۳ـ ۷۰ : توزیع سرعت $V=(۳٫۵-r)V_{max}/۲٫۵$ است.

$$Q=\int_A(\vec{V}\cdot\vec{n})\,dA=\int_{۱}^{۳٫۵}\frac{۳٫۵-r}{۲٫۵}V_{max}(h\,dr)=۱٫۲۵\,h\,V_{max}\ ;\ (۰٫۰۳\ m^3/s)=۱٫۲۵(۰٫۰۰۶\,m)V_{max}$$

$V_{max}=۴\ m/s$ که در آن h ارتفاع شکاف است

$$T_{mm}=\int_{cs}(\vec{r}\times\vec{V})\rho(\vec{V}\cdot\hat{n})\,dA=-\rho\int_{cs}rV^2\,dA=-\int_{۱}^{۳٫۵}r\left(\frac{۳٫۵-r}{۲٫۵}V_{max}\right)^2\rho(h\,dr)$$

$$=-\rho h\left[۱۵٫۶۸\,r^2+۰٫۶۴\,r^4-۵٫۸۷\,r^3\right]_{۱}^{۳٫۵}\Rightarrow M_m=-(۱۰۰۰\,kg/m^3)(۰٫۰۰۶\,m)(۱۹۷۲٫۸)\Rightarrow M_m=-۱۳۱\ N$$

۳-۷۳:

$\omega = 30 \ rpm$ $\qquad$ $\omega = 30 \times \frac{2\pi}{60} = \pi$

$V_1 = 15 \ \frac{m}{s}$ $\qquad$ $\dot{W}_{shaft} = U V_\theta \dot{m}$

$\dot{W} = ?$

$V_\theta = 15 \times \cos 30 \cong 13 \ \frac{m}{s}$

$U = r.\omega = 0/6 \times \pi = 0/6\pi = 1/885 \ \frac{m}{s}$

$\dot{m} = 1000 \times 15 \times \sin 30^\circ \times 2\pi \times 0/6 \times 0/2 = 8482 \ \frac{Kg}{s}$

$\dot{W}_{shaft} = 13 \times 1/885 \times 8482 = 20/785 \times 10^4 \ W$

(۳-۷

$$\dot{w}_L = \sum_{out} \dot{m}\left(\frac{P}{\rho} + \frac{v^2}{2}\right) - \sum_{in} \dot{m}\left(\frac{P}{\rho} + \frac{v^2}{2}\right)$$

$$= \dot{m}_3\left(\frac{P_3}{\rho} + \frac{v_3^2}{2}\right) + \dot{m}_2\left(\frac{P_2}{\rho} + \frac{v_2^2}{2}\right)$$

$$- \dot{m}_1\left(\frac{P_1}{\rho} + \frac{v_1^2}{2}\right)$$

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \begin{cases} \dot{m}_1 = \rho v_1 A = \text{۱۰۰۰} \times \text{۵} \times \text{۰٫۴} = \text{۲۰۰۰} \\ \dot{m}_3 = \rho v_3 A = \text{۱۰۰۰} \times \text{۶} \;\; \text{۰٫۱۸} = \text{۱۰۸۰} \end{cases}$$

$$\rho v_1 A = \rho v_2 A + \rho v_3 A \qquad v_2 = \frac{v_1 A_1 - v_3 A_3}{A_2}$$

$$= \frac{5 \times \text{۰٫}4 - \text{۶} \times \text{۰٫۱۸}}{\text{۰٫۲۸}} = 3{,}29 \ \tfrac{m}{s}$$

