سوال نمونه از فصل ۱۸

۱ـ یک موج تخت تکفام با میدان الکتریکی بفرم زیر بصورت مایل از هوا به سطح یک دی‌الکتریک با ضریب شکست $n = \sqrt{5}$ برخورد می‌کند. مرز دی‌الکتریک در صفحه xy قرار داشته و دی‌الکتریک در ناحیه $z > 0$ قرار دارد.

الف) بردار عدد موجِ موج فرودی را بدست آورید. زاویه فرودی چقدر است؟مقدار η بر حسب ثابتهای بنیادی و فرکانس چقدر است؟

ب)قطبش موج فرودی از چه نوعی است؟ (p یا s یا ترکیبی از p و s)

پ)میدان الکتریکی موج منعکس شده و عبوری را بدست آورید؟

ت)میدان مغناطیسی هر یک از سه موج فرودی، منعکس شده و عبوری را بدست آورید.

$$\mathbf{E}(\mathbf{r}, t) = E_0(\hat{x} + 2\hat{y} - 5\hat{z})e^{i[\eta(x+2y+z)-\omega t]}$$

حل:

الف) با مقایسه عبارت بالا با شکل کلی میدان موج تخت فرودی $\mathbf{E}_1(\mathbf{r}, t) = \hat{\mathbf{E}}_1 e^{i(\kappa_1 \cdot \mathbf{r} - \omega t)}$ داریم

$$\kappa_1 \cdot \mathbf{r} = \eta(x + 2y + z) \qquad => \qquad \kappa_1 = \eta(\hat{x} + 2\hat{y} + \hat{z})$$

بردار یکه عمود بر فصل مشترک دو محیط $\hat{n} = \hat{z}$ و داریم $\mathbf{u_1} \cdot \hat{n} = \cos\theta_1$ و $\mathbf{u_1} = \dfrac{\kappa_1}{|\kappa_1|} = \dfrac{\hat{x} + 2\hat{y} + \hat{z}}{\sqrt{6}}$

بنابراین $\cos\theta_1 = 1/\sqrt{6}$.

برای یک موج تخت داریم $|\kappa_1| = n_1 \dfrac{\omega}{c}$ بنابراین $\sqrt{6}\eta = \dfrac{\omega}{c}$ یعنی $\eta = \dfrac{\omega}{\sqrt{6}c}$

ب) بردار یکه عمود بر صفحه فرود عبارتست از $\hat{n}' = \dfrac{\hat{n} \times \kappa_1}{|\hat{n} \times \kappa_1|} = \dfrac{\hat{y} - 2\hat{x}}{\sqrt{5}}$ و $\hat{\mathbf{E}}_1 = E_0(\hat{x} + 2\hat{y} - 5\hat{z})$

بنابراین $\hat{n}' \cdot \hat{\mathbf{E}}_1 = 0$ و $\hat{n} \cdot \hat{\mathbf{E}}_1 \neq 0$ بنابراین قطبش موج از نوع p است.

پ) داریم $n_1 = 1$ و $n_2 = \sqrt{5}$ و $\sin\theta_1 = \sqrt{1-\frac{1}{6}} = \sqrt{5/6}$ و $n_1 \sin\theta_1 = n_2 \sin\theta_2$ بنابراین

$$\sqrt{5}\sin\theta_2 = \sqrt{5/6} \qquad => \qquad \cos\theta_2 = \sqrt{5/6}$$

همچنین داریم

$$r_{12p} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{\sqrt{5}/\sqrt{6} - \sqrt{5/6}}{\sqrt{5}/\sqrt{6} + \sqrt{5/6}} = 0 \qquad => \qquad \hat{E}'_{1p} = r_{12p}\hat{E}_{1p} = 0$$

در واقع زاویه فرودی زاویه بروستر است $\tan\theta_1 = \sqrt{\frac{1}{\cos^2\theta_1} - 1} = \sqrt{5} = \frac{n_2}{n_1}$ بنابراین انعکاس برای

