

STAT 319 – Probability and Statistics For Engineers

LECTURE 07

Hypothesis Testing

*Engineering College, Hail University, Saudi Arabia*

**How to Conduct a Hypothesis Testing**

Suppose that a company produces computer circuit boards, wants to test a hypothesis made by an engineer that exactly 20 percent of the defects found on the boards are traceable to the CPU socket.

Because the company produces thousands of boards a day, it would not be possible to test every single board to validate or reject that statement,

So a sample of boards is analyzed and statistics computed. Based on the results found and some decision rules, the hypothesis is or is not rejected.

## How to Conduct a Hypothesis Testing

If exactly 10 percent or 29 percent of the defects on the sample taken are actually traced to the CPU socket, the hypothesis will certainly be rejected, but what if 19.95 percent or 20.05 percent of the defects are actually traced to the CPU socket? Should the 0.05 percent difference be attributed to a sampling error? Should we reject the statement in this case? To answer these questions, we must understand how a hypothesis testing is conducted. There are six steps in the process of testing a hypothesis to determine if it is to be rejected or not.

### Step 1: Null hypothesis

The first step consists in stating the hypothesis. The hypothesis would be:

*“On average, exactly 20 percent of the defects on the circuit board are traceable to the CPU socket.”*

This statement is called the null hypothesis, denoted  $H_0$

The statement will be written as:

$$H_0 : \mu = 20\%$$

## Step 2: Alternate hypothesis

If the hypothesis is not rejected, exactly 20 percent of the defects will actually be traced to the CPU socket.

But if enough evidence is statistically provided that the null hypothesis is untrue, an *alternate hypothesis* should be assumed to be true.

The alternate hypothesis, denoted  $H_1$ , *tells what should be concluded if  $H_0$  is rejected.*

$$H_1 : \mu \neq 20\%$$

## Step 3: Test statistic

Quantity based on sample data and null hypothesis used to test between null and alternative hypotheses

To test the mean  $\mu$ , the Z formula is used when the sample sizes are greater than 30,

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

and the *t formula* is used when the samples are smaller,

$$t = \frac{\bar{X} - \mu}{s / \sqrt{n}}$$

These two equations look alike but, remember that the tables that are used to compute the Z-statistic and t-statistic are different.

### Step 4: Level of significance or level of risk

There are risks that you will reach an incorrect conclusion when using a sample statistic to make decisions about a population parameter.

In fact, you can make two different types of errors when applying hypothesis-testing methodology: Type I and Type II.

### Hypothesis Testing

Test Result –	$H_0$ True	$H_0$ False
True State $H_0$ True	Correct Decision	Type I Error
$H_0$ False	Type II Error	Correct Decision

$$\alpha = P(\text{Type I Error}) \quad \beta = P(\text{Type II Error})$$

Goal: Keep  $\alpha$ ,  $\beta$  reasonably small•

## Errors in hypothesis testing

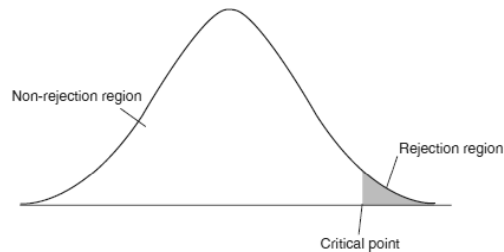
A *type I error* consists of rejecting the null hypothesis  $H_0$  when it was true.

A *type II error* involves not rejecting  $H_0$  when  $H_0$  is false.

## Step 5: Decision rule determination

The *decision rule* determines the conditions under which the null hypothesis is rejected or not.

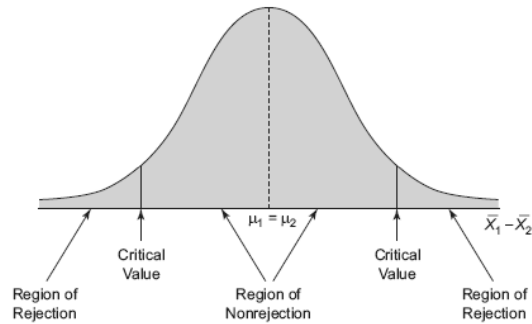
The one-tailed (right-tailed) graph shows the region of rejection.



The critical value is the dividing point between the area where  $H_0$  is rejected and the area where it is assumed to be true.

## Regions of Rejection and Nonrejection

The sampling distribution of the test statistic is divided into two regions:  
a region of rejection and a region of nonrejection.