$$\dot{m}_2 = \rho v_2 A_2 = \text{۱۰۰۰} \times 3{,}29 \times \text{۰٫۲۸}$$

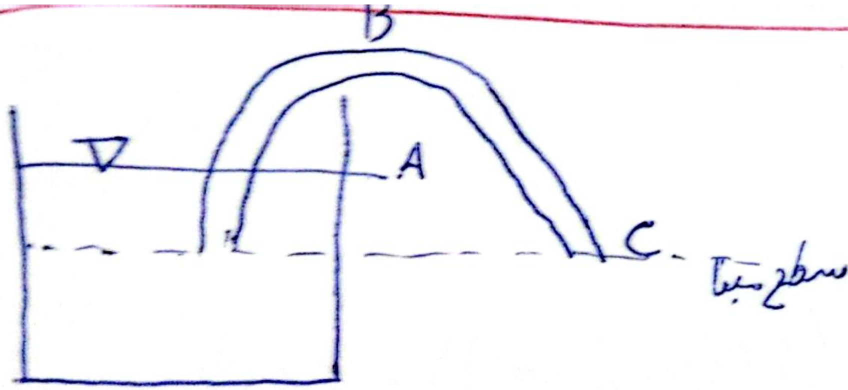
$$m_2 = 921 \ \tfrac{kg}{s}$$

$$\dot{w}_L = (\text{۴۹۳۵۵۰۴۵۸}\,w) + (\text{۱۰۵۴۹۲۴}\,w) - (\text{۱۹۰۵۰۰۰}\,w)$$

$$= \text{۲۱۵۰۳۴}\,w \qquad \boxed{w_L = 215 \ kw}$$

6

(۸۴-۳)



A و C $= \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A + h_{shaft}$

$= \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + h_L \Rightarrow 0 + 0 + h + 0 = 0 + \frac{V_C^2}{2g} + \dots$ ①

B و C: $\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_{shaft} = \frac{P_C}{\gamma} + \frac{V_C^2}{2g} + Z_C + h_L$

$-6 + 0 + (1+h) + 0 = 0 + 0 + 0 + 0$ ②

① $\Rightarrow V_C = \sqrt{2gh}$ ۲ $\Rightarrow h = 5\,m$

$\Rightarrow V_C = \sqrt{2 \times 10 \times 5} = 10\ m/s$ و $Q = A V_C = \pi \times \left(\frac{0/2}{2}\right)^2 \times 10 = 0/3142\ m^3/s = 314{,}2\ L/s$

گزینه ی ۳

$Q = ?$ $= ۸۷.۳$

$\frac{\cancel{P_A}}{\gamma} + \frac{\cancel{V_A^2}}{۲g} + Z_A + h_p = \frac{\cancel{P_B}}{\gamma} + \frac{\cancel{V_B^2}}{۲g} + Z_B + h_L$

$P_A = P_B$

$h_f = ۲۰ \frac{v^2}{۲g}$

$A = \left(\frac{\pi}{٤} \times ۰.۱۵^2\right)$

$Z_A + h_p = Z_B + h_L \rightarrow Q$

$\rightarrow h_p = Z_A - Z_B + h_L \rightarrow h_p = ٤۰ - h_L$

$h_L = ۲۰ \frac{V^2}{۲g} = ۲۰ \frac{\left(\frac{Q}{A}\right)^2}{۲g} \rightarrow ۲۰ \times \frac{\left(\frac{Q}{۰.۰۱۷}\right)^2}{۲g} \Rightarrow ۲۰ \times \frac{Q^2}{۲g \times ۰.۰۱۷^2} \Rightarrow \boxed{h_L = ۳۴۲٤ \cdot Q^2}$

شیب $\Rightarrow \frac{\Delta y}{\Delta x} = \frac{۵۰}{۲} = ۲۵ \rightarrow h_p = +۵۰ - ۲۵Q$

