza Neto et al., 2008): (a) FE (b) PINN with 400,000 collocation points, (c) PINN with 200,000 collocation points.

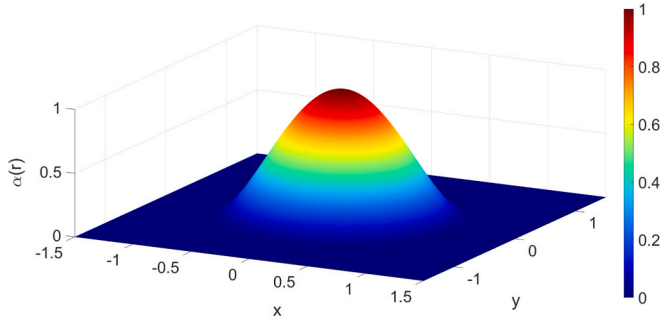


Fig. 16. Bell-shaped truncated quartic polynomial weight function $\alpha(r)$ from Eq. (21), with $\ell_r = 1$.

materials remains challenging.

5.1.3. Nonlocal formulations in integral-type approaches

Nonlocal extensions of constitutive models are achieved by replacing specific local variables with their spatially averaged counterparts. Key approaches include:

- Nonlocal Damage (D):** The damage variable D is averaged over a neighborhood using a weighting function (e.g., Gaussian or truncated quartic). This directly regularizes damage evolution and is effective for ductile materials where microvoid growth dominates failure.
- Nonlocal Energy Release Rate ($-\bar{Y}$):** The thermodynamic force driving damage ($-Y$) is nonlocally averaged. This approach better captures stress triaxiality effects and avoids nonphysical damage localization under complex loading.
- Nonlocal Hardening Variables (e.g., $\bar{R}$):** Variables like isotropic hardening (R) are averaged to model interactions between dislocation networks or other microstructural barriers. This is less common

but useful for materials where hardening mechanisms are influenced by neighboring regions.

Each of these method introduces a characteristic length and requires iterative global algorithms (e.g., Newton–Raphson) to ensure consistency, equilibrium, and objectivity in simulations.

5.2. PINN in nonlocal damage

The applicability of PINN for predicting nonlocal damage was further examined using two geometries, which are presented in the following sections. It should be noted that the network does not learn l_r or any smoothing effect. Once trained with local data, it can be employed during the inference phase in two ways: (i) by directly feeding the strain paths to predict local damage, or (ii) by using a weighted average of strain paths from neighboring integration points to predict nonlocal damage. The latter approach constitutes an advantage of the proposed method in capturing nonlocal effects. The proposed PINN framework is designed as a data-driven approximation for integral-type nonlocal damage formulations (de Almeida César de Sá et al., 2010), rather than as a new theoretical model. In this setting, the weighted averaging of strain paths from neighboring Gauss points Eq. (21) serves as a discrete approximation of the nonlocal integral equation.

In this framework, the neural network does not represent the mechanism responsible for nonlocal interactions. Instead, it acts as a surrogate constitutive operator that maps the strain history to the corresponding damage response. The nonlocal character of the formulation is introduced externally during the inference stage through the spatial interaction among neighboring integration points. In this way, the PINN provides a flexible data-driven representation of the local damage evolution law, while the nonlocal coupling is incorporated through the distance-weighted averaging of strain histories.