## Step 6: Decision making

Only two decisions are considered, either the null hypothesis is rejected or it is not.

The decision to reject a null hypothesis or not depends on the level of significance.

This level often varies between 0.01 and 0.10.

## Testing for a Population Mean

### Large sample with known $\sigma$

When the sample size is greater than 30 and  $\sigma$  is known, the *Z formula* can be used to test a null hypothesis about the mean.

Test Statistic:

$$Z = \frac{\bar{X} - \mu_0}{S / \sqrt{n}}$$

## Testing for a Population Mean

### Example 7.1:

An old survey had found that the average income of operations managers for Fortune 500 companies was \$80,000 a year. A pollster wants to test that figure to determine if it is still valid. He takes a random sample of 150 operations managers to determine if their average income is \$80,000.

The mean of the sample is found to be \$78,000 with a standard deviation assumed to be \$15,000. The level of significance is set at 5 percent.

Should He reject \$80,000 as the average income or not?

## Testing for a Population Mean

### Solution:

The null hypothesis will be \$80,000 and the alternate hypothesis will be anything other than \$80,000,

$$H_0 : \mu = \$80,000$$

$$H_1 : \mu \neq \$80,000$$

Because the sample size  $n$  is larger than 30, we can use the Z formula to test the hypothesis.

Because the significance level is set at 5 percent (in other words,  $\alpha = 0.05$ ) and we are dealing with a two-tailed test, the area under each tail of the distribution will be  $\alpha/2 = 0.025$ .

The area between the mean  $\mu$  and the critical value on each side will be 0.4750 (or  $0.05 - 0.025$ ).

The critical Z-value is obtained from the Z score table by using the 0.4750 area under the curve.

## Testing for a Population Mean

### Solution:

A value of  $Z_{\alpha/2} = \pm 1.96$  corresponds to 0.4750.

The null hypothesis will not be rejected if  $-1.96 \leq Z \leq +1.96$  and rejected otherwise.

$$Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{78000 - 80000}{\frac{15000}{\sqrt{150}}} = \frac{-2000}{1224.745} = -1.633$$

Because  $Z$  is within the interval  $\pm 1.96$ , the statistical decision should be to not reject the null hypothesis.

A salary of \$78,000 is just the sample mean; if a confidence interval were determined, \$80,000 would have been the estimate point.



Table 1 Cumulative Standard Normal Distribution (continued)

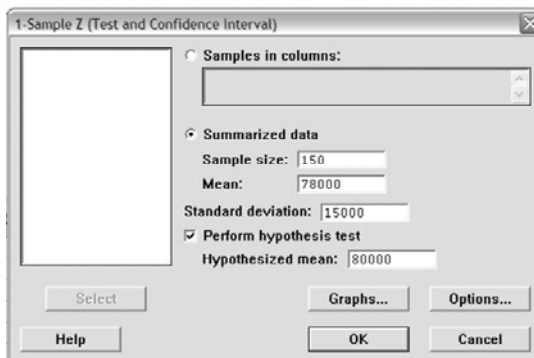
$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.523922	0.527903	0.531881	0.535856	0.0
0.1	0.539828	0.543795	0.547758	0.551717	0.555670	0.559618	0.563559	0.567495	0.571424	0.575345	0.1
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092	0.2
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732	0.3
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933	0.4
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405	0.5
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903	0.6
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236	0.7
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267	0.8
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913	0.9
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143	1.0
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977	1.1
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475	1.2
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736	1.3
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888	1.4
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083	1.5
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486	1.6
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273	1.7
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621	1.8
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705	1.9
2.0	0.977350	0.977884	0.978308	0.978722	0.979125	0.979518	0.980001	0.980474	0.980937	0.981391	2.0
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738	2.1
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989	2.2
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576	2.3
2.4	0.991803	0.992034	0.992260	0.992481	0.992696	0.992905	0.993108	0.993304	0.993493	0.993676	2.4
2.5	0.993850	0.993993	0.994132	0.994267	0.994397	0.994521	0.994641	0.994756	0.994865	0.994969	2.5
2.6	0.995139	0.995243	0.995343	0.995439	0.995531	0.995618	0.995701	0.995780	0.995855	0.995927	2.6
2.7	0.996053	0.996136	0.996216	0.996292	0.996365	0.996435	0.996501	0.996568	0.996631	0.996691	2.7
2.8	0.996748	0.996803	0.996859	0.996912	0.996962	0.997010	0.997056	0.997100	0.997143	0.997183	2.8
2.9	0.997224	0.997263	0.997300	0.997335	0.997368	0.997400	0.997430	0.997458	0.997484	0.997509	2.9
3.0	0.997532	0.997558	0.997582	0.997605	0.997626	0.997645	0.997662	0.997678	0.997692	0.997705	3.0
3.1	0.997717	0.997729	0.997739	0.997748	0.997755	0.997761	0.997766	0.997770	0.997773	0.997775	3.1
3.2	0.997776	0.997778	0.997779	0.997779	0.997779	0.997778	0.997777	0.997775	0.997773	0.997770	3.2
3.3	0.997767	0.997764	0.997761	0.997757	0.997753	0.997748	0.997743	0.997738	0.997732	0.997726	3.3
3.4	0.997719	0.997713	0.997707	0.997700	0.997693	0.997686	0.997678	0.997670	0.997662	0.997654	3.4
3.5	0.997645	0.997636	0.997627	0.997618	0.997608	0.997600	0.997590	0.997581	0.997571	0.997561	3.5
3.6	0.997549	0.997538	0.997528	0.997517	0.997506	0.997495	0.997484	0.997472	0.997461	0.997449	3.6
3.7	0.997436	0.997423	0.997410	0.997397	0.997384	0.997370	0.997356	0.997342	0.997328	0.997313	3.7
3.8	0.997298	0.997283	0.997268	0.997253	0.997238	0.997223	0.997207	0.997192	0.997176	0.997160	3.8
3.9	0.997143	0.997126	0.997109	0.997092	0.997075	0.997058	0.997040	0.997023	0.997005	0.996987	3.9

