به‌نام نامی الله

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☑ تمرینات تحویلی

Exercises

1. Degree Sequence

If G has vertices $v_1,v_2,...,v_n$, the sequence $(d(v_1),d(v_2),...,d(v_n))$ is called a degree sequence of G. Let $d:=(d_1,d_2,...,d_n)$ be a nonincreasing sequence of nonnegative integers, that is, $d_1 \geq d_2 \geq \cdots \geq d_n \geq 0$. Show that:

a. there is a graph with degree sequence d if and only if $\sum_{i=1}^{n} d_i$ is even,
b. there is a loopless graph with degree sequence d if and only if $\sum_{i=1}^{n} d_i$ is even and $d_1 \leq \sum_{i=2}^{n} d_i$

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2. Complement of a Graph

Let G be a simple graph. The complement $\bar{G}$ of G is the simple graph whose vertex set is V and whose edges are the pairs of nonadjacent vertices of G.

a. Express the degree sequence of $\bar{G}$ in terms of the degree sequence of G.
b. Show that if G is disconnected, then $\bar{G}$ is connected. Is the converse true?

3.

- **n-Cube** The n-cube Q_n $(n \geq 1)$ is the graph whose vertex set is the set of all n-tuples of 0s and 1s, where two n-tuples are adjacent if they differ in precisely one coordinate.
- **boolean lattice** The boolean lattice BLn $(n \geq 1)$ is the graph whose vertex set is the set of all subsets of {1, 2,...,n}, where two subsets X and Y are adjacent if their symmetric difference has precisely one element.

Show that the n-cube Qn and the boolean lattice BLn are isomorphic.

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4. Self-Complementary Graph

A simple graph is self-complementary if it is isomorphic to its complement. Show that:

a. each of the graphs P_4 and C_5 is self-complementary,
b. every self-complementary graph is connected,
c. if G is self-complementary, then $n \equiv 0, 1 \pmod 4$,
d. every self-complementary graph on $4k + 1$ vertices has a vertex of degree $2k$.

5.

a. Show that every nontrivial acyclic graph has at least two vertices of degree less than two.
b. Deduce that every nontrivial connected acyclic graph has at least two vertices of degree one. When does equality hold?

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6. An(X,Y)-path is a path which starts at a vertex of X, ends at a vertexof Y , and whose internal vertices belong to neither X nor Y.

Show that a graph G is connected if and only if there is an (X,Y)-path in G for any two nonempty subsets X and Y of V .

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7. Prove the following theorems
    - In a tree, any two vertices are connected by exactly one path.
    - Every nontrivial tree has at least two leaves.
    - If T is a tree, then $e(T)= v(T) − 1$.

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8. Show that every tree is a bipartite graph.

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9. Show that the graph G is a forest if and only if every connected subgraph of G be induced subgraph

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10. Is $k_{2,3}$ isomorphic with the line graph of another graph (in other word, there exists G such that L(G) is isomorphic with this $k_{2,3}$)?
