11. Solve the problem in the disk $r < a$ subject to u_0 (given) on the upper semicircle and $-u_0$ on the lower semicircle.

$$u = \frac{4u_0}{\pi}\left(\frac{r}{a}\sin\theta + \frac{1}{3a^3}r^3\sin 3\theta + \frac{1}{5a^5}r^5\sin 5\theta + \cdots\right)$$

13. Increase by a factor $\sqrt{2}$

15. $T = 6.826\rho R^2 f_1^2$

17. No

25. $\alpha_{11}/(2\pi) = 0.6098$; See Table A1 in App. 5.

Problem Set 12.11, page 598

5. $A_4 = A_6 = A_8 = A_{10} = 0, \quad A_5 = 605/16, \quad A_7 = -4125/128, \quad A_9 = 7315/256$

9. $\nabla^2 u = u'' + 2u'/r = 0, \quad u''/u' = -2/r, \quad \ln|u'| = -2\ln|r| + c_1,$
$u' = \tilde{c}/r^2, u = c/r + k$

13. $u = 320/r + 60$ is smaller than the potential in Prob. 12 for $2 < r < 4$.

17. $u = 1$

19. $\cos 2\phi = 2\cos^2\phi - 1, 2w^2 - 1 = \frac{4}{3}P_2(w) - \frac{1}{3}, u = \frac{4}{3}r^2 P_2(\cos\phi) - \frac{1}{3}$

25. Set $1/r = \rho$. Then $u(\rho, \theta, \phi) = rv(r, \theta, \phi), u_\rho = (v + rv_r)(-1/\rho^2),$
$u_{\rho\rho} = (2v_r + rv_{rr})(1/\rho^4) + (v + rv_r)(2/\rho^3), u_{\rho\rho} + (2/\rho)u_\rho = r^5(v_{rr} + (2/r)v_r).$
Substitute this and $u_{\phi\phi} = rv_{\phi\phi}$ etc. into (7) [written in terms of ρ] and divide by r^5.

Problem Set 12.12, page 602

5. $W = \frac{c(s)}{x^s} + \frac{x}{s^2(s+1)}, W(0, s) = 0, c(s) = 0, w(x, t) = x(t - 1 + e^{-t})$

7. $w = f(x)g(t), xf'g + f\dot{g} = xt$, take $f(x) = x$ to get $g = ce^{-t} + t - 1$ and $c = 1$ from $w(x, 0) = x(c - 1) = 0$.

11. Set $x^2/(4c^2\tau) = z^2$. Use z as a new variable of integration. Use $\text{erf}(\infty) = 1$.

Chapter 12 Review Questions and Problems, page 603

17. $u = c_1(x)e^{-3y} + c_2(x)e^{2y} - 3$

19. Hyperbolic, $f_1(x) + f_2(y + x)$

21. Hyperbolic, $f_1(y + 2x) + f_2(y - 2x)$

23. $\frac{3}{4}\cos 2t \sin x - \frac{1}{4}\cos 6t \sin 3x$

25. $\sin 0.01\pi x\, e^{-0.001143t}$

27. $\frac{3}{4}\sin 0.01\pi x\, e^{-0.001143t} - \frac{1}{4}\sin 0.03\pi x\, e^{-0.01029t}$

29. $100\cos 2x\, e^{-4t}$

39. $u = (u_1 - u_0)(\ln r)/\ln(r_1/r_0) + (u_0 \ln r_1 - u_1 \ln r_0)/\ln(r_1/r_0)$

Problem Set 13.1, page 612

1. $1/i = i/i^2 = -i, \quad 1/i^3 = i/i^4 = i$

3. $4.8 - 1.4i$

5. $x - iy = -(x + iy), \quad x = 0$

9. $-117, \quad 4$

11. $-8 - 6i$

13. $-120 - 40i$

15. $3 - i$

17. $-4x^2y^2$

19. $(x^2 - y^2)/(x^2 + y^2), \quad 2xy/(x^2 + y^2)$

Problem Set 13.2, page 618

1. $\sqrt{2}\,(\cos\frac{1}{4}\pi + i\sin\frac{1}{4}\pi)$

3. $2(\cos\frac{1}{2}\pi + i\sin\frac{1}{2}\pi), \quad 2(\cos\frac{1}{2}\pi - i\sin\frac{1}{2}\pi)$

5. $\frac{1}{2}(\cos \pi + i \sin \pi)$

7. $\sqrt{1 + \frac{1}{4}\pi^2}\,(\cos \arctan \frac{1}{2}\pi + i \sin \arctan \frac{1}{2}\pi)$

9. $3\pi/4$

11. $\pm\arctan(\frac{4}{3}) = \pm 0.9273$

13. -1024. *Answer:* π

15. $-3i$

17. $2 + 2i$

21. $\sqrt[6]{2}\,(\cos \frac{1}{12}k\pi + i \sin \frac{1}{12}\pi)$, $k = 1, 9, 17$

23. 6, $-3 \pm 3\sqrt{3}i$

25. $\cos(\frac{1}{8}\pi + \frac{1}{2}k\pi) + i \sin(\frac{1}{8}\pi + \frac{1}{2}k\pi)$, $k = 0, 1, 2, 3$

27. $\cos \frac{1}{5}\pi \pm i \sin \frac{1}{5}\pi$, $\cos \frac{3}{5}\pi \pm i \sin \frac{3}{5}\pi$, -1

29. i, $-1 - i$

31. $\pm(1 - i)$, $\pm(2 + 2i)$

33. $|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2)$. Multiply out and use $\text{Re}\, z_1\bar{z}_2 \leqq |z_1\bar{z}_2|$ (Prob. 34).
$z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 = |z_1|^2 + 2\,\text{Re}\, z_1\bar{z}_2 + |z_2|^2 \leqq |z_1|^2 + 2|z_1||z_2| + |z_2|^2 = (|z_1| + |z_2|)^2$. Hence $|z_1 + z_1|^2 \leqq (|z_1| + |z_2|)^2$. Taking square roots gives (6).

