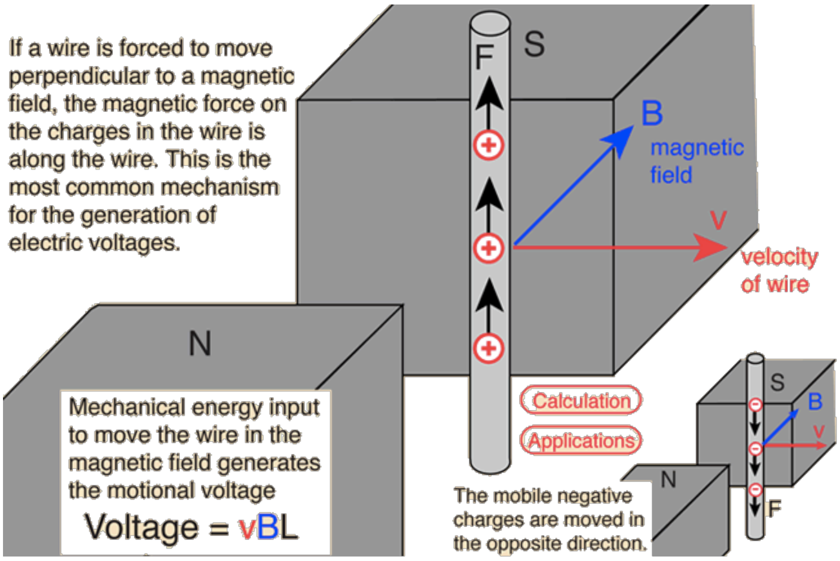
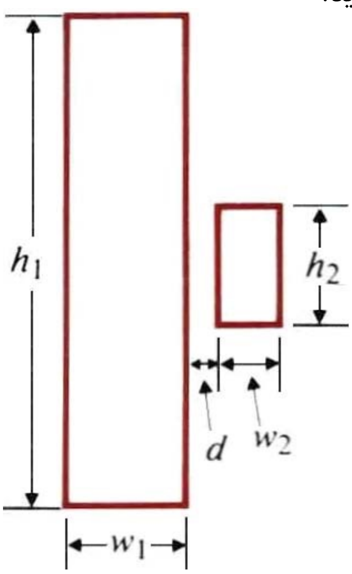
۱ـ از قانون القا فاراده $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ می دانیم که میدان مغناطیسی متغیر با زمان عامل ایجاد میدان الکتریکی القایی است. در شکل زیر سیم عمودی با سرعت v عمود بر میدان مغناطیسی یک اهنربای دائمی در حال حرکت است. به بارهای الکتریکی در این سیم نیرو وارد می‌شود. توضیح دهید از نظر ناظر ثابت نسبت به آهنربا و سپس از نظر ناظر ثابت نسبت به سیم عامل این نیرو چیست و چه روابطی را باید بکار ببریم تا از نظر هر یک از دو ناظر این نیرو را بدست آوریم.



۲ـ الف) ضریب القای متقابل بین دو حلقه سیم مستطیلی هم صفحه و با اضلاع موازی به شکل زیر را بدست آورید. فرض کنید $h_1 \gg h_2$ $\;(h_2 > w_2 > d)$. ب) اگر جریان یکسان و ثابت I از هر دو حلقه در جهت عقربه‌های ساعت عبور کند، از روش انرژی نیرویی که دو حلقه به یکدیگر وارد می‌کنند را بدست آورید؟



۳ـ میدان الکتریکی القایی توسط $\dot{\mathbf{B}} = -\partial \mathbf{B}/\partial t$ را می‌توان به طور صریح به صورت زیر نوشت

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi}\int \frac{(\mathbf{r}-\mathbf{r}')\times \dot{\mathbf{B}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}$$

با مشتق‌گیری در داخل انتگرال صحت روابط $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$ و $\nabla \cdot \mathbf{E} = 0$ را نشان دهید.