In practical terms, this nonlocal behavior is introduced through a preprocessing step in which the strain history at each Gauss point is replaced by a distance-weighted average of the strain histories from neighboring points. This procedure can be interpreted as a discrete

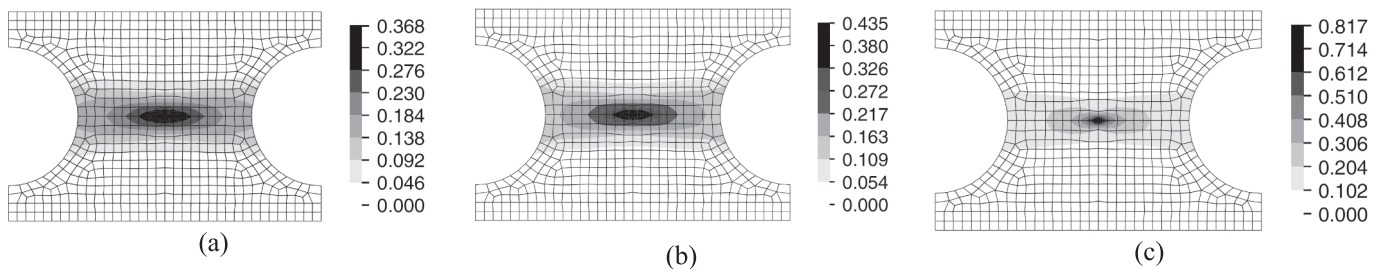


Fig. 17. Nonlocal damage predicted by FE (de Almeida César de Sá et al., 2010) using a characteristic length of 0.6325 mm. a) Nonlocal formulation based on $\bar{D}$, b) Nonlocal formulation based on $-\bar{Y}$, c) Nonlocal formulation based on $\bar{R}$