$-۲۵Q + ۵۰ = ٤۰ - ۳۴۲٤ Q^2 \xrightarrow{\text{ماشین}} ۰.۰۵۷ \ \frac{m^3}{s}$

⑤

6

$$\alpha = \frac{1}{A}\int\left(\frac{v}{V}\right)^3 dA \;,\; dA = 2\pi r\,dr \qquad v = \frac{r}{R} = \frac{r}{R}V_{max} \qquad A = \pi R^2$$

$$\bar{V} = \frac{\int_0^R \frac{r}{R} V_{max} \times 2\pi r\,dr}{\pi R^2} = \frac{2}{3} V_{max} \;,\; \alpha = \frac{1}{\pi R^2} \times \frac{2\pi \times V_{max}^3}{\frac{8}{27} V_{max}^3 \times R^3} \times \int_0^R r^4\,dr = \frac{27}{20}$$

$\sum \rho Q V_x = \sum F_x \rightarrow R_x = -P_1 A_1 - P_2 A_2 - \rho V_1^2 A_1 - \rho V_2^2 A_2 = -P_1 A_1 - P_2 A_2 - \rho Q (V_1 + V_2)$ ★ (۹۷-۲

$V_1 = \frac{Q}{A_1} = ۳٫۵۴ \ m/s, \quad V_2 = \frac{Q}{A_2} = ۱۴٫۱۵ \ m/s \quad / \quad \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$

$\rightarrow \frac{۱۵۰۰۰۰}{۹۸۱۰} + \frac{۳٫۵۴^۲}{۲ \times ۹٫۸۱} + ۰٫۱۴ = \frac{P_2}{۹۸۱۰} + \frac{۱۴٫۱۵^۲}{۲ \times ۹٫۸۱} + ۰٫۰۷۵ \rightarrow P_2 = ۵۹۳۰۰ \ Pa$

★ $\rightarrow R_x = -۱۵۰۰۰۰ \times ۰٫۰۷۰۷ - ۵۹۳۰۰ \times ۰٫۰۱۷۷ - ۱۰۰۰ \times ۰٫۱۲۵ \times (۳٫۵۴ + ۱۴٫۱۵) \rightarrow -۱۴۰۷۷ \ N$

$\sum F_y = ۰ \rightarrow R_y - ۵۰۰ - ۰٫۱ \times ۹۸۱۰ = ۰ \rightarrow R_y = ۱۴۸۱ \ N$

$R = \sqrt{۱۴۰۷۷^۲ + ۱۴۸۱^۲} = \boxed{۱۴۱۴۵ \ N}$

6

$V_1 = \frac{Q}{A_1} = \frac{0.02}{\frac{\pi}{4} \times 0.075^2} = 4.53 \ m/s$ 53.5

5

$V_2 = \frac{Q}{A_2} = \frac{0.02}{\frac{\pi}{2} \times 0.04^2} = 15.92 \ m/s$

$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \rightarrow \frac{P_1}{9810} + \frac{4.53^2}{2 \times 9.81} = \frac{15.92^2}{2 \times 9.81} \rightarrow P_1 = \underset{Pa}{116463} = 116 \ KPa$

(5-57

$$\frac{P_2}{\gamma_w}+\frac{V_2^2}{2g}+Z_2=\frac{P_3}{\gamma_w}+\frac{V_3^2}{2g}+Z_3 \longrightarrow \begin{cases} Z_2=Z_3 \\ P_3=0 \end{cases} \longrightarrow V_2A_2=V_3A_3 \longrightarrow V_2\,\tfrac{\pi}{4}(0.02)^2=V_3\,\tfrac{\pi}{4}(0.05)^2 \longrightarrow V_2=6.25V_3$$

$$P_2=-\gamma_w h=-9810\times0.2 \longrightarrow P_2=-1962 \qquad \frac{-1962}{12.1}+\frac{(6.25V_3)^2}{2g}+0=0+\frac{V_3^2}{2\times9.81}+0 \longrightarrow V_3=9.14$$

5

$$Q=A_3V_3=\tfrac{\pi}{4}(0.05)^2\times9.14 \longrightarrow Q=0.018 \xrightarrow{V_1=V_3} \frac{P_1}{\gamma}+\cancel{\frac{V_1^2}{2g}}+Z_1=\frac{P_3}{\gamma}+\cancel{\frac{V_3^2}{2g}}+Z_3 \longrightarrow P_1=P_3=0$$