## Testing for a Population Mean

Large sample with known  $\sigma$

Using Minitab. Open Minitab and from the Stat menu, select "Basic Statistics" and then select "1-Sample Z. . ." The "1-Sample Z" dialog box appears, and values are entered.



## Testing for a Population Mean

### Large sample with known $\sigma$

After selecting "OK," the Minitab output should show as shown below.

The Minitab output suggests that for a 95 percent confidence level, the mean is expected to fall within the interval 75,599.5 and 80,400.5.

Because the mean obtained from the sample is 78,000, we cannot reject the null hypothesis.

One-Sample Z						
Test of mu = 80000 vs not = 80000						
The assumed standard deviation = 15000						
N	Mean	SE Mean	95% CI	Z	P	
150	78000.0	1224.7	(75599.5, 80400.5)	-1.63	0.102	

## Testing for a Population Mean

### Small samples with unknown $\sigma$

The *Z test statistic* is used when the population is normally distributed or when the sample sizes are greater than 30.

If the sample being analyzed is small ( $n \leq 30$ ), the *Z test statistic* would not be appropriate; the *t test* should be used instead.

The formula for the *t test* resembles the one for the *Z test* but the tables used to compute the values for *Z* and *t* are different.

Because  $\sigma$  is unknown, it will be replaced by *s*, the sample standard deviation.

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

$$df = n - 1$$

## Testing for a Population Mean

Small samples with unknown  $\sigma$

**Example :** A machine used to produce gaskets has been stable and operating under control for many years, but lately the thickness of the gaskets seems to be smaller than they once were. The mean thickness was historically 0.070 inches.

A Quality Assurance manager wants to determine if the age of the machine is causing it to produce poorer quality gaskets. He takes a sample of 10 gaskets for testing and finds a mean of 0.074 inches and a standard deviation of 0.008 inches.

Test the hypothesis that the machine is working properly with a significance level of 0.05.

## Testing for a Population Mean

Solution:

The null hypothesis should state that the population mean is still 0.070 inches (i.e. the machine is still working properly) and the alternate hypothesis should state that the mean is different from 0.070.

$$H_0 : \mu = 0.070$$

$$H_1 : \mu \neq 0.070$$

We have an equality, therefore we are faced with a two-tailed test and we will have  $\alpha/2 = 0.025$  on each side. The degree of freedom ( $n - 1$ ) is equal to 9. The value of  $t$  that we will be looking for is  $t_{0.025,9} = 2.26$ . If the computed value  $t$  falls within the interval  $[-2.26, +2.26]$ , we will not reject the null hypothesis;

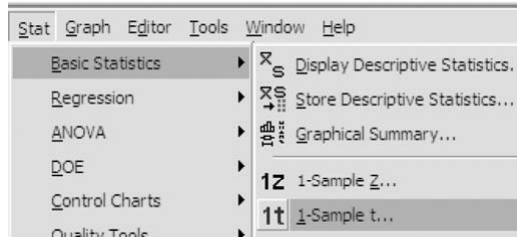
$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{0.074 - 0.07}{0.008/\sqrt{10}} = 1.012$$

The computed value of  $t$  is 1.012, therefore it falls within the interval  $[-2.262, +2.262]$ . We conclude that we cannot reject the null hypothesis.