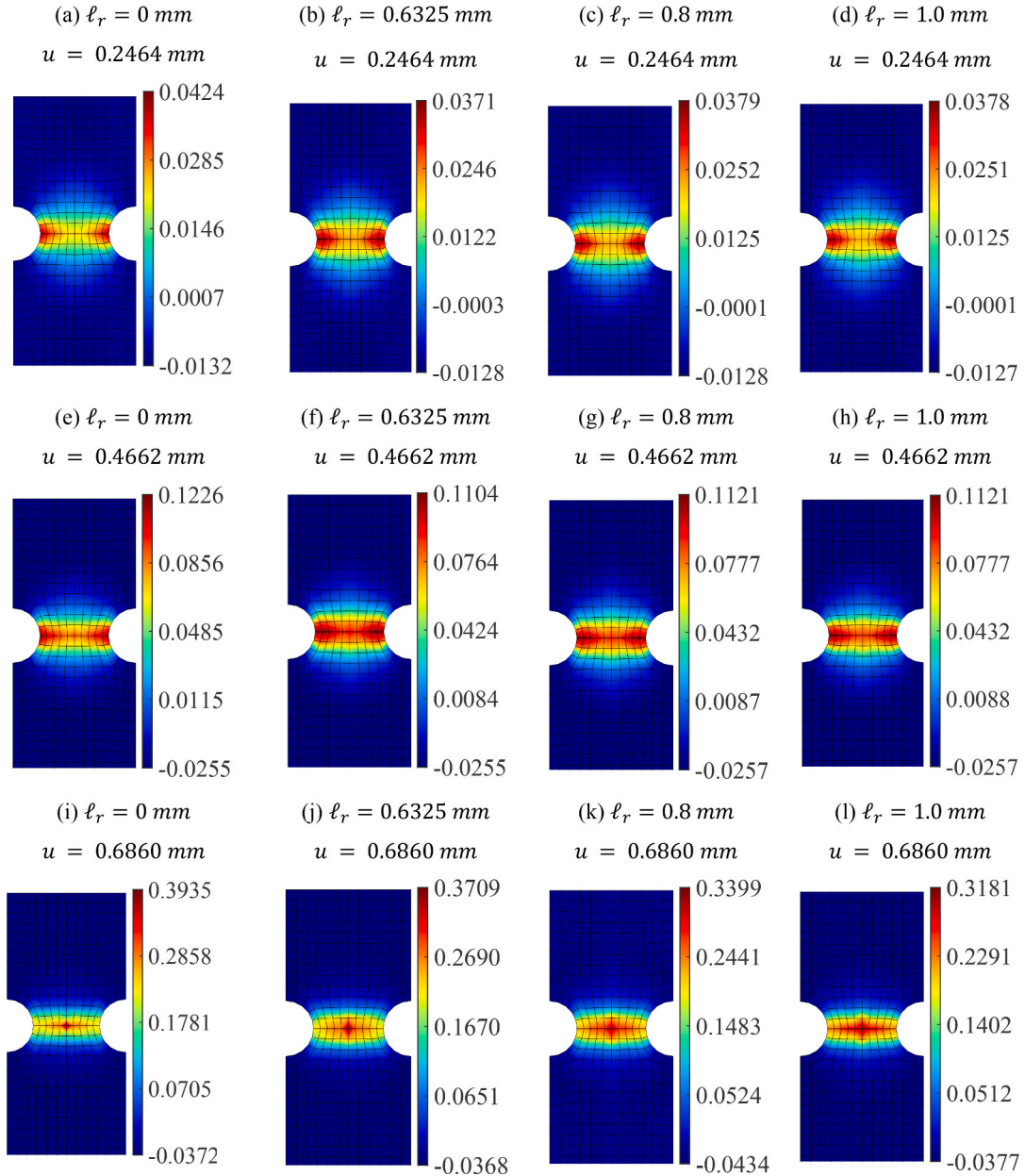


Fig. 18. Nonlocal PINN-predicted damage evolution in an aluminum cylindrical notched bar using a coarse mesh for different characteristic lengths (u = Edge Displacement).

realization of the integral-type nonlocal regularization commonly used in classical damage models. The neural network then operates on these processed inputs to learn the constitutive mapping between the averaged strain histories and the resulting damage evolution. This separation highlights that the network itself does not infer spatial nonlocality, but rather functions within an externally imposed nonlocal framework.

Within this formulation, the characteristic length ℓ_r governs the spatial extent of the averaging kernel and therefore controls the effective interaction domain among neighboring points. Consistent with classical integral nonlocal damage models, increasing ℓ_r enlarges the averaging region, leading to stronger regularization of the damage field and a more diffuse process zone. The numerical results presented in this study confirm this expected trend, showing that larger neighborhood radii produce smoother damage distributions and improved mesh objectivity.

It is important to note that the present framework does not modify the underlying variational structure typically adopted in nonlocal damage formulations. The governing equations and the overall

numerical implementation remain unchanged. The neural network is used only to approximate the constitutive response at the material point level, effectively replacing the conventional explicit damage evolution law with a data-driven surrogate representation. As a consequence, the regularization of the damage field and the mitigation of pathological strain localization originate from the nonlocal averaging procedure rather than from the neural network architecture itself. Accordingly, the mesh-objective behavior observed in the simulations is primarily inherited from the underlying nonlocal formulation, while the PINN provides an efficient approximation of the constitutive response.

5.2.1. Aluminum notched bar

The model was evaluated using the same cylindrical notched bar introduced in section 4.4, this time made of aluminum, and the results were compared with those reported by [de Almeida César de Sá et al. \(2010\)](#). Accordingly, the PINN was retrained using the aluminum material properties listed in [Table 7](#).