۹۱-

نقطه ۱ : سطح آزاد

نقطه ۲ : نقطه با فاصله x از سطح مجرای خروجی

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \mathrm{I}$$

$$\mathrm{I} \Rightarrow 0+0+h+\frac{b}{2}=0+\frac{V_2^2}{2\times 10}+b-x$$

$$V_2^2=20\left(h+x-\frac{b}{2}\right) \qquad V_2=2\sqrt{5\left(h+x-\frac{b}{2}\right)}$$

$$V_{av}=\frac{1}{A}\int_A v\,dA \qquad , dA=b\,dx \qquad A=b^2$$

$$V_{av}=\frac{1}{A}\int_0^b 2\sqrt{5\left(h+x-\frac{b}{2}\right)}\,b\,dx=\frac{2\sqrt{5}}{b}\int_0^b\left(h+x-\frac{b}{2}\right)^{\frac{1}{2}}dx=\frac{2\sqrt{5}}{2}\times\frac{2}{3}\left[\sqrt{\left(h+x-\frac{b}{2}\right)^3}\right]_0^b$$

$$\boxed{=\frac{4\sqrt{5}}{3b}\left[\sqrt{\left(h+\frac{b}{2}\right)^3}-\sqrt{\left(h-\frac{b}{2}\right)^3}\right]}$$

5

۵-۶۴
پس

سرعت افقی $= v = \sqrt{2gh_1} \Rightarrow x = vt = \sqrt{2gh_1}\, t$

$$v_{iy} - v_y = gt \Rightarrow v_i \sin\alpha = gt \Rightarrow t = \frac{v_i \sin\alpha}{g} = \frac{\sqrt{2g(h_1+y_1)}\,\sin\alpha}{g}$$

$$\cos\alpha = \frac{v_{ix}}{v_i} = \frac{\sqrt{2gh_1}}{\sqrt{2g(h_1+y_1)}} \Rightarrow \sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1 - \frac{2gh_1}{2g(h_1+y_1)}} = \sqrt{\frac{y_1}{h_1+y_1}}$$

$$x = \frac{\sqrt{2g(h_1+y_1)}\sqrt{y_1/(h_1+y_1)}}{g}\sqrt{2gh_1} \Rightarrow x = 2\sqrt{h_1 y_1}$$

$$2\sqrt{h_1 y_1} = 2\sqrt{h_2 y_2} \Rightarrow h_1 y_1 = h_2 y_2$$

6

$Q_A = Q_B \quad V_A \times 0{,}4 = V_B \times 0{,}5 \rightarrow V_B = \frac{4}{5} V_A$ (68-

$\sqrt{2gh_A} \times \frac{\pi}{4} \times D_A^2 = \sqrt{2gh_B} \times \frac{\pi}{4} \times D_B^2 \quad h_A = \left(\frac{D_B}{D_A}\right)^4 \times h_B = \left(\frac{0{,}5}{0{,}4}\right)^4 \times 2 = 4{,}88\,m$ 6

$A_1 V_1 = A_2 V_2 \quad 2Lr\left(-\frac{dh}{dt}\right) = A_2\sqrt{2gh} \rightarrow 2L\sqrt{R^2-(R-h)^2}\,dh = A_2\sqrt{2gh}\,dt$ (74-5

$dt = \frac{2L\sqrt{2Rh-h^2}}{A_2\sqrt{2gh}}\,dh \rightarrow \Delta t = (275,95)\frac{2}{3}\left[(2-h)^{\frac{3}{2}}\right]_1^h \rightarrow \Delta t = 163,97\left[(2-h)^{\frac{3}{2}}-1\right]$ 5