## Testing for a Population Mean

Small samples with unknown  $\sigma$

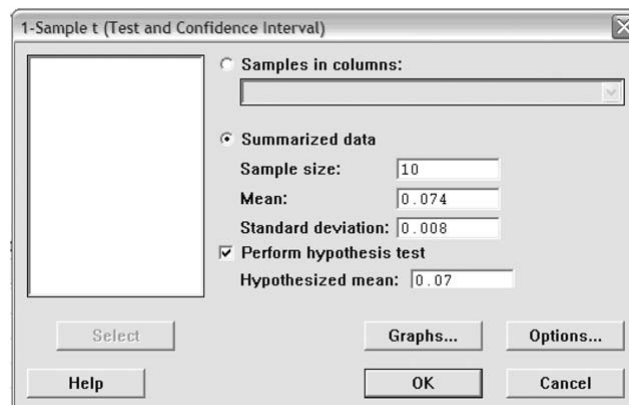
Using Minitab. From the Stat menu, select “Basic Statistics” and from the drop-down list, select “1-Sample t . . .”



## Testing for a Population Mean

Small samples with unknown  $\sigma$

Fill out the “1-Sample t” dialog box as shown.



## Testing for a Population Mean

Small samples with unknown  $\sigma$

Select "OK" to get the result shown.

One-Sample T						
Test of $\mu = 0.07$ vs not = 0.07						
N	Mean	StDev	SE Mean	95% CI	T	P
10	0.074000	0.008000	0.002530	(0.068277, 0.079723)	1.58	0.148

**Interpretation of the results.** The *p-value of 0.148 is greater than the value  $\alpha = 0.05$ . The confidence interval is [0.068277, 0.079723] and the sample mean is 0.074. The mean falls within the confidence interval, therefore we cannot reject the null hypothesis.*

## P - value

The *P-value* is the smallest level of significance at which  $H_0$  would be rejected when a specified test procedure is used on a given data set.