35. $[(x_1 + x_2)^2 + (y_1 + y_2)^2] + [(x_1 - x_2)^2 + (y_1 - y_2)^2] = 2(x_1^2 + y_1^2 + x_2^2 + y_2^2)$

Problem Set 13.3, page 624

1. Closed disk, center $-1 + 5i$, radius $\frac{3}{2}$

3. Annulus (circular ring), center $4 - 2i$, radii π and 3π

5. Domain between the bisecting straight lines of the first quadrant and the fourth quadrant.

7. Half-plane extending from the vertical straight line $x = -1$ to the right.

11. $u(x, y) = (1 - x)/((1 - x)^2 + y^2)$, $u(1, -1) = 0$,
$v(x, y) = y((1 - x)^2 + y^2)$, $v(1, -1) = -1$

15. Yes, since $\text{Im}(|z|^2/z) = \text{Im}(|z|^2\bar{z}/(z\bar{z})) = \text{Im}\,\bar{z} = -r \sin\theta \to 0$.

17. Yes, because $\text{Re}\, z = r\cos\theta \to 0$ and $1 - |z| \to 1$ as $r \to 0$.

19. $f'(z) = 8(z - 4i)^7$. Now $z - 4i = 3$, hence $f'(3 + 4i) = 8 \cdot 3^7 = 17{,}496$.

21. $n(1 - z)^{-n-1}i$, ni

23. $3iz^2/(z + i)^4$, $-3i/16$

Problem Set 13.4, page 629

1. $r_x = x/r = \cos\theta$, $r_y = \sin\theta$, $\theta_x = -(\sin\theta)/r$, $\theta_y = (\cos\theta)/r$
(a) $0 = u_x - v_y = u_r \cos\theta + u_\theta(-\sin\theta)/r - v_r \sin\theta - v_\theta(\cos\theta)/r$
(b) $0 = u_y + v_x = u_r \sin\theta + u_\theta(\cos\theta)/r + v_r \cos\theta + v_\theta(-\sin\theta)/r$
Multiply (a) by $\cos\theta$, (b) by $\sin\theta$, and add. Etc.

3. Yes

5. No, $f(z) = \overline{(z^2)}$

7. Yes, when $z \neq 0$. Use (7).

9. Yes, when $z \neq 0$, $-2\pi i$, $2\pi i$

11. Yes

13. $f(z) = -\frac{1}{2}i(z^2 + c)$, c real

15. $f(z) = 1/z + c$ (c real)

17. $f(z) = z^2 + z + c$ (c real)

19. No

21. $a = \pi$, $v = e^{\pi x} \sin \pi y$

23. $a = 0$, $v = \frac{1}{2}b(y^2 - x^2) + c$

27. $f = u + iv$ implies $if = -v + iu$.

29. Use (4), (5), and (1).

Problem Set 13.5, page 632

3. $e^{2\pi i}e^{-2\pi} = e^{-2\pi} = 0.001867$

5. $e^2(-1) = -7.389$

7. $e^{\sqrt{2}}i = 4.113i$

9. $5e^{i \arctan(3/4)} = 5e^{0.644i}$

11. $6.3e^{\pi i}$

13. $\sqrt{2}e^{\pi i/4}$

15. $\exp(x^2 - y^2)\cos 2xy, \quad \exp(x^2 - y^2)\sin 2xy$
17. $\text{Re}(\exp(z^3)) = \exp(x^3 - 3xy^2)\cos(3x^2y - y^3)$
19. $z = 2n\pi i, \quad n = 0, 1, \cdots$

Problem Set 13.6, page 636

1. Use (11), then (5) for e^{iy}, and simplify.
7. $\cosh 1 = 1.543,\ i \sinh 1 = 1.175i$
9. Both $-0.642 - 1.069i$. Why?
11. $i \sinh \pi = 11.55i$, both
15. Insert the definitions on the left, multiply out, and simplify.
17. $z = \pm(2n + 1)i/2$
19. $z = \pm n\pi i$

Problem Set 13.7, page 640

5. $\ln 11 + \pi i$
7. $\frac{1}{2}\ln 32 - \pi i/4 = 1.733 - 0.785i$
9. $i \arctan(0.8/0.6) = 0.927i$
11. $\ln e + \pi i/2 = 1 + \pi i/2$
13. $\pm 2n\pi i, \quad n = 0, 1, \cdots$
15. $\ln|e^i| + i \arctan \dfrac{\sin 1}{\cos 1} \pm 2n\pi i = 0 + i + 2n\pi i, \quad n = 0, 1, \cdots$
17. $\ln(i^2) = \ln(-1) = (1 \pm 2n)\pi i, \quad 2\ln i = (1 \pm 4n)\pi i, n = 0, 1, \cdots$
19. $e^{4-3i} = e^4(\cos 3 - i \sin 3) = -54.05 - 7.70i$
21. $e^{0.6}e^{0.4i} = e^{0.6}(\cos 0.4 + i \sin 0.4) = 1.678 + 0.710i$
23. $e^{(1-i)\,\text{Ln}\,(1+i)} = e^{\ln\sqrt{2} + \pi i/4 - i\ln\sqrt{2} + \pi/4} = 2.8079 + 1.3179i$
25. $e^{(3-i)(\ln 3 + \pi i)} = 27e^{\pi}(\cos(3\pi - \ln 3) + i \sin(3\pi - \ln 3)) = -284.2 + 556.4i$
27. $e^{(2-i)\,\text{Ln}(-1)} = e^{(2-i)\pi i} = e^{\pi} = 23.14$

Chapter 13 Review Questions and Problems, page 641

1. $2 - 3i$
3. $27.46e^{0.9929i}, \quad 7.616e^{1.976i}$
11. $-5 + 12i$
13. $0.16 - 0.12i$
15. i
17. $4\sqrt{2}e^{-3\pi i/4}$
19. $15e^{-\pi i/2}$
21. $\pm 3, \quad \pm 3i$
23. $(\pm 1 \pm i)/\sqrt{2}$
25. $f(z) = -iz^2/2$
27. $f(z) = e^{-2z}$
29. $f(z) = e^{-z^2/2}$
31. $\cos 3 \cosh 1 + i \sin 3 \sinh 1 = -1.528 + 0.166i$
33. $i \tanh 1 = 0.7616i$
35. $\cosh \pi \cos \pi + i \sinh \pi \sin \pi = -11.592$