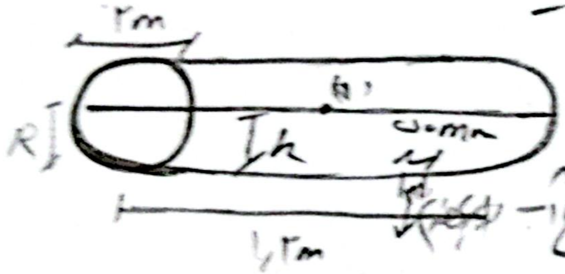
$$\frac{y_1}{y_2} = \frac{v_{02}}{v_{01}^2}$$

۷۴_ن

$g = 10\ m/s^2$ $\quad \pi = 3$

$u_2 = \sqrt{2gh}$ حرکت

طبق معادله پیوستگی داریم ، $A_1 u_1 = A_2 u_2$



$$(l \times x) \times \left(-\frac{dh}{dt}\right) = \frac{\pi}{4}(5\times 10^{-3})^2 \times \sqrt{2gh}$$

طول استوانه

عرض در ارتفاع h

کاهش ارتفاع آب

$$-l x\, dh = 8.38\times 10^{-3}\sqrt{h}\, dt$$

$$x = \sqrt{R^2-(R-h)^2}$$

$$\rightarrow -1.2\frac{\sqrt{R^2-(R-h)^2}}{\sqrt{h}}\, dh = 8.38\times 10^{-3}\, dt$$

$$\rightarrow \int \frac{\sqrt{2Rh-h^2}}{\sqrt{h}}\, dh = \int -6.98\times 10^{-3}\, dt \rightarrow \int_1^h \sqrt{2R-h}\, dh = \int_0^t -6.98\times 10^{-3}\, dt$$

$$\rightarrow \int_1^h \sqrt{2-h}\, dh = -6.98\times 10^{-3}\, t \rightarrow \frac{2}{3}(2-h)^{3/2}\Big]_1^h = -6.98\times 10^{-3}\, t$$

6

$$\rightarrow \frac{2}{3}\left((2-h)^{3/2} - 1\right) = -6.98\times 10^{-3}\, t \rightarrow t = 183.97\left[(2-h)^{3/2} - 1\right]$$

$$V=\sqrt{۲gh} \qquad Q=\frac{d\forall}{dt}$$

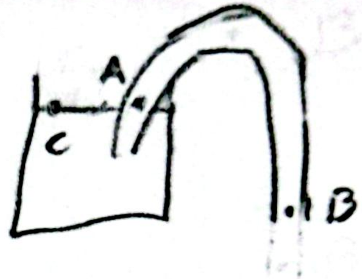
$$x=\sqrt{R^{۲}-(h-R)^{۲}}=\sqrt{۲hR-h^{۲}}$$

$$\pi x^{۲}\times\left(\frac{-dh}{dt}\right)=\frac{\pi V}{۴}d^{۲} \longrightarrow \frac{-dh}{dt}=\frac{d^{۲}}{۴}\times\frac{\sqrt{۲gh}}{۲hR-h^{۲}}$$

$$\longrightarrow \frac{dh}{dt}=-1/11\,\frac{d^{۲}\sqrt{h}}{۲hR-h^{۲}}$$

6

۳ـ ۸۱:



$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + g z_A = \frac{P_B}{\rho} + \frac{V_B^2}{2} + g z_B$$

$$\left.\begin{array}{l} A_A = A_B \\ Q_A = Q_B \end{array}\right\} \rightarrow V_A = V_B$$

$$\frac{P_A}{\rho} + \cancel{\frac{V_A^2}{2}} + g \times 1 = \frac{P_B}{\rho} + \cancel{\frac{V_A^2}{2}} + 0 \rightarrow \frac{P_A}{\rho} + g = \frac{P_B}{\rho} \quad \mathrm{I}$$

اگر جریان خروجی از B را ثابت در نظر بگیریم $\Rightarrow P_B = P_a$

5

$$\mathrm{I} \Rightarrow P_A = P_a - \gamma$$