$$1. P\text{-value} \leq \alpha$$

$\Rightarrow$  reject  $H_0$  at a level of  $\alpha$

$$2. P\text{-value} > \alpha$$

$\Rightarrow$  do not reject  $H_0$  at a level of  $\alpha$

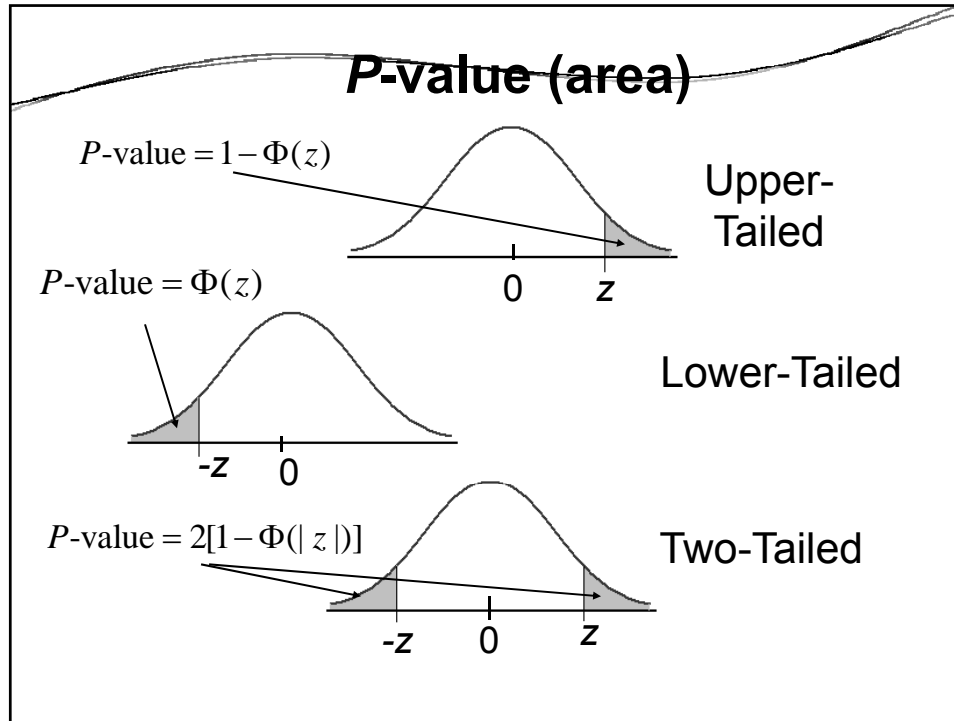
## ***P* - value**

The *P-value* is the probability, calculated assuming  $H_0$  is true, of obtaining a test statistic value at least as contradictory to  $H_0$  as the value that actually resulted. The smaller the *P*-value, the more contradictory is the data to  $H_0$ .

## ***P*-values for a *z*-test**

*P*-value:

$$P = \begin{cases} 1 - \Phi(z) & \text{upper-tailed test} \\ \Phi(z) & \text{lower-tailed test} \\ 2[1 - \Phi(|z|)] & \text{two-tailed test} \end{cases}$$



## Statistical Inference about Two Populations

Very often, it is not enough to be able to make statistical inference about one population. We sometimes want to compare two populations.

A quality controller might want to compare data from a production line to see what effect the aging machines are having on the production process over a certain period of time.

A manager might want to know how the productivity of her employees compares to the average productivity in the industry.

In this section, we will learn how to test and estimate the difference between two population means, proportions, and variances.

## Statistical Inference about Two Populations

### Inference about the difference between two means

The best estimator for the population mean  $\mu$  was the sample mean  $\bar{X}$ , so the best estimator of the difference between the population means  $(\mu_1 - \mu_0)$  will be the difference between the samples' means  $(\bar{X}_1 - \bar{X}_0)$ .

The Central Limit Theorem applies in this case, too. When the two populations are normal,  $(\bar{X}_1 - \bar{X}_0)$  will be normally distributed and it will be approximately normal if the samples sizes are large ( $n \geq 30$ ).

## Statistical Inference about Two Populations

### Inference about the difference between two means

The standard deviation for  $(\bar{X}_1 - \bar{X}_0)$  will be

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_0^2}{n_0}}$$

and its expected value

$$E(\bar{X}_1 - \bar{X}_0) = (\mu_1 - \mu_0)$$

Therefore,

$$Z = \frac{(\bar{X}_1 - \bar{X}_0) - (\mu_1 - \mu_0)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_0^2}{n_0}}}$$



## Statistical Inference about Two Populations

### Inference about the difference between two means

This equation can be transformed to obtain the confidence interval

$$(\bar{X}_1 - \bar{X}_0) - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_0^2}{n_0}} \leq (\mu_1 - \mu_0) \leq (\bar{X}_1 - \bar{X}_0) + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_0^2}{n_0}}$$

## Statistical Inference about Two Populations

### Inference about the difference between two means

**Example:** In December, the average productivity per employee at Company A was 150 machines per hour with a standard deviation of 15 machines.

For the same month, the average productivity per employee at Company B was 135 machines per hour with a standard deviation of 9 machines.

If 45 employees at Company A and 39 at Company B were randomly sampled, what is the probability that the difference in sample averages would be greater than 20 machines?

## Statistical Inference about Two Populations

Inference about the difference between two means

### Solution

$$\mu_1 = 150, \quad \sigma_1 = 15 \quad n_1 = 45$$

$$\mu_0 = 135, \quad \sigma_0 = 9 \quad n_0 = 39$$

$$\bar{X}_1 - \bar{X}_0 = 20$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_0) - (\mu_1 - \mu_0)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_0^2}{n_0}}} = \frac{20 - (150 - 135)}{\sqrt{\frac{15^2}{45} + \frac{9^2}{39}}} = \frac{5}{2.66} = 1.88$$

## Statistical Inference about Two Populations

Inference about the difference between two means

### Solution

From the *Z score table*, the probability for *Z to be larger than 1.88 will be 0.0301*.

Therefore, the probability that the difference in the sample averages will be greater than 20 machines is 0.0301.

In other words, there exists a 3.01 percent chance that the difference would be at least 20 machines.

**Thank You**  
**Any Questions ?**

**STAT 319 – Probability and Statistics For Engineers**

Dr Mohamed AICHOUNI  
&  
Dr Mustapha BOUKENDAKDJI

<http://faculty.uoh.edu.sa/m.aichouni/stat319/>  
**Email:** [m.aichouni@uoh.edu.sa](mailto:m.aichouni@uoh.edu.sa)