(۴-۴

$$\Delta P = f(\mu, D, \rho, K, L, V) \rightarrow \Delta P \doteq M L^{-1} T^{-2}$$

$$\mu \doteq M L^{-1} T^{-1},\quad D \doteq L,\quad \rho \doteq M L^{-3},\quad L \doteq L,\quad V \doteq L T^{-1}$$

(فهرست نویسی) $\Rightarrow M L^{-1} T^{-2} = (M L^{-1} T^{-1})^{a} (L)^{b} (M L^{-3})^{c} (L)^{d} (L T^{-1})^{e} K$

$$\begin{cases} a + c = 1 \\ -a + b - 3c + d + e = -1 \\ a + e = 2 \end{cases} \qquad d = 1 \qquad \Rightarrow \quad -a + b - 3c + e = -2 \qquad \begin{cases} e = 2 - a \\ c = 1 - a \\ b = -a - 1 \end{cases}$$

5

$$\rightarrow \Delta P = f(\mu^{a}, D^{-a-1}, \rho^{1-a}, L^{1}, K^{a}, V^{2-a}) \longrightarrow \Delta P = \frac{\rho V^{2} L}{D} f\left(\frac{\mu}{\rho V D}, K\right)$$

$$\Delta P = f(D_1, D_2, V, \rho, \mu)$$

6-11

$\Delta P \to FL^{-2}$, $D_1 \to L$, $D_2 \to L$, $V \to LT^{-1}$, $\rho \to FL^{-4}T^{2}$, $\mu \to FL^{-2}T$

$$\left.\begin{array}{l} n = 6 \\ j = 3 \end{array}\right\} n - j = 3$$

$$\Pi_1 = \Delta P\, D_1^{a_1} V^{b_1} \mu^{c_1}$$

$$\Pi_2 = D_2\, D_1^{a_2} V^{b_2} \mu^{c_2} \;;$$

$$\Pi_3 = \rho\, D_1^{a_3} V^{b_3} \mu^{c_3}$$

$$\begin{cases} \Pi_1 \doteq \left\{(FL^{-2})(L)^{a_1}(LT^{-1})^{b_1}(FL^{-2}T)^{c_1}\right\} ; \dfrac{\Delta P}{D_1 V \mu} \\ \Pi_2 \doteq \left\{(L)(L)^{a_2}(LT^{-1})^{b_2}(FL^{-2}T)^{c_2}\right\} ; \dfrac{D_2}{D_1} \\ \Pi_3 \doteq \left\{(FL^{-4}T^{2})(L)^{a_3}(LT^{-1})^{b_3}(FL^{-2}T)^{c_3}\right\} ; \dfrac{D_1 V \rho}{\mu} \end{cases}$$

5

۱۵-۶

$$f_k = f(V , \rho , \mu , D) \rightarrow n = 5$$

$$f_k \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad \mu \doteq FL^{-2}T \quad D \doteq L$$

D و V و ρ متغیرهای تکراری هستند

تعداد متغیرهای بی بعد ۲ = ۳ - ۵

$$\Pi_1 = f_k D^{a_1} V^{b_1} \rho^{c_1} \qquad \Pi_1 \doteq (T^{-1})(L)^{a_1}(LT^{-1})^{b_1}(FL^{-4}T^{2})^{c_1}$$

$$\rightarrow$$

$$\Pi_2 = \mu D^{a_2} V^{b_2} \rho^{c_2} \qquad \Pi \doteq (FL^{-2}T)(L)^{a_2}(LT^{-1})^{b_2}(FL^{-4}T^{2})^{c_2}$$

$$\Pi_1 \rightarrow \begin{cases} F: c_1 = 0 \\ L: a_1 + b_1 - 4c_1 = 0 \rightarrow a_1 = +1 \\ T: -1 - b_1 + 2c_1 = 0 \rightarrow b_1 = -1 \end{cases} \rightarrow \Pi_1 = f_k D^{+1} V^{-1}$$

$$\Pi_2 \rightarrow \begin{cases} F: 1 + c_2 = 0 \rightarrow c_2 = -1 \\ L: -2 + a_2 + b_2 - 4c_2 = 0 \rightarrow a_2 = -1 \\ T: 1 - b_2 + 2c_2 = 0 \rightarrow b_2 = -1 \end{cases} \rightarrow \Pi_2 = \mu D^{-1} V^{-1} \rho^{-1}$$

$$\rightarrow \Pi_1 = f(\Pi_2) \rightarrow \boxed{\frac{f_k D}{V} = f\left(\frac{\mu}{D V \rho}\right)}$$

5

(19 _ ۴

$f_k = (V, \rho, \mu, D, c)$ $\qquad n = 6 \qquad k = n - 3 = 3$

$f_k \doteq T^{-1} \quad V \doteq TL^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad \mu \doteq FL^{-2}T \quad D \doteq L \quad C \doteq LT^{-1}$

$$\pi_1 = (T^{-1})(L)^{a_1}(LT^{-1})^{b_1}(FL^{-4}T^{2})^{c_1} \Rightarrow \pi_1 = \frac{f_k D}{V}$$