قطبش p باید صفر باشد.

$$t_{12p} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2} = \frac{2/\sqrt{6}}{\sqrt{5}/\sqrt{6} + \sqrt{5/6}} = \frac{\sqrt{5}}{5} \qquad => \quad \hat{E}_{2p} = t_{12p}\hat{E}_{1p} = \frac{\sqrt{5}}{5}\hat{E}_{1p}$$

برای محاسبه میدان الکتریکی عبوری باید بردارهای κ_2 و $\hat{\mathbf{E}}_{2p}$ را بدست آوریم

داریم $|\kappa_2| = n_2\frac{\omega}{c}$ یا $\frac{|\kappa_2|}{n_2} = \frac{|\kappa_1|}{n_1} = \frac{\omega}{c} = \sqrt{6}\eta$ یعنی $|\kappa_2| = \sqrt{30}\eta$ همچنین $\mathbf{u_2}\cdot\hat{n} = \cos\theta_2$

و $\mathbf{u_2}\cdot\hat{n}' = 0$ و $\mathbf{u_2} = \frac{\kappa_2}{|\kappa_2|}$

اگر $\kappa_2 = \kappa_{2x}\hat{x} + \kappa_{2y}\hat{y} + \kappa_{2z}\hat{z}$ در نظر بگیریم روابط بالا نتیجه می‌دهند

$$\kappa_{2x} = \eta \qquad \kappa_{2y} = 2\eta \qquad \kappa_{2z} = 5\eta$$

بنابراین $\kappa_2 = \eta(\hat{x} + 2\hat{y} + 5\hat{z})$

برای محاسبه $\hat{\mathbf{E}}_{2p}$ از این واقعیت استفاده می‌کنیم که برای قطبش p داریم $\frac{\hat{\mathbf{E}}_{2p}}{|\hat{\mathbf{E}}_{2p}|} = \hat{n}'\times\mathbf{u_2}$ و در محاسبه بالا

بدست آوردیم $\hat{E}_{2p} = |\hat{\mathbf{E}}_{2p}| = \frac{\sqrt{5}}{5}\hat{E}_{1p} = \frac{\sqrt{5}}{5}\sqrt{30}E_0 = \sqrt{6}E_0$ بنابراین

$$\hat{\mathbf{E}}_{2p} = E_0(\hat{x} + 2\hat{y} - \hat{z})$$

و میدان الکتریکی عبوری برابر خواهد بود با

$$\mathbf{E}_{2p}(\mathbf{r},t) = E_0(\hat{x} + 2\hat{y} - \hat{z})e^{i[\eta(x+2y+5z)-\omega t]}$$

$$\hat{\mathbf{B}} = \frac{n}{c}\mathbf{u} \times \hat{\mathbf{E}} \quad , \quad \kappa = n\frac{\omega}{c}$$

$$r_{12s} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \quad , \quad t_{12s} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}$$

$$r_{12p} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} \quad , \quad t_{12p} = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}$$

$$r_{12s} = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)} \quad , \quad t_{12s} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_2 + \theta_1)}$$

$$r_{12p} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)} \quad , \quad t_{12p} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}$$

موفق باشید

Maxwell's equations (using *D* and *H*)

Differential form:		Integral form:	
$\nabla \cdot \boldsymbol{D} = \rho_{\text{free}}$	(7.58)	$\oint_{\text{closed surface}} \boldsymbol{D} \cdot d\boldsymbol{s} = \int_{\text{volume}} \rho_{\text{free}}\, d\tau$	(7.59)
$\nabla \cdot \boldsymbol{B} = 0$	(7.60)	$\oint_{\text{closed surface}} \boldsymbol{B} \cdot d\boldsymbol{s} = 0$	(7.61)
$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$	(7.62)	$\oint_{\text{loop}} \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\Phi}{dt}$	(7.63)
$\nabla \times \boldsymbol{H} = \boldsymbol{J}_{\text{free}} + \frac{\partial \boldsymbol{D}}{\partial t}$	(7.64)	$\oint_{\text{loop}} \boldsymbol{H} \cdot d\boldsymbol{l} = I_{\text{free}} + \int_{\text{surface}} \frac{\partial \boldsymbol{D}}{\partial t} \cdot d\boldsymbol{s}$	(7.65)