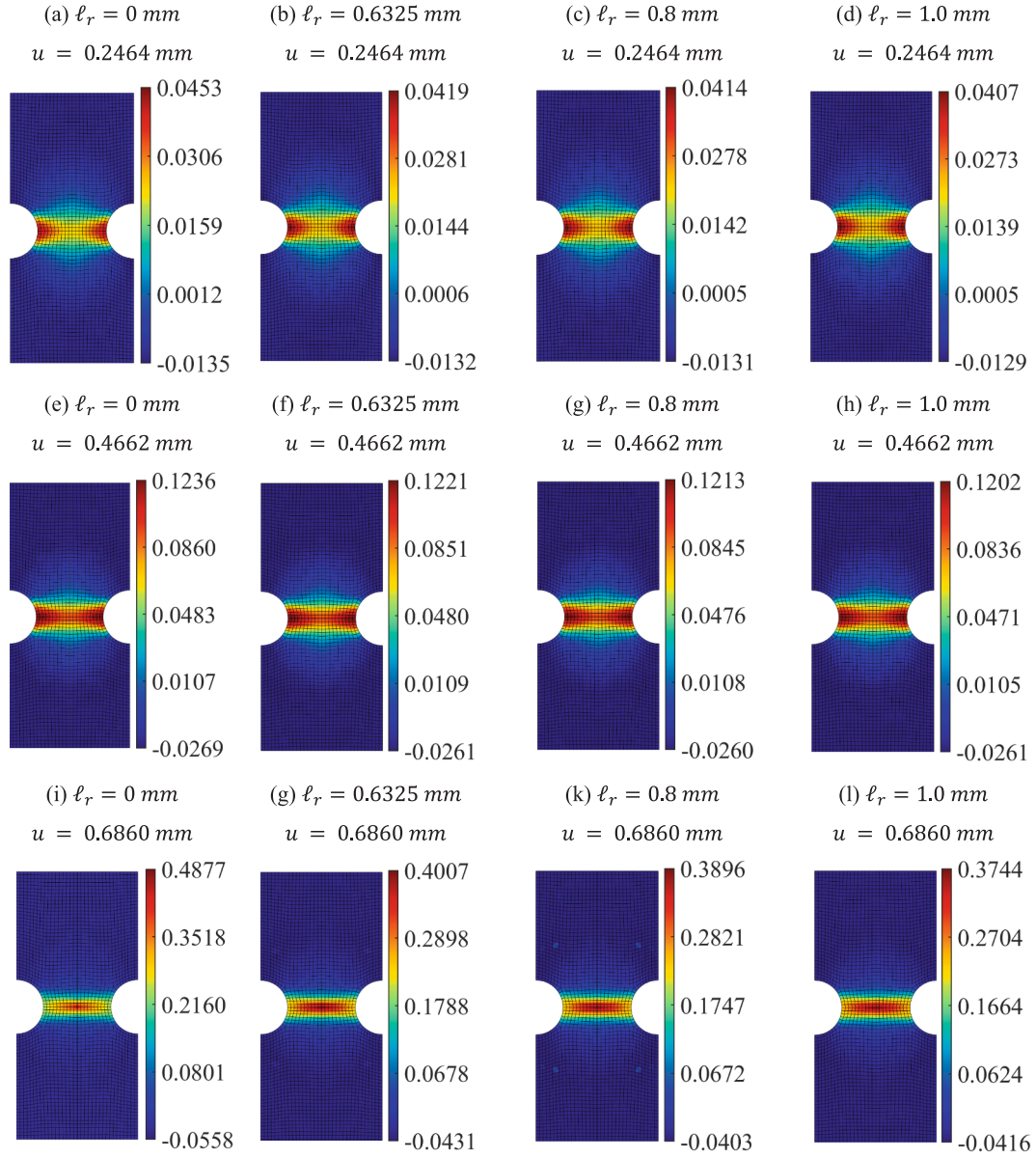


Fig. 19. Nonlocal PINN-predicted damage evolution in an aluminum cylindrical notched bar using a fine mesh for different characteristic lengths.

Table 8

Material properties of Aluminum 7075-T6 used in the double-notched specimen (Ranjbar et al., 2017).

E	71.7 GPa
ν	0.33
Hardening curve	$\sigma_y(R) = 491.5 + 500(1 - \exp(-1.05 R))$ MPa
s	2.0
r	2.175 MPa
Characteristic length	1 mm

Similar to nonlocal damage modeling, the characteristic length ℓ_r was adapted. To predict the damage at a given Gauss point, the weighted average of the strain paths from neighboring Gauss points within its vicinity was used as input to the network. For the weight function $\alpha(r)$, a bell-shaped truncated quartic polynomial was employed (Fig. 16), as it is commonly used in classical nonlocal damage modeling. It is defined as:

$$\alpha(r) = \begin{cases} \left(1 - \frac{r^2}{\ell_r^2}\right)^2 & \text{if } r \leq \ell_r \\ 0 & \text{if } r \geq \ell_r \end{cases} \quad (21)$$

where r is the distance from a neighboring Gauss point to the Gauss point of interest.