Problem Set 14.1, page 651

1. Straight segment from (2, 1) to (5, 2.5).
3. Parabola $y = x^2$ from (1, 2) to (2, 8).
5. Circle through (0, 0), center (3, −1), radius $\sqrt{10}$, oriented clockwise.
7. Semicircle, center 2, radius 4.
9. Cubic parabola $y = x^3 \quad (-2 \leqq x \leqq 2)$
11. $z(t) = t + (2 + t)i \quad (-1 \leqq t \leqq 1)$
13. $z(t) = 2 - i + 2e^{it} \quad (0 \leqq t \leqq \pi)$

15. $z(t) = 2\cosh t + i\sinh t \ (-\infty < t < \infty)$
17. Circle $z(t) = -a - ib + re^{-it} \quad (0 \leqq t \leqq 2\pi)$
19. $z(t) = t + (1 - \frac{1}{4}t^2)i \quad (-2 \leqq t \leqq 2)$
21. $z(t) = (1 + i)t \quad (1 \leqq t \leqq 3), \quad \text{Re}\, z = t, \quad z'(t) = 1 + i$. *Answer:* $4 + 4i$
23. $e^{2\pi i} - e^{\pi i} = 1 - (-1) = 2$
25. $\frac{1}{2}\exp z^2|_1^i = \frac{1}{2}(e^{-1} - e^1) = -\sinh 1$
27. $\tan \frac{1}{4}\pi i - \tan \frac{1}{4} = i \tanh \frac{1}{4} - 1$
29. $\text{Im}\, z^2 = 2xy = 0$ on the axes. $z = 1 + (-1 + i)t \quad (0 \leqq t \leqq 1)$,
$(\text{Im}\, z^2)\,\dot{z} = 2(1 - t)y(-1 + i)$ integrated: $(-1 + i)/3$.
35. $|\text{Re}\, z| = |x| \leqq 3 = M$ on C, $L = \sqrt{8}$

Problem Set 14.2, page 659

1. Use (12), Sec. 14.1, with $m = 2$. **3.** Yes **5.** 5
7. **(a)** Yes. **(b)** No, we would have to move the contour across $\pm 2i$.
9. 0, yes
11. πi, no
13. 0, yes
15. $-\pi$, no
17. 0, no
19. 0, yes
21. $2\pi i$
23. $1/z + 1/(z - 1)$, hence $2\pi i + 2\pi i = 4\pi i$.
25. 0 (Why?)
27. 0 (Why?)
29. 0

Problem Set 14.3, page 663

1. $2\pi i z^2/(z - 1)|_{z=-1} = -\pi i$
3. 0
5. $2\pi i(\cos 3z)/6|_{z=0} = \pi i/3$
7. $2\pi i(i/2)^3/2 = \pi/8$
11. $2\pi i \cdot \left.\dfrac{1}{z + 2i}\right|_{z=2i} = \dfrac{\pi}{2}$
13. $2\pi i(z + 2)|_{z=2} = 8\pi i$
15. $2\pi i \cosh(-\pi^2 - \pi i) = -2\pi i \cosh \pi^2 = -60{,}739i$ since $\cosh \pi i = \cos \pi = -1$ and $\sinh \pi i = i \sin \pi = 0$.
17. $2\pi i \left.\dfrac{\text{Ln}\,(z + 1)}{z + i}\right|_{z=i} = 2\pi i \dfrac{\text{Ln}\,(1 + i)}{2i} = \pi(\ln\sqrt{2} + i\pi/4) = 1.089 + 2.467i$
19. $2\pi i e^{2i}/(2i) = \pi e^{2i}$

Problem Set 14.4, page 667

1. $(2\pi i/3!)(-\cos 0) = -\pi i/3$
3. $(2\pi i/(n - 1)!)e^0$
5. $\dfrac{2\pi i}{3!}(\cosh 2z)''' = \dfrac{\pi i}{3} \cdot 8 \sinh 1 = 9.845i$
7. $(2\pi i/(2n)!)\,(\cos z)^{(2n)}|_{z=0} = (2\pi i/(2n)!)(-1)^n \cos 0 = (-1)^n 2\pi i/(2n)!$
9. $-2\pi i(\tan \pi z)'\Big|_{z=0} = \left.\dfrac{-2\pi i \cdot \pi}{\cos^2 \pi z}\right|_{z=0} = -2\pi^2 i$
11. $\dfrac{2\pi i}{4}((1 + z)\sin z)'\Big|_{z=1/2} = \frac{1}{2}\pi i(\sin z + (1 + z)\cos z)|_{z=1/2}$
$= \frac{1}{2}\pi i(\sin \frac{1}{2} + \frac{3}{2}\cos \frac{1}{2})$
$= 2.821i$

13. $2\pi i \cdot \left.\frac{1}{z}\right|_{z=2} = \pi i$

15. 0. Why?

17. 0 by Cauchy's integral theorem for a doubly connected domain; see (6) in Sec. 14.2.

19. $(2\pi i/2!)4^{-3}(e^{3z})''|_{z=\pi i/4} = -9\pi(1 + i)/(64\sqrt{2})$

Chapter 14 Review Questions and Problems, page 668

21. $\frac{1}{2}\cosh(-\frac{1}{4}\pi^2) - \frac{1}{2} = 2.469$

23. $2\pi i(e^z)^{(4)}|_{z=0} = ie^z/12|_{z=0} = \pi i/12$ by Cauchy's integral formula.