$\boldsymbol{D}$	displacement field	$\boldsymbol{E}$	electric field
ρ_{free}	free charge density (in the sense of $\rho = \rho_{\text{induced}} + \rho_{\text{free}}$)	$d\boldsymbol{s}$	surface element
$\boldsymbol{B}$	magnetic flux density	$d\tau$	volume element
$\boldsymbol{H}$	magnetic field strength	$d\boldsymbol{l}$	line element
$\boldsymbol{J}_{\text{free}}$	free current density (in the sense of $\boldsymbol{J} = \boldsymbol{J}_{\text{induced}} + \boldsymbol{J}_{\text{free}}$)	Φ	linked magnetic flux ($= \int \boldsymbol{B} \cdot d\boldsymbol{s}$)
		I_{free}	linked free current ($= \int \boldsymbol{J}_{\text{free}} \cdot d\boldsymbol{s}$)
		t	time

$D = \varepsilon E$

$H = B/\mu$

Laplacian (scalar)

Rectangular coordinates	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ (2.45)	f scalar field
Cylindrical coordinates	$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$ (2.46)	ρ distance from the z axis
Spherical polar coordinates	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$ (2.47)	
General orthogonal coordinates	$\nabla^2 f = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial f}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_3 h_1}{h_2}\frac{\partial f}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1 h_2}{h_3}\frac{\partial f}{\partial q_3}\right)\right]$ (2.48)	q_i basis; h_i metric elements

Differential operator identities

$\nabla(fg) \equiv f\nabla g + g\nabla f$	(2.49)	
$\nabla \cdot (f\boldsymbol{A}) \equiv f\nabla \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \nabla f$	(2.50)	
$\nabla \times (f\boldsymbol{A}) \equiv f\nabla \times \boldsymbol{A} + (\nabla f) \times \boldsymbol{A}$	(2.51)	
$\nabla(\boldsymbol{A} \cdot \boldsymbol{B}) \equiv \boldsymbol{A} \times (\nabla \times \boldsymbol{B}) + (\boldsymbol{A} \cdot \nabla)\boldsymbol{B} + \boldsymbol{B} \times (\nabla \times \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A}$	(2.52)	
$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) \equiv \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B})$	(2.53)	f, g scalar fields
$\nabla \times (\boldsymbol{A} \times \boldsymbol{B}) \equiv \boldsymbol{A}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{A}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{A} - (\boldsymbol{A} \cdot \nabla)\boldsymbol{B}$	(2.54)	$\boldsymbol{A}, \boldsymbol{B}$ vector fields
$\nabla \cdot (\nabla f) \equiv \nabla^2 f \equiv \triangle f$	(2.55)	
$\nabla \times (\nabla f) \equiv \boldsymbol{0}$	(2.56)	
$\nabla \cdot (\nabla \times \boldsymbol{A}) \equiv 0$	(2.57)	
$\nabla \times (\nabla \times \boldsymbol{A}) \equiv \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$	(2.58)	