When the characteristic length is set to a value close to zero, the results correspond to the local case, with maximum damage values of 0.4877 and 0.3935 for the fine and coarse meshes, respectively, as shown in Figs. 18 and 19. FE nonlocal damage results for this problem, reported by de Almeida César de Sá et al. (2010), are shown in Fig. 17, where three nonlocal formulations based on D, $-Y$, and R were employed with a characteristic length of 0.6325 mm.

Using the PINN, selecting an appropriate characteristic length ℓ_r resulted in excellent agreement with the finite element results. As shown in Figs. 18 and 19, four characteristic lengths were considered across two mesh configurations. The results indicate that increasing the neighborhood radius produces a more uniform damage distribution and effectively suppresses damage localization at the center of the specimen.

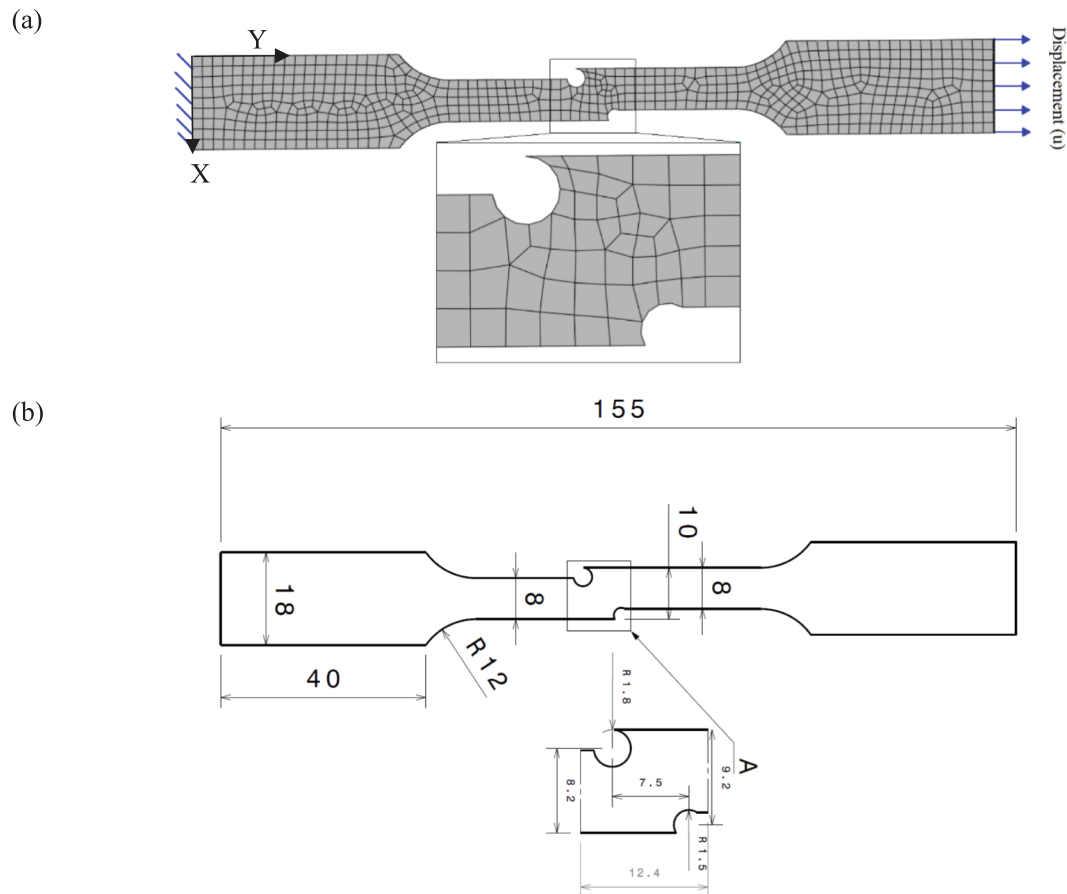


Fig. 20. Double-notched specimen subjected to monotonic axial loading: (a) boundary conditions and FE mesh, (b) geometry (all dimensions in mm; thickness = 1 mm).