25. $-2\pi i(\tan \pi z)'|_{z=1} = -2\pi^2 i/\cos^2 \pi z|_{z=1} = -2\pi^2 i$

27. 0 since $z^2 + \bar{z} - 2 = 2(x^2 - y^2)$ and $y = x$

29. $-4\pi i$

Problem Set 15.1, page 679

1. $z_n = (2i/2)^n$; bounded, divergent, ± 1, $\pm i$

3. $z_n = -\frac{1}{2}\pi i/(1 + 2/(ni))$ by algebra; convergent to $-\pi i/2$

5. Bounded, divergent, $\pm 1 + 10i$

7. Unbounded, hence divergent

9. Convergent to 0, hence bounded

17. Divergent; use $1/\ln n > 1/n$.

19. Convergent; use $\Sigma 1/n^2$.

21. Convergent

23. Convergent

25. Divergent

29. By absolute convergence and Cauchy's convergence principle, for given $\epsilon > 0$ we have for every $n > N(\epsilon)$ and $p = 1, 2, \cdots$

$$|z_{n+1}| + \cdots + |z_{n+p}| < \epsilon,$$

hence $|z_{n+1} + \cdots + z_{n+p}| < \epsilon$ by (6*), Sec. 13.2, hence convergence by Cauchy's principle.

Problem Set 15.2, page 684

1. No! Nonnegative integer powers of z (or $z - z_0$) only!

3. At the center, in a disk, in the whole plane

5. $\Sigma a_n z^{2n} = \Sigma a_n (z^2)^n$, $|z^2| < R = \lim |a_n/a_{n+1}|$; hence $|z| < \sqrt{R}$.

7. $\pi/2$, ∞

9. i, $\sqrt{3}$

11. $0, \sqrt{\frac{26}{5}}$

13. $-i$, $\frac{1}{2}$

15. $2i$, 1

17. $1/\sqrt{2}$

Problem Set 15.3, page 689

3. $f = \sqrt[n]{n}$. Apply l'Hôpital's rule to $\ln f = (\ln n)/n$.

5. 2

7. $\sqrt{3}$

9. $1/\sqrt{2}$

11. $\sqrt{\frac{7}{3}}$

13. 1

15. $\frac{3}{4}$

Problem Set 15.4, page 697

3. $2z^2 - \frac{(2z^2)^3}{3!} + \cdots = 2z^2 - \frac{4}{3}z^6 + \frac{4}{15}z^{10} - + \cdots, \quad R = \infty$

5. $\frac{1}{2} - \frac{1}{4}z^4 + \frac{1}{8}z^8 - \frac{1}{16}z^{12} + \frac{1}{32}z^{16} - + \cdots, \quad R = \sqrt[4]{2}$

7. $\frac{1}{2} + \frac{1}{2}\cos z = 1 - \frac{1}{2 \cdot 2!}z^2 + \frac{1}{2 \cdot 4!}z^4 - \frac{1}{2 \cdot 6!}z^6 + - \cdots, \quad R = \infty$

9. $\int_0^z \left(1 - \frac{1}{2}t^2 + \frac{1}{8}t^4 - + \cdots\right) dt = z - \frac{1}{6}z^3 + \frac{1}{40}z^5 - + \cdots, \quad R = \infty$

11. $z^3/(1!3) - z^7/(3!7) + z^{11}/(5!11) - + \cdots, \quad R = \infty$

13. $(2/\sqrt{\pi})(z - z^3/3 + z^5/(2!5) - z^7/(3!7) + \cdots), \quad R = \infty$

17. Team Project. **(a)** $(\mathrm{Ln}\,(1 + z))' = 1 - z + z^2 - + \cdots = 1/(1 + z)$.

(c) Use that the terms of $(\sin iy)/(iy)$ are all positive, so that the sum cannot be zero.

19. $\frac{1}{2} + \frac{1}{2}i + \frac{1}{2}i(z - i) + (-\frac{1}{4} + \frac{1}{4}i)(z - i)^2 - \frac{1}{4}(z - i)^3 + \cdots, \quad R = \sqrt{2}$

21. $1 - \frac{1}{2!}\left(z - \frac{1}{2}\pi\right)^2 + \frac{1}{4!}\left(z - \frac{1}{2}\pi\right)^4 - \frac{1}{6!}\left(z - \frac{1}{2}\pi\right)^6 + - \cdots, \quad R = \infty$

23. $-\frac{1}{4} - \frac{2}{8}i(z - i) + \frac{3}{16}(z - i)^2 + \frac{4}{32}i(z - i)^3 - \frac{5}{64}(z - i)^4 + \cdots, \quad R = 2$

25. $2\left(z - \frac{1}{2}i\right) + \frac{2^3}{3!}\left(z - \frac{1}{2}i\right)^3 + \frac{2^5}{5!}\left(z - \frac{1}{2}i\right)^5 + \cdots, \quad R = \infty$

Problem Set 15.5, page 704

3. $|z + i| \leqq \sqrt{3} - \delta, \quad \delta > 0$

5. $|z + \frac{1}{2}i| \leqq \frac{1}{4} - \delta, \quad \delta > 0$

7. Nowhere

9. $|z - 2i| \leqq 2 - \delta, \quad \delta > 0$

11. $|z^n| \leqq 1$ and $\Sigma 1/n^2$ converges. Use Theorem 5.

13. $|\sin^n |z|| \leqq 1$ for all z, and $\Sigma 1/n^2$ converges. Use Theorem 5.

15. $R = 4$ by Theorem 2 in Sec. 15.2; use Theorem 1.

17. $R = 1/\sqrt{\pi} > 0.56$; use Theorem 1.