Maxwell's equations (using $\boldsymbol{D}$ and $\boldsymbol{H}$)

Differential form:		Integral form:	
$\nabla \cdot \boldsymbol{D} = \rho_{\text{free}}$	(7.58)	$\oint_{\text{closed surface}} \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{s} = \int_{\text{volume}} \rho_{\text{free}} \, \mathrm{d}\tau$	(7.59)
$\nabla \cdot \boldsymbol{B} = 0$	(7.60)	$\oint_{\text{closed surface}} \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s} = 0$	(7.61)
$\nabla \times \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}$	(7.62)	$\oint_{\text{loop}} \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\dfrac{\mathrm{d}\Phi}{\mathrm{d}t}$	(7.63)
$\nabla \times \boldsymbol{H} = \boldsymbol{J}_{\text{free}} + \dfrac{\partial \boldsymbol{D}}{\partial t}$	(7.64)	$\oint_{\text{loop}} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{\text{free}} + \int_{\text{surface}} \dfrac{\partial \boldsymbol{D}}{\partial t} \cdot \mathrm{d}\boldsymbol{s}$	(7.65)

$\boldsymbol{D}$	displacement field	$\boldsymbol{E}$	electric field
ρ_{free}	free charge density (in the sense of $\rho = \rho_{\text{induced}} + \rho_{\text{free}}$)	$\mathrm{d}\boldsymbol{s}$	surface element
$\boldsymbol{B}$	magnetic flux density	$\mathrm{d}\tau$	volume element
$\boldsymbol{H}$	magnetic field strength	$\mathrm{d}\boldsymbol{l}$	line element
$\boldsymbol{J}_{\text{free}}$	free current density (in the sense of $\boldsymbol{J} = \boldsymbol{J}_{\text{induced}} + \boldsymbol{J}_{\text{free}}$)	Φ	linked magnetic flux ($= \int \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{s}$)
		I_{free}	linked free current ($= \int \boldsymbol{J}_{\text{free}} \cdot \mathrm{d}\boldsymbol{s}$)
		t	time

Gradient

Rectangular coordinates	$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$ (2.25)	f scalar field; $\hat{}$ unit vector
Cylindrical coordinates	$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$ (2.26)	ρ distance from the z axis
Spherical polar coordinates	$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$ (2.27)	
General orthogonal coordinates	$\nabla f = \frac{\hat{\boldsymbol{q}}_1}{h_1}\frac{\partial f}{\partial q_1} + \frac{\hat{\boldsymbol{q}}_2}{h_2}\frac{\partial f}{\partial q_2} + \frac{\hat{\boldsymbol{q}}_3}{h_3}\frac{\partial f}{\partial q_3}$ (2.28)	q_i basis; h_i metric elements

Divergence

Rectangular coordinates	$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ (2.29)	$\boldsymbol{A}$ vector field; A_i ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$ (2.30)	ρ distance from the z axis
Spherical polar coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$ (2.31)	
General orthogonal coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}(A_1 h_2 h_3) + \frac{\partial}{\partial q_2}(A_2 h_3 h_1) + \frac{\partial}{\partial q_3}(A_3 h_1 h_2)\right]$ (2.32)	q_i basis; h_i metric elements

Curl

Rectangular coordinates	$\nabla\times\boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$ (2.33)	$\hat{}$ unit vector; $\boldsymbol{A}$ vector field; A_i ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla\times\boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{\rho}}/\rho & \hat{\boldsymbol{\phi}} & \hat{\boldsymbol{z}}/\rho \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$ (2.34)	ρ distance from the z axis
Spherical polar coordinates	$\nabla\times\boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{r}}/(r^2\sin\theta) & \hat{\boldsymbol{\theta}}/(r\sin\theta) & \hat{\boldsymbol{\phi}}/r \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ A_r & rA_\theta & rA_\phi \sin\theta \end{vmatrix}$ (2.35)	
General orthogonal coordinates	$\nabla\times\boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} \hat{\boldsymbol{q}}_1 h_1 & \hat{\boldsymbol{q}}_2 h_2 & \hat{\boldsymbol{q}}_3 h_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$ (2.36)	q_i basis; h_i metric elements

Radial forms[a]

$\nabla r = \frac{\boldsymbol{r}}{r}$	(2.37)	$\nabla(1/r) = \frac{-\boldsymbol{r}}{r^3}$	(2.41)
$\nabla \cdot \boldsymbol{r} = 3$	(2.38)	$\nabla \cdot (\boldsymbol{r}/r^2) = \frac{1}{r^2}$	(2.42)
$\nabla r^2 = 2\boldsymbol{r}$	(2.39)	$\nabla(1/r^2) = \frac{-2\boldsymbol{r}}{r^4}$	(2.43)
$\nabla \cdot (r\boldsymbol{r}) = 4r$	(2.40)	$\nabla \cdot (\boldsymbol{r}/r^3) = 4\pi\delta(\boldsymbol{r})$	(2.44)

[a]Note that the curl of any purely radial function is zero. $\delta(\boldsymbol{r})$ is the Dirac delta function.