5.

$$\pi_2 = (FL^{-2}T)(L)^{a_2}(LT^{-1})^{b_2}(FL^{-4}T^{2})^{c_2} \Rightarrow \frac{\mu}{\rho V D} = \pi_2$$

$$\pi_3 = (LT^{-1})(L)^{a_3}(LT^{-1})^{b_3}(FL^{-4}T^{2})^{c_3} \Rightarrow \pi_3 = \frac{C}{V}$$

-۶-۲۸

$f_k = f(V, \rho, \mu, D)$ $\quad f_k \doteq T^{-1} \quad V \doteq LT^{-1} \quad \rho \doteq FL^{-۴}T^{۲} \quad \mu \doteq FL^{-۲}T \quad D \doteq L$

ρ, V, D را انتخاب می‌کنیم

$F: f_k = f(V, \rho, \mu, D) \Rightarrow f_k = f_1\left(V, \frac{\rho}{\rho}, \frac{\mu}{\rho}, D\right) = f_1\left(V, \frac{\mu}{\rho}, D\right)$

$\frac{f_k}{V} = f_۲\left(\frac{V}{V}, \frac{\mu}{\rho V}, D\right) = f_۲\left(\frac{\mu}{\rho V}, D\right)$

5

$\frac{f_k D}{V} = f_۳\left(\frac{\mu}{\rho V D}, \frac{D}{D}\right) \Rightarrow \frac{f_k D}{V} = f_۳\left(\frac{\mu}{\rho V D}\right)$

$\Delta P = f(\sigma, R)$ / $\Delta P \doteq FL^{-2}$ / $\sigma \doteq FL^{-1}$ / $R \doteq L$

$$L: \underbrace{\Delta P R^{2}}_{F} = f_1(\underbrace{\sigma R}_{F}) \Rightarrow F: \Delta P R^{2} (\sigma R)^{-1} = f_1\left[(\sigma R)(\sigma R)^{-1}\right]$$

$$\Rightarrow \frac{\Delta P R}{\sigma} = C$$

مقادیر بی بعد را جدول می شود :

۴۳٫۴۶	۴۳٫۵۷	۴۳٫۵۸	۴۴٫۱۵	۴۴٫۵۳	$\Delta PR/\sigma$

از جدول فوق مقدار متوسط $C = \text{۴۳٫۹}$ بدست می آید.

$$\Rightarrow \frac{\Delta P R}{\sigma} = \text{۴۳٫۹} \Rightarrow \Delta P = \text{۴۳٫۹}\,\frac{\sigma}{R}$$

۵

وال ۶- ۴۵ :

$V_m = \frac{V_p}{L_r}$; $V_p = L_r V_m = \frac{1}{۵} \times ۱٫۵$; $V_p = ۰٫۳ \ \frac{cm}{s}$

5

۶-۶۰ :

$V_m = ۱۵ \ \frac{m}{s} \longrightarrow T_m = ۱۸۰ \ N.m$

$\frac{T_p}{T_m} = \frac{F_{D_p} l_p}{F_{D_m} l_m} = \frac{l_p}{l_m} = \frac{1}{L_r}$; $T_p = T_m \frac{1}{L_r} = ۱۸۰ \times ۲۰$; $T_p = ۳۶۰۰$

5

$L\ F\ T \ /\ \omega \doteq T^{-1} \ /\ \mu \doteq FL^{-2}T \ /\ k \doteq FL \ /\ D_1 \doteq D_2 \doteq l \doteq L$ ۶-۴۴.

$$\begin{array}{l} l \doteq L \\ \omega \doteq T^{-1} \\ k \doteq FL \end{array} \Rightarrow \begin{cases} L \doteq l \\ T \doteq \omega^{-1} \\ F = kL^{-1} = k\,l^{-1} \end{cases}$$

چون θ بی بعد است، خودش یکی از متغیرهای پی مسأله است

$$\Rightarrow \mu \doteq FL^{-2}T = (k\,l^{-1})\, l^{-2}\, (\omega^{-1}) = k\, l^{-3}\, \omega^{-1}$$

$$\Rightarrow \Pi_1 = \theta \Rightarrow \Pi_2 = \frac{\mu\omega l^3}{k} \ /\ \Pi_3 = \frac{D_1}{l} \ /\ \Pi_4 = \frac{D_2}{l}$$

هر متغیر غیرتکراری (مانند D_1 و D_2) که بعد متغیر تکراری را داشته باشد، از تقسیم آن بر متغیر تکراری عدد بی بعد است

$$\theta = f'\left(\frac{\mu\omega l^3}{k}, \frac{D_1}{l}, \frac{D_2}{l}\right)$$

$$\Rightarrow \theta = \phi'\left(\frac{\mu\omega l^3}{k}\right)$$

۵

$$\Rightarrow \theta = 1787.4\left(\frac{\mu\omega l^3}{k}\right) \Rightarrow 1787.4\,\frac{\mu\omega(0.3)^3}{13.6} \Rightarrow \theta = 3.549\,\mu\omega$$