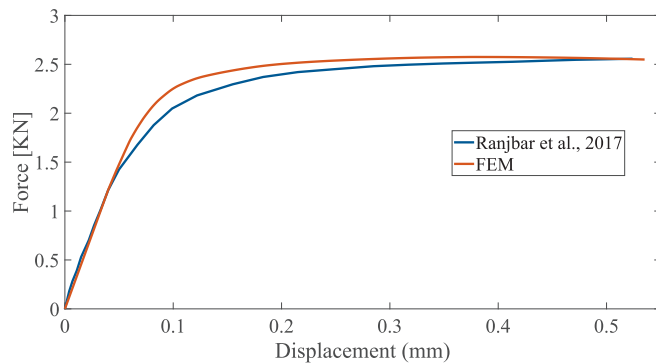


Fig. 21. Reaction force versus displacement obtained from FEM, p-FEM, for the double-notched specimen.

Figs. 18 and 19 show that the discrepancies in the predicted nonlocal damage between the two mesh configurations are 0.0298, 0.0497, and 0.0563 for characteristic lengths of 0.6325, 0.8, and 1.0, respectively. These results clearly demonstrate that the neural network's predictions of nonlocal damage are effectively mesh-independent. Notably, despite being trained on local data, the PINN exhibited excellent performance in predicting nonlocal damage. These results demonstrated the potential of PINN to capture nonlocal effects, highlighting its capability for more advanced damage predictions. This behavior further underscores the nonlocal model's ability to capture size effects and mitigate the mesh dependency often observed in classical local models.

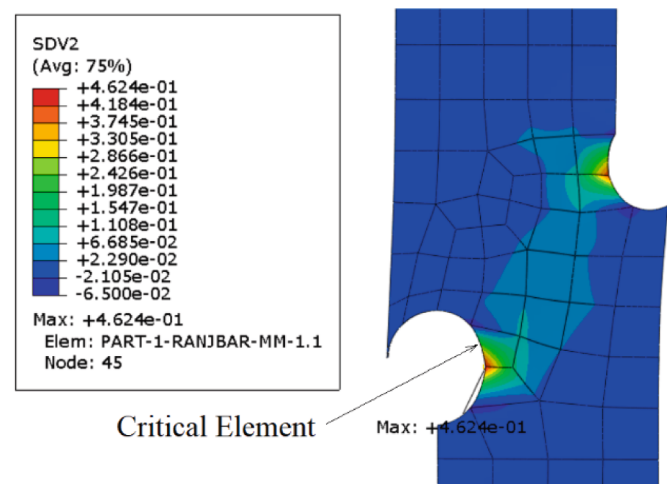


Fig. 22. Finite element prediction of local damage in the double-notched specimen at 1 mm edge displacement.(using a mesh similar to that of [Ranjbar et al. \(2017\)](#)).