Laplacian (scalar)

Rectangular coordinates	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ (2.45)	f scalar field
Cylindrical coordinates	$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$ (2.46)	ρ distance from the z axis
Spherical polar coordinates	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$ (2.47)	
General orthogonal coordinates	$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial f}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial q_3}\right)\right]$ (2.48)	q_i basis; h_i metric elements

Differential operator identities

$$\nabla(fg) \equiv f\nabla g + g\nabla f \qquad (2.49)$$

$$\nabla\cdot(f\boldsymbol{A}) \equiv f\nabla\cdot\boldsymbol{A} + \boldsymbol{A}\cdot\nabla f \qquad (2.50)$$

$$\nabla\times(f\boldsymbol{A}) \equiv f\nabla\times\boldsymbol{A} + (\nabla f)\times\boldsymbol{A} \qquad (2.51)$$

$$\nabla(\boldsymbol{A}\cdot\boldsymbol{B}) \equiv \boldsymbol{A}\times(\nabla\times\boldsymbol{B}) + (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} + \boldsymbol{B}\times(\nabla\times\boldsymbol{A}) + (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} \qquad (2.52)$$

$$\nabla\cdot(\boldsymbol{A}\times\boldsymbol{B}) \equiv \boldsymbol{B}\cdot(\nabla\times\boldsymbol{A}) - \boldsymbol{A}\cdot(\nabla\times\boldsymbol{B}) \qquad (2.53)$$

$$\nabla\times(\boldsymbol{A}\times\boldsymbol{B}) \equiv \boldsymbol{A}(\nabla\cdot\boldsymbol{B}) - \boldsymbol{B}(\nabla\cdot\boldsymbol{A}) + (\boldsymbol{B}\cdot\nabla)\boldsymbol{A} - (\boldsymbol{A}\cdot\nabla)\boldsymbol{B} \qquad (2.54)$$

$$\nabla\cdot(\nabla f) \equiv \nabla^2 f \equiv \triangle f \qquad (2.55)$$

$$\nabla\times(\nabla f) \equiv \boldsymbol{0} \qquad (2.56)$$

$$\nabla\cdot(\nabla\times\boldsymbol{A}) \equiv 0 \qquad (2.57)$$

$$\nabla\times(\nabla\times\boldsymbol{A}) \equiv \nabla(\nabla\cdot\boldsymbol{A}) - \nabla^2\boldsymbol{A} \qquad (2.58)$$

f, g	scalar fields
$\boldsymbol{A}, \boldsymbol{B}$	vector fields

Vector integral transformations

Gauss's (Divergence) theorem	$\int_V (\nabla\cdot\boldsymbol{A})\,\mathrm{d}V = \oint_{S_c} \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{s}$ (2.59)	
Stokes's theorem	$\int_S (\nabla\times\boldsymbol{A})\cdot\mathrm{d}\boldsymbol{s} = \oint_L \boldsymbol{A}\cdot\mathrm{d}\boldsymbol{l}$ (2.60)	
Green's first theorem	$\oint_S (f\nabla g)\cdot\mathrm{d}\boldsymbol{s} = \int_V \nabla\cdot(f\nabla g)\,\mathrm{d}V$ (2.61) $= \int_V [f\nabla^2 g + (\nabla f)\cdot(\nabla g)]\,\mathrm{d}V$ (2.62)	
Green's second theorem	$\oint_S [f(\nabla g) - g(\nabla f)]\cdot\mathrm{d}\boldsymbol{s} = \int_V (f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}V$ (2.63)	

$\boldsymbol{A}$	vector field
$\mathrm{d}V$	volume element
S_c	closed surface
V	volume enclosed
S	surface
$\mathrm{d}\boldsymbol{s}$	surface element
L	loop bounding S
$\mathrm{d}\boldsymbol{l}$	line element
f, g	scalar fields