In the name of God

Permutation Groups
(February, 23, 20011)

1. Let G be a finite group acting on a finite set Ω. Then G has m orbits on Ω where $m|G| = \sum_{x \in G} |\mathrm{fix}(x)|$.

2. Let G be an arbitrary group with a normal subgroup N, and put $K := G/N$. Then there is an embedding $\phi : G \longrightarrow N\ wr K$ such that ϕ maps N onto $\mathrm{Im}\phi \cap B$ where B is the base group of $N wr K$.

3. Suppose that H and K are nontrivial groups acting on the sets Γ and Δ, respectively. Then the wreath product $W := K\ wr\ H$ is primitive in the product action on $\Omega := \mathrm{Fun}(\Gamma, \Delta)$ if and only if:
(i) K acts primitively but not regularly on Δ; and
(ii) Γ is finite and H acts transitively on Γ.

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Permutation Groups
(January, 08, 2013)

Answer to 4 questions

1. Let F be a field and $d \geq d$. Show that $AGL_d(F)$ is a split extension of a regular normal subgroup by a subgroup isomorphic to $GL_d(F)$; and it is a 2-transitive subgroup of $Sym(F^d)$. Also show that if $d \geq 2$, then $ASL_d(F)$ acts 2-transitively on the set of points of $AG_d(F)$.

2. Let $n \geq 5$. If $G \leq S_n$ and $G \neq A_n$ or S_n, then $|S_n : G| \geq n$.

3. Let G be a finite primitive permutation subgroup of $Sym(\Omega)$ of degree n and rank $r > 2$ with subdegrees $n_l = 1 \leq n2 \leq \cdots \leq n_r$. Assume that G is not regular. Then $n_{i+l} \leq n_i(n_2 - 1)$ for all $i \geq 2$.

4. Let G be a primitive subgroup of $Sym(\Omega)$. If G contains a 3-cycle, then $G \geq Alt(\Omega)$.

5. Let G be a permutation group of degree n containing a regular subgroup R. Suppose that R is abelian and has a cyclic Sylow p- subgroup for some prime p with $p < n$. Then G is either imprimitive or 2-transitive.

In the name of God

Permutation Groups
(December, 23, 2012)

1. Suppose that the group G acts transitively on the two sets Ω and Γ, and let H be a stabilizer of a point in the first action. Then the actions are equivalent if and only if H is the stabilizer of some point in the second action.

2. Using definition (only) to prove that a if G be a group acting transitively on a set n with at least two points, then G is primitive if and only if each point stabilizer G_α is a maximal subgroup of G.

3. Let G be an arbitrary group with a normal subgroup N, and put $K := G/N$. Then there is an embedding $\phi : G \longrightarrow N\ wrK$ such that ϕ maps N onto $\mathrm{Im}\phi \cap B$ where B is the base group of $NwrK$.