

به نام نای الله

- تمرین تحویلی سری سوم
- درس نظریه گراف – آقای دکتر عین الله زاده
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- ارادتمند شما سعید حمیدی

تمرینات تحویلی

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### Exercises

1. Let  $G$  be a graph and  $e$  a link of  $G$ . Then

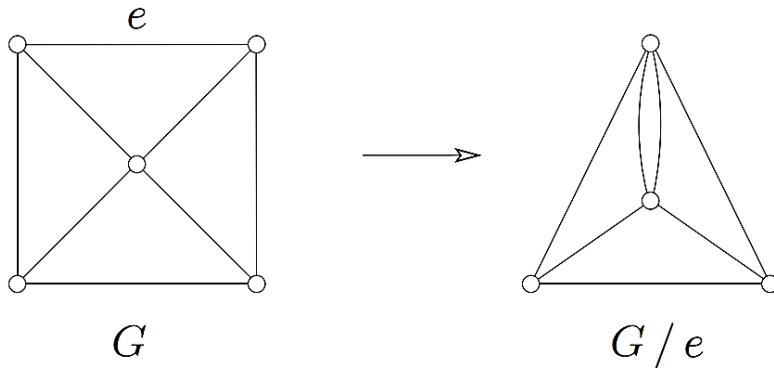
$$t(G) = t(G \setminus e) + t(G/e)$$

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**Note:**

- We denote the number of spanning trees in an arbitrary graph  $G$  by  $t(G)$ . Cayley's Formula says that  $t(K_n) = n^{n-2}$ .
- An edge with identical ends is called a loop, and an edge with distinct ends a link.
- **Edge Contraction:**

To contract an edge  $e$  of a graph  $G$  is to delete the edge and then (if the edge is a link) identify its ends. The resulting graph is denoted by  $G/e$  (see Figure below).



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2. Show that the graph  $G$  is a forest if and only if every induced subgraph of  $G$  has a vertex of degree at most one.

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3. Let the maximum degree of a vertex in a tree be  $\Delta$ . Show that there are at least  $\Delta$  leaves in the tree.

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