5.2.2. Double-Notched specimen

The model was further evaluated on the double-notched aluminum specimen studied by [Ranjbar et al. \(2017\)](#), and the results were compared with their nonlocal integral damage model implemented using the p-FEM ($p = 3$). Consequently, the PINN was retrained using the aluminum material properties listed in [Table 8](#). The FE analysis was

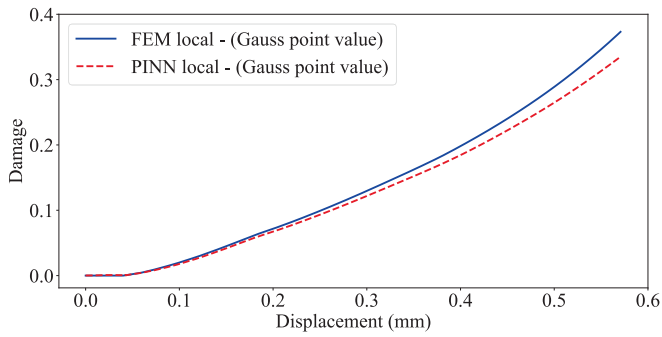


Fig. 23. Comparison of local damage at the critical integration point in the double-notched specimen: PINN vs. FEM.

performed in Abaqus using 638 C3D20R elements (20-node hexahedral elements with reduced integration). The boundary conditions, FE mesh, and specimen geometry are shown in Fig. 20.

The reaction force and local damage evolution of the specimen are presented in Figs. 21 and 22, respectively. The y-direction displacement in Fig. 21 was evaluated from the displacement difference over the 12.4 mm gauge length, as illustrated in Fig. 20(b).

The strain path of the most critical integration point in the specimen was used as input to the PINN to predict the corresponding local damage evolution. In addition, by considering a characteristic length of 1 mm, the nonlocal damage was predicted for the entire specimen. As shown in Figs. 23 and 24, the PINN predictions for both local and nonlocal cases are in good agreement with the FEM and p-FEM, respectively, where in Fig. 24(a) the values at the integration points were used for visualization. The nonlocal damage distribution in Fig. 24(a), as predicted by the PINN, provides a compact demonstration of the principal benefits of nonlocal damage modeling: mesh-objective (regularized) localization, a physically interpretable finite process-zone width governed by a characteristic length, and a damage morphology that closely matches experimental failure paths for the double-notched specimen.

6. Conclusion

This study demonstrated the effectiveness of PINNs in modeling damage evolution in ductile materials. Comparisons with finite element simulations confirmed the accuracy and reliability of PINN predictions across various loading paths and scenarios. By embedding damage evolution equations into the network's loss function, the proposed PINN framework ensures physical consistency and significantly mitigates thermodynamic violations. The PINN approach clearly demonstrated that, compared to regular neural networks using standard loss function,

it can achieve outstanding performance even outside the training domain.

The application of PINNs in nonlocal damage mechanics was also investigated. The network showed excellent performance in predicting nonlocal damage despite being trained solely on local data, and the influence of the selected characteristic length on the PINN predictions was demonstrated. This suggests that PINNs can effectively capture nonlocal effects, an essential feature for mitigating mesh dependency in damage mechanics. Furthermore, applying the trained network to benchmark problems validated its generalization capabilities, demonstrating that PINNs can predict damage evolution accurately, making them a promising tool for data-driven modeling of both local and nonlocal damage.

The numerical examples presented in this study were designed as controlled benchmark problems in order to clearly demonstrate the capabilities of the proposed PINN based surrogate constitutive framework. The objective of these examples is to illustrate the ability of the neural network to learn the mapping between strain histories and the corresponding damage evolution, rather than to address highly complex large scale problems. The results show that the trained model can accurately reproduce the damage evolution for strain paths that differ from those used in the training dataset, indicating a reasonable level of generalization within the considered range of loading conditions.