Chapter 15 Review Questions and Problems, page 706

11. 1

13. 3

15. $\frac{1}{2}$

17. $\infty, \quad e^{2z}$

19. $\infty, \quad \cosh \sqrt{z}$

21. $\sum_{n=0}^{\infty} \frac{z^{4n}}{(2n + 1)!}, \quad R = \infty$

23. $\frac{1}{2} + \frac{1}{2}\cos 2z = 1 + \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}(2z)^{2n}, \quad R = \infty$

25. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} z^{2n-2}, \quad R = \infty$

27. $\cos\,[(z - \frac{1}{2}\pi) + \frac{1}{2}\pi] = -(z - \frac{1}{2}\pi) + \frac{1}{6}(z - \frac{1}{2}\pi)^3 - + \cdots = -\sin\,(z - \frac{1}{2}\pi)$

29. $\ln 3 + \frac{1}{3}(z - 3) - \frac{1}{2 \cdot 9}(z - 3)^2 + \frac{1}{3 \cdot 27}(z - 3)^3 - + \cdots, \quad R = 3$

Problem Set 16.1, page 714

1. $z^{-4} - \frac{1}{2}z^{-2} + \frac{1}{24} - \frac{1}{720}z^2 + - \cdots, \quad 0 < |z| < \infty$

3. $z^{-3} + z^{-1} + \frac{1}{2}z + \frac{1}{6}z^3 + \frac{1}{24}z^5 + \cdots, \quad 0 < |z| < \infty$

5. $z^{-2} + z^{-1} + 1 + z + z^2 + \cdots, \quad 0 < |z| < 1$

7. $z^3 + \frac{1}{2}z + \frac{1}{24}z^{-1} + \frac{1}{720}z^3 + \cdots, \quad 0 < |z| < \infty$

9. $\exp[1 + (z-1)](z-1)^{-2} = e \cdot [(z-1)^{-2} + (z-1)^{-1} + \frac{1}{2} + \frac{1}{6}(z-1) + \cdots],$
$0 < |z - 1| < \infty$

11. $$\frac{[\pi i + (z - \pi i)]^2}{(z - \pi i)^4} = \frac{(\pi i)^2}{(z - \pi i)^4} + \frac{2\pi i}{(z - \pi i)^3} + \frac{1}{(z - \pi i)^2}$$

13. $$i^{-3}\left(1 + \frac{z - i}{i}\right)^{-3}(z - i)^{-2} = \sum_{n=0}^{\infty} \binom{-3}{n} i^{-3-n}(z - i)^{n-2} = i(z - i)^{-2}$$
$-3(z - i)^{-1} - 6i + 10(z - i) + \cdots, \quad 0 < |z - i| < 1$

15. $(-\cos(z - \pi))(z - \pi)^{-2} = -(z - \pi)^{-2} + \frac{1}{2} - \frac{1}{24}(z - \pi)^2 + - \cdots,$
$0 < |z - \pi| < \infty$

19. $$\sum_{n=0}^{\infty} z^{2n}, \quad |z| < 1, \qquad -\sum_{n=0}^{\infty} \frac{1}{z^{2n+2}}, \quad |z| > 1$$

21. $-(z + \frac{1}{2}\pi)^{-1}\cos(z + \frac{1}{2}\pi) = -(z + \frac{1}{2}\pi)^{-1} + \frac{1}{2}(z + \frac{1}{2}\pi) - \frac{1}{24}(z + \frac{1}{2}\pi)^3 + - \cdots,$
$|z + \frac{1}{2}\pi| > 0$

23. $z^8 + z^{12} + z^{16} + \cdots, \quad |z| < 1, \quad -z^4 - 1 - z^{-4} - z^{-8} - \cdots, \quad |z| > 1$

25. $$\frac{i}{(z - i)^2} + \frac{1}{z - i} + i + (z - i)$$

Section 16.2, page 719

1. $0 \pm 2\pi, \pm 4\pi, \cdots$, fourth order

3. $-81i$, fourth order

5. $\pm 1, \pm 2, \cdots$, second order

7. $\pm(2 + 2i), \pm i$, simple

9. $\frac{1}{2}\sin 4z$, $z = 0, \pm\pi/4, \pm\pi/2, \cdots$, simple

11. $f(z) = (z - z_0)^n g(z)$, $g(z_0) \neq 0$, hence $f^2(z) = (z - z_0)^{2n} g^2(z)$.

13. Second-order poles at i and $-2i$

15. Simple pole at ∞, essential singularity at $1 + i$

17. Fourth-order poles at $\pm n\pi i$, $n = 0, 1, \cdots$, essential singularity at ∞

19. $e^z(1 - e^z) = 0$, $e^z = 1$, $z = \pm 2n\pi i$ simple zeros. *Answer:* simple poles at $\pm 2n\pi i$, essential singularity at ∞

21. $1, \infty$ essential singularities, $\pm 2n\pi i$, $n = 0, 1, \cdots$, simple poles

Section 16.3, page 725

3. $\frac{4}{15}$ at 0

5. $\pm 4i$ at $\mp i$

7. $1/\pi$ at $0, \pm 1, \cdots$

9. -1 at $\pm 2n\pi i$

11. $(e^z)''/2!|_{z=\pi i} = -\frac{1}{2}$ at $z = \pi i$

15. Simple pole at $\frac{1}{4}$ inside C, residue $-1/(2\pi)$. *Answer:* $-i$

17. Simple poles at $\pi/2$, residue $e^{\pi/2}/(-\sin \pi/2)$, and at $-\pi/2$, residue $e^{-\pi/2}/\sin \pi/2 = e^{-\pi/2}$. *Answer:* $-4\pi i \sinh \pi/2$

19. $2\pi i(\sinh \frac{1}{2}i)/2 = -\pi \sin \frac{1}{2}$

21. $z^{-5}\cos \pi z = \cdots + \pi^4/(4!z) - + \cdots$. *Answer:* $2\pi^5 i/24$

23. Residues $\frac{1}{2}$ at $z = \frac{1}{2}$, 2 at $z = \frac{1}{3}$. *Answer:* $5\pi i$

25. Simple poles inside C at $2i, -2i, 3i, -3i$, residues $(2i \cosh 2i)/(4z^3 + 26z)|_{z=2i} = \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}$, respectively. *Answer:* $2\pi i \cdot \frac{4}{10}$

Problem Set 16.4, page 733

1. $2\pi/\sqrt{k^2 - 1}$

3. $\pi/\sqrt{2}$

5. $5\pi/12$

7. $2a\pi/\sqrt{a^2 - 1}$

9. 0. Why? (Make a sketch.)

11. $\pi/2$

13. 0. Why?

15. $\pi/3$

17. 0. Why?