Vector integral transformations

Gauss's (Divergence) theorem	$\int_V (\nabla \cdot \boldsymbol{A})\,\mathrm{d}V = \oint_{S_c} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{s}$ (2.59)	$\boldsymbol{A}$ vector field; $\mathrm{d}V$ volume element; S_c closed surface; V volume enclosed
Stokes's theorem	$\int_S (\nabla \times \boldsymbol{A}) \cdot \mathrm{d}\boldsymbol{s} = \oint_L \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$ (2.60)	S surface; $\mathrm{d}\boldsymbol{s}$ surface element; L loop bounding S; $\mathrm{d}\boldsymbol{l}$ line element
Green's first theorem	$\oint_S (f\nabla g) \cdot \mathrm{d}\boldsymbol{s} = \int_V \nabla \cdot (f\nabla g)\,\mathrm{d}V$ (2.61) $= \int_V [f\nabla^2 g + (\nabla f) \cdot (\nabla g)]\,\mathrm{d}V$ (2.62)	f, g scalar fields
Green's second theorem	$\oint_S [f(\nabla g) - g(\nabla f)] \cdot \mathrm{d}\boldsymbol{s} = \int_V (f\nabla^2 g - g\nabla^2 f)\,\mathrm{d}V$ (2.63)	

Gradient

Rectangular coordinates	$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	(2.25)	f scalar field; $\hat{}$ unit vector
Cylindrical coordinates	$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	(2.26)	ρ distance from the z axis
Spherical polar coordinates	$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$	(2.27)	
General orthogonal coordinates	$\nabla f = \frac{\hat{\boldsymbol{q}}_1}{h_1}\frac{\partial f}{\partial q_1} + \frac{\hat{\boldsymbol{q}}_2}{h_2}\frac{\partial f}{\partial q_2} + \frac{\hat{\boldsymbol{q}}_3}{h_3}\frac{\partial f}{\partial q_3}$	(2.28)	q_i basis; h_i metric elements

Divergence

Rectangular coordinates	$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	(2.29)	$\boldsymbol{A}$ vector field; A_i ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	(2.30)	ρ distance from the z axis
Spherical polar coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$	(2.31)	
General orthogonal coordinates	$\nabla \cdot \boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}(A_1 h_2 h_3) + \frac{\partial}{\partial q_2}(A_2 h_3 h_1) + \frac{\partial}{\partial q_3}(A_3 h_1 h_2)\right]$	(2.32)	q_i basis; h_i metric elements

Curl

Rectangular coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$	(2.33)	$\hat{}$ unit vector; $\boldsymbol{A}$ vector field; A_i ith component of $\boldsymbol{A}$
Cylindrical coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{\rho}}/\rho & \hat{\boldsymbol{\phi}} & \hat{\boldsymbol{z}}/\rho \\ \partial/\partial \rho & \partial/\partial \phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$	(2.34)	ρ distance from the z axis
Spherical polar coordinates	$\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{r}}/(r^2\sin\theta) & \hat{\boldsymbol{\theta}}/(r\sin\theta) & \hat{\boldsymbol{\phi}}/r \\ \partial/\partial r & \partial/\partial \theta & \partial/\partial \phi \\ A_r & rA_\theta & rA_\phi \sin\theta \end{vmatrix}$	(2.35)	
General orthogonal coordinates	$\nabla \times \boldsymbol{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} \hat{\boldsymbol{q}}_1 h_1 & \hat{\boldsymbol{q}}_2 h_2 & \hat{\boldsymbol{q}}_3 h_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$	(2.36)	q_i basis; h_i metric elements

Radial forms[a]

$\nabla r = \frac{\boldsymbol{r}}{r}$	(2.37)	$\nabla(1/r) = \frac{-\boldsymbol{r}}{r^3}$	(2.41)
$\nabla \cdot \boldsymbol{r} = 3$	(2.38)	$\nabla \cdot (\boldsymbol{r}/r^2) = \frac{1}{r^2}$	(2.42)
$\nabla r^2 = 2\boldsymbol{r}$	(2.39)	$\nabla(1/r^2) = \frac{-2\boldsymbol{r}}{r^4}$	(2.43)
$\nabla \cdot (r\boldsymbol{r}) = 4r$	(2.40)	$\nabla \cdot (\boldsymbol{r}/r^3) = 4\pi\delta(\boldsymbol{r})$	(2.44)

[a]Note that the curl of any purely radial function is zero. $\delta(\boldsymbol{r})$ is the Dirac delta function.