Probability and Statistics Final Exam

Name: _____

Please show your work on all problems.

1. At a certain university, 60% of the students are enrolled in a math course, 50% are enrolled in an English course, and 40% are enrolled in both. What percentage of the students are enrolled in an English course and/or a math course?

2. Let A and B be events such that $P(A) = 0.4$ and $P(B) = 0.3$. Find $P(A \cup B)$ assuming

   (a) A and B are mutually exclusive.

   (b) A and B are statistically independent.

3. Pizzas sold at a certain restaurant can be small, medium, or large, and they can have original crust or crispy crust. If there are eight toppings available, how many different four topping pizzas can be made? (The toppings are thoroughly mixed together before being applied to the pizza.)

4. One in 2,000 people have a certain disease. There is a test for the disease that correctly diagnoses patients 99% of the time. In other words, the test is positive 99% of the time for patients with the disease, and it is negative 99% of the time for patients without the disease. If a randomly selected person tests positive for the disease, what is the chance he or she has the disease?

5. In a club with 100 members, 20 are seniors. Consider a random sample of size 12 without replacement from this club.

   (a) Find the probability that exactly 3 people in this sample are seniors.

   (b) Find the expected value and standard deviation of the number of people in this sample who are seniors.

6. A coin is weighted so that it has a 70% chance of landing heads up when flipped. In a sequence of 10 independent flips, let X be the number of flips where the coin lands face up. What type of distribution does X have? Write the probability mass function for X. Find $P(X = 6)$.

7. On average, an automobile paint job has 2 flaws in a 10 ft^2 area. Find the probability that a 16 ft^2 area has at least one flaw. Assume that the flaws on the paint job form a Poisson process.

8. A sample of radioactive material emits α-particles at an average rate of 4 per second according to a Poisson process.

    (a) Let W be the waiting time in seconds until the first particle is emitted. Find the probability density function for W.

    (b) Let W_{10} be the waiting time until the 10$^{\text{th}}$ α-particle is emitted. What type of distribution does W_{10} have? Find the expected value of W_{10}.

9. At a doctor's office, the time a patient waits between arriving and seeing a doctor is uniformly distributed between 10 minutes and 25 minutes.

    (a) Find the expected value of a patient's waiting time.

    (b) If a patient arrives at the doctor's office at 4:00, find the probability that the patient sees a doctor before 4:17.

10. Apples in a certain orchard have weights that are normally distributed with a mean of 120 grams and a standard deviation of 20 grams. Find the probability that a randomly selected apple weighs between 90 grams and 130 grams.