19. Simple poles at ± 1, i (and $-i$); $2\pi i \cdot \frac{1}{4}i + \pi i(-\frac{1}{4} + \frac{1}{4}) = -\frac{1}{2}\pi$

21. Simple poles at 1 and $\pm 2\pi i$, residues i and $-i$. *Answer:* $\frac{\pi}{5}(\cos 1 - e^{-2})$

23. $-\pi/2$

25. 0

27. Let $q(z) = (z - a_1)(z - a_2) \cdots (z - a_k)$. Use (4) in Sec. 16.3 to form the sum of the residues $1/q'(a_1) + \cdots + 1/q'(a_k)$ and show that this sum is 0; here $k > 1$.

Chapter 16 Review Questions and Problems, page 733

11. $6\pi i$

13. $2\pi i(-10 - 10)$

15. $2\pi i(25z^2)'|_{z=5} = 500\pi i$

17. 0 (n even), $(-1)^{(n-1)/2} 2\pi i/(n - 1)!$ (n odd)

19. $\pi/6$

21. $\pi/60$

23. 0. Why?

25. $\operatorname{Res}_{z=i} e^{iz}/(z^2 + 1) = 1/(2ie)$. *Answer:* π/e.

Problem Set 17.1, page 741

5. Only in size

7. $x = c, w = -y + ic; \quad y = k, w = -k + ix$

9. Parallel displacement; each point is moved 2 to the right and 1 up.

11. $|w| \leqq \frac{1}{4}, \quad -\pi/4 < \operatorname{Arg} w < \pi/4$

13. $-5 \leqq \operatorname{Re} z \leqq -2$

15. $u \geqq 1$

17. Annulus $\frac{1}{2} \leqq |w| \leqq 4$

19. $0 < u < \ln 4, \quad \pi/4 < v \leqq 3\pi/4$

21. $z^3 + az^2 + bz + c, \quad z = -\frac{1}{3}(a \pm \sqrt{a^2 - 3b})$

23. $z = (-1 \pm \sqrt{3})/2$

25. $\sinh z = 0$ at $z = 0, \quad \pm\pi i, \pm 2\pi i, \cdots$

29. $M = |z| = 1$ on the unit circle, $J = |z|^2$

31. $|w'| = 1/|z|^2 = 1$ on the unit circle, $J = 1/|z|^4$

33. $M = e^x = 1$ for $x = 0$, the y-axis, $J = e^{2x}$

35. $M = 1/|z| = 1$ on the unit circle, $J = 1/|z|^2$

Problem Set 17.2, page 745

7. $z = \dfrac{w + i}{2w}$

9. $z = \dfrac{4w + i}{-3iw + 1}$

11. $z = 0, \quad 1/(a + ib)$

13. $z = 0, \quad \pm\frac{1}{2}, \pm = \pm i/2$

15. $z = i, 2i$ **17.** $w = \dfrac{az}{cz + a}$ **19.** $w = \dfrac{az + b}{-bz + a}$

Problem Set 17.3, page 750

3. Apply the inverse g of f on both sides of $z_1 = f(z_1)$ to get $g(z_1) = g(f(z_1)) = z_1$.
9. $w = iz$, a rotation. Sketch to see.
11. $w = (z + i)/(z - i)$
13. $w = 1/z$, almost by inspection
15. $w = 1/z - 1$
17. $w = (2z - i)/(-iz - 2)$
19. $w = (z^4 - i)(-iz^4 + 1)$

Problem Set 17.4, page 754

1. Circle $|w| = e^c$
3. Annulus $1/\sqrt{e} \leqq |w| \leqq \sqrt{e}$
5. w-plane without $w = 0$
7. $1 < |w| < e, v > 0$
9. $\pm(2n + 1)\pi/2, \quad n = 0, 1, \cdots$
11. $u^2/\cosh^2 2 + v^2/\sinh^2 2 < 1, \quad u > 0, v > 0$
13. Elliptic annulus bounded by $u^2/\cosh^2 1 + v^2/\sinh^2 1 = 1$ and $u^2/\cosh^2 3 + v^2/\sinh^2 3 = 1$
15. $\cosh z = \cos iz = \sin(iz + \frac{1}{2}\pi)$
17. $0 < \text{Im } t < \pi$ is the image of R under $t = z^2/2$. *Answer:* $e^t = e^{z^2/2}$.
19. Hyperbolas $u^2/\cos^2 c - v^2/\sin^2 c = \cosh^2 c - \sinh^2 c = 1$ when $c \neq 0, \pi$, and $u = \pm\cosh y$ (thus $|u| \geqq 1$), $v = 0$ when $c = 0, \pi$.
21. Interior of $u^2/\cosh^2 2 + v^2/\sinh^2 2 = 1$ in the fourth quadrant, or map $\pi/2 < x < \pi, 0 < y < 2$ by $w = \sin z$ (why?).
23. $v < 0$
25. The images of the five points in the figure can be obtained directly from the function w.

Problem Set 17.5, page 756

1. w moves once around the circle $|w| = \frac{1}{2}$.
3. Four sheets, branch point at $z = -1$
5. $-i/4$, three sheets
7. z_0, n sheets
9. $\sqrt{z(z - i)(z + i)}$, $0, \pm i$, two sheets

Chapter 17 Review Questions and Problems, page 756

11. $1 < |w| < 4$, $|\arg w| < \pi/4$
13. Horizontal strip $-8 < v < 8$
15. $u = 1 - \frac{1}{4}v^2$, same (why?)
17. $|w| > 1$
19. $\frac{1}{3} < |w| < \frac{1}{2}, \quad v < 0$
21. $w = 1 + iv, \quad v < 0$
23. $w = \dfrac{10z + 5i}{z + 2i}$
25. Rotation $w = iz$
27. $w = 1/z$
29. $z = 0$
31. $z = 2 \pm \sqrt{6}$
33. $z = 0, \pm i, \pm 3i$
35. $w = e^{4z}$
37. $w = iz^2 + 1$
39. $w = z^2/(2c)$

Problem Set 18.1, page 762

1. 2.5 mm = 0.25 cm; $\Phi = \text{Re}\, 110(1 + (\text{Ln}\, z)/\ln 4)$

3. $\Phi = \text{Re}\left(30 - \dfrac{20}{\ln 10}\,\text{Ln}\, z\right)$

5. $\Phi(x) = \text{Re}\,(375 + 25z)$

7. $\Phi(r) = \text{Re}\,(32 - z)$

13. Use Fig. 391 in Sec. 17.4 with the z- and w-planes interchanged and $\cos z = \sin(z + \frac{1}{2}\pi)$.

15. $\Phi = 220(x^3 - 3xy^2) = \text{Re}\,(220z^3)$

Problem Set 18.2, page 766

3. $w = iz^2$ maps R onto the strip $-2 \leqq u \leqq 0$; and $\Phi^* = U_2 + (U_1 - U_2)(1 + \frac{1}{2}u) = U_2 + (U_1 - U_2)(1 - xy)$.