Although the proposed PINN framework demonstrates strong predictive accuracy, the optimization process presents several inherent challenges. First, incorporating multiple physics-based residual terms into the loss function requires careful tuning of weighting factors to balance data fidelity against physical constraint satisfaction; inappropriate weighting can lead to dominance of certain loss components, degrading overall accuracy. Second, the computational cost of evaluating residual terms—particularly when automatic differentiation is used for higher-order derivatives—can increase training time. Third, recurrent architectures such as GRUs, while effective for capturing history dependence, involve higher computational costs compared to standard perceptron-based architectures. Finally, generalization performance is dependent on the distribution and density of collocation points; inadequate coverage can result in poor enforcement of physics constraints in unseen regions of the strain space. Another promising direction is the Deep Energy Method (DEM), which employs the principle of minimum potential energy by defining the total potential energy of the system as the loss function. The network then learns the equilibrium configuration by minimizing directly this energy functional. Although the integration of DEM into our framework was beyond the scope of the present study, it represents a valuable avenue for future research to further improve optimization stability and convergence speed.

It should be noted that neural networks are known to exhibit a spectral bias (Rahaman et al., 2019), meaning that smooth components

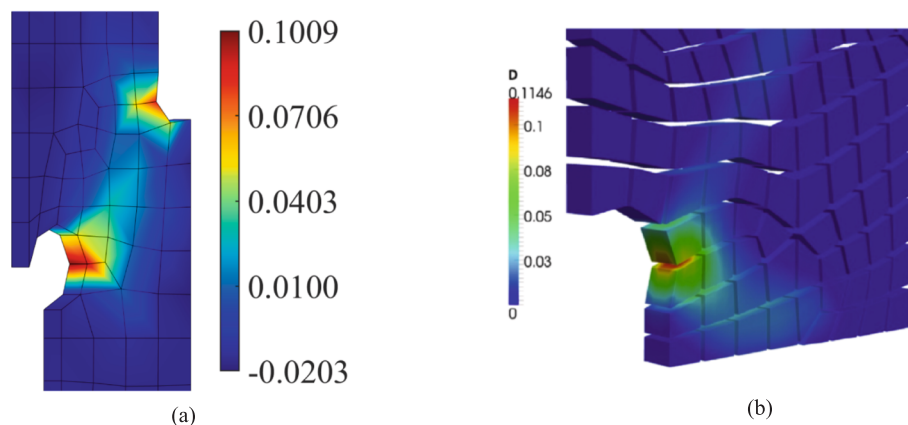


Fig. 24. Nonlocal damage distribution with a characteristic length of $\ell_r = 1$ mm at a displacement of 0.54 mm, predicted by (a) PINN and (b) the p-FEM (Ranjbar et al., 2017).

of a function are typically learned more easily than high-frequency features. In the present study, the damage evolution is governed by a continuous internal variable formulation, which leads to a smooth relationship between the strain history and the damage variable. Consequently, the training data does not contain discontinuities, and the neural network can accurately approximate the underlying constitutive mapping. For problems characterized by highly localized responses, sharp gradients, or discontinuities in the material behavior, additional strategies may be required to ensure accurate approximation. Possible approaches include adaptive sampling of the training data, the use of modified network architectures, or specialized neural network formulations designed to better capture non-smooth responses.

The above discussion highlights that while the proposed PINN demonstrates strong predictive capability, it has challenges in scenarios involving highly localized phenomena with steep gradients or discontinuities in the strain or stress fields such as stress concentrations near crack tips or abrupt changes in loading conditions especially when such cases are not represented in the training data. Since the present network was trained on smooth strain paths, its extrapolation to such sharp-gradient cases may be limited. A promising way to mitigate this issue is to perform high-fidelity finite element simulations of representative geometries and loading scenarios that naturally give rise to complex, localized deformation patterns, and to extract their corresponding strain paths for inclusion in the training dataset. Enriching the training set with such challenging strain histories could substantially improve the network's ability to handle complex, localized problems in future extensions of this work.

CRedit authorship contribution statement

Mohammad Sadegh Mirzaei: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Mohammad Mashayekhi:** Writing – review & editing, Supervision, Resources, Methodology, Investigation, Conceptualization. **Mohsen Mirkhalaf:** Writing – review & editing, Supervision, Methodology, Investigation, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

We have shared a link to the data and codes of this study.

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