5. (a) $\dfrac{(x-2)(2x-1) + 2y^2}{(x-2)^2 + y^2} = c$, **(b)** $x^2 - y^2 = c$, $xy = c$, $e^x \cos y = c$

7. See Fig. 392 in Sec. 17.4. $\Phi = \text{Re}\,(\sin^2 z)$, $\sin^2 x\ (y = 0)$, $\sin^2 x \cosh^2 1 - \cos^2 x \sinh^2 1\ (y = 1)$, $-\sinh^2 y\ (x = 0, \pi)$.

9. $\Phi(x, y) = \cos^2 x \cosh^2 y - \sin^2 x \sinh^2 y$; $\cosh^2 y\ (x = 0)$, $-\sinh y\ (x = \frac{\pi}{2})$, $\cos^2 x\ (y = 0)$, $\cos^2 x \cosh^2 1 - \sin^2 x \sinh^2 1\ (y = 1)$

13. Corresponding rays in the w-plane make equal angles, and the mapping is conformal.

15. Apply $w = z^2$.

17. $z = (2Z - i)/(-iZ - 2)$ by (3) in Sec. 17.3.

19. $\Phi = \dfrac{5}{\pi}\,\text{Arg}\,(z - 2)$, $F = -\dfrac{5i}{\pi}\,\text{Ln}\,(z - 2)$

Problem Set 18.3, page 769

1. $(80/d)y + 20$. Rotate through $\pi/2$.

5. $\dfrac{80}{\pi}\arctan\dfrac{y}{x} = \text{Re}\left(-\dfrac{80i}{\pi}\,\text{Ln}\, z\right)$

7. $T_1 + \dfrac{2}{\pi}(T_2 - T_1)\arctan\dfrac{y}{x} = \text{Re}\left(T_1 - \dfrac{2i}{\pi}(T_2 - T_1)\,\text{Ln}\, z\right)$

9. $\dfrac{T_1}{\pi}\left(\arctan\dfrac{y}{x-b} - \arctan\dfrac{y}{x-a}\right) = \text{Re}\left(\dfrac{iT_1}{\pi}\,\text{Ln}\,\dfrac{z-a}{z-b}\right)$

11. $\dfrac{100}{\pi}(\text{Arg}\,(z-1) - \text{Arg}\,(z+1)) = \text{Re}\left(\dfrac{100i}{\pi}\,\text{Ln}\,\dfrac{z+1}{z-1}\right)$

13. $\dfrac{100}{\pi}[\text{Arg}\,(z^2 - 1) - \text{Arg}\,(z^2 + 1)]$ from $w = z^2$ and Prob. 11.

15. $-20 + (320/\pi)\,\text{Arg}\, z = \text{Re}\left(-20 - \dfrac{320i}{\pi}\,\text{Ln}\, z\right)$

17. $\text{Re}\, F(z) = 100 + (200/\pi)\,\text{Re}\,(\arcsin z)$

Problem Set 18.4, page 776

1. $V(z)$ continuously differentiable.

3. $|F'(iy)| = 1 + 1/y^2$, $|y| \geqq 1$, is maximum at $y = \pm 1$, namely, 2.

5. Calculate or note that ∇^2 = div grad and curl grad is the zero vector; see Sec. 9.8 and Problem Set 9.7.
7. Horizontal parallel flow to the right.
9. $F(z) = z^4$
11. Uniform parallel flow upward, $V = \overline{F'} = iK, V_1 = 0, V_2 = K$
13. $F(z) = z^3$
15. $F(z) = z/r_0 + r_0/z$
17. Use that $w = \arccos z$ gives $z = \cos w$ and interchanging the roles of the z- and w-planes.
19. $y/(x^2 + y^2) = c$ or $x^2 + (y - k)^2 = k^2$

Problem Set 18.5, page 781

5. $\Phi = \frac{3}{2} r^3 \sin 3\theta$
7. $\Phi = \frac{1}{2} a + \frac{1}{2} a r^8 \cos 8\theta$
9. $\Phi = 3 - 4r^2 \cos 2\theta + r^4 \cos 4\theta$
11. $\Phi = \dfrac{2}{\pi}\left(r \sin \theta - \dfrac{1}{2} r^2 \sin 2\theta + \dfrac{1}{3} r^3 \sin 3\theta - + \cdots\right)$
13. $\Phi = \dfrac{2}{\pi} r \sin \theta + \dfrac{1}{2} r^2 \sin 2\theta - \dfrac{2}{9\pi} r^3 \sin 3\theta - \dfrac{1}{4} r^4 \sin 4\theta + + - \cdots$
15. $\Phi = \dfrac{1}{2} + \dfrac{2}{\pi}\left(r \cos \theta - \dfrac{1}{3} r^3 \cos 3\theta + \dfrac{1}{5} r^5 \cos 5\theta - + \cdots\right)$
17. $\Phi = \dfrac{1}{3} - \dfrac{4}{\pi^2}\left(r \cos \theta - \dfrac{1}{4} r^2 \cos 2\theta + \dfrac{1}{9} r^3 \cos 3\theta - + \cdots\right)$

Problem Set 18.6, page 784

1. Use (2). $F(z_0 + e^{i\alpha}) = (\frac{7}{2} + e^{i\alpha})^3$, etc. $F(\frac{5}{2}) = \frac{343}{8}$
3. Use (2). $F(z_0 + e^{i\alpha}) = (2 + 3e^{i\alpha})^2$, etc. $F(4) = 100$
5. No, because $|z|$ is not analytic.
7.
$$\Phi(2, -2) = -3 = \frac{1}{\pi}\int_0^1\int_0^{2\pi} (1 + r\cos\alpha)(-3 + r\sin\alpha) r\, dr\, d\alpha$$
$$= \frac{1}{\pi}\int_0^1\int_0^{2\pi} (-3r + \cdots)\, dr\, d\alpha = \frac{1}{\pi}\left(-\frac{3}{2}\right)\cdot 2\pi$$
9.
$$\Phi(1, 1) = 3 = \frac{1}{\pi}\int_0^1\int_0^{2\pi} (3 + r\cos\alpha + r\sin\alpha + r^2\cos\alpha\sin\alpha) r\, dr\, d\alpha$$
$$= \frac{1}{\pi}\cdot\frac{3}{2}\cdot 2\pi$$
13. $|F(z)| = [\cos^2 x + \sinh^2 y]^{1/2}, \quad z = \pm i, \quad \text{Max} = [1 + \sinh^2 1]^{1/2} = 1.543$
15. $|F(z)|^2 = \sinh^2 2x \cos^2 2y + \cosh^2 2x \sin^2 2y = \sinh^2 2x + 1 \cdot \sin^2 2y, \quad z = 1,$ Max $= \sinh 2 = 3.627$
17. $|F(z)|^2 = 4(2 - 2\cos 2\theta), \quad z = \pi/2, \quad 3\pi/2, \quad \text{Max} = 4$
19. No. Make up a counterexample.

Chapter 18 Review Questions and Problems, page 785

11. $\Phi = 10(1 - x + y), \quad F = 10 - 10(1 + i)z$

13. $\Phi = \text{Re}\,(220 - 95.54 \text{ Ln } z) = 220 - \dfrac{220}{\ln 10} \ln r = 220 - 95.54 \ln r.$

17. $2(1 - (2/\pi) \text{ Arg } z)$
19. $30(1 - (2/\pi) \text{ Arg } (z - 1))$
21. $\Phi = x + y = \text{const}, \quad V = \overline{F'(z)} = 1 - i,$ parallel flow
23. $T(x, y) = x(2y + 1) = \text{const}$
25. $\overline{F'(z)} = \bar{z} + 1 = x + 1 - iy$

Problem Set 19.1, page 796

1. $0.84175 \cdot 10^2, \quad -0.52868 \cdot 10^3, \quad 0.92414 \cdot 10^{-3}, \quad -0.36201 \cdot 10^6$
3. 6.3698, 6.794, 8.15, impossible
5. Add first, then round.
7. 29.9667, 0.0335; 29.9667, 0.0333704 (6S-exact)
9. 29.97, 0.035; 29.97, 0.03337; 30, 0.0; 30, 0.033
11. $|\epsilon| = |x + y - (\tilde{x} + \tilde{y})| = |(x - \tilde{x}) + (y - \tilde{y})| = |\epsilon_x + \epsilon_y|$
$\leqq |\epsilon_x| + |\epsilon_y| = \beta_x + \beta_y$

13. $\dfrac{a_1}{a_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2 + \epsilon_2} = \dfrac{\tilde{a}_1 + \epsilon_1}{\tilde{a}_2}\left(1 - \dfrac{\epsilon_2}{\tilde{a}_2} + \dfrac{\epsilon_2^2}{\tilde{a}_2^2} - + \cdots\right) \approx \dfrac{\tilde{a}_1}{\tilde{a}_2} + \dfrac{\epsilon_1}{\tilde{a}_2} - \dfrac{\epsilon_2}{\tilde{a}_2} \cdot \dfrac{\tilde{a}_1}{\tilde{a}_2},$

hence $\left|\left(\dfrac{a_1}{a_2} - \dfrac{\tilde{a}_1}{\tilde{a}_2}\right)\Big/\left|\dfrac{a_1}{a_2}\right|\right| \approx \left|\dfrac{\epsilon_1}{a_1} - \dfrac{\epsilon_2}{a_2}\right| \leqq |\epsilon_{r1}| + |\epsilon_{r2}| \leqq \beta_{r1} + \beta_{r2}$

15. **(a)** $1.38629 - 1.38604 = 0.00025$, **(b)** $\ln 1.00025 = 0.000249969$ is 6S-exact.
19. In the present case, (b) is slightly more accurate than (a) (which may produce nonsensical results; cf. Prob. 20).
21. $c_4 \cdot 2^4 + \cdots + c_0 \cdot 2^0 = (1\,0\,1\,1\,1.)_2$, NOT $(1\,1\,1\,0\,1.)_2$
23. The algorithm in Prob. 22 repeats 0011 infinitely often.
25. $n = 26$. The beginning is 0.09375 ($n = 1$).
27. $I_{14} = 0.1812$ (0.1705 4S-exact), $I_{13} = 0.1812$ (0.1820), $I_{12} = 0.1951$ (0.1951), $I_{11} = 0.2102$ (0.2103), etc.
29. $-0.126 \cdot 10^{-2}, -0.402 \cdot 10^{-3}; \quad -0.266 \cdot 10^{-6}, -0.847 \cdot 10^{-7}$

Problem Set 19.2, page 807

3. $g = 0.5 \cos x, \quad x = 0.450184$ ($= x_{10}$, exact to 6S)
5. Convergence to 4.7 for all these starting values.
7. $x = x/(e^x \sin x)$; 0.5, 0.63256, $\cdots$ converges to 0.58853 (5S-exact) in 14 steps.
9. $x = x^4 - 0.12; \quad x_0 = 0, x_3 = -0.119794$ (6S-exact)
11. $g = 4/x + x^3/16 - x^5/576; \quad x_0 = 2, x_n = 2.39165$ ($n \geqq 6$), 2.405 4S-exact
13. This follows from the intermediate value theorem of calculus.
15. $x_3 = 0.450184$
17. Convergence to $x = 4.7, 4.7, 0.8, -0.5$, respectively. Reason seen easily from the graph of f.