★ سؤال ⑨

$$u = -\frac{U_0}{\left(1-\frac{r_0^4}{x^4}\right)}$$

$\frac{Du}{Dt} = a$ یافتن شتاب در راستای محور x یعنی:

$$\frac{Du}{Dt} = u\frac{du}{dx} \longrightarrow -U_0\left(1-\frac{r_0^4}{x^4}\right)^{-1} \xrightarrow[u\sim]{\text{مشتق نسبت}} U_0 \times \frac{4r_0^4}{x^5}\left(1-\frac{r_0^4}{x^4}\right)^{-2}$$

$$a(x) = u\frac{du}{dx} = -U_0^2\left(\frac{4r_0^4}{x^5}\right)\left(1-\frac{r_0^4}{x^4}\right)^{-3} = -\frac{4U_0^2 r_0^4}{x^5\left(1-\frac{r_0^4}{x^4}\right)^3}$$

⑤

★ سؤال ⑩

$$\overline{V} = \frac{1}{A}\int_A v\,dA \longrightarrow \overline{V} = \frac{1}{r_0}\int_0^{r_0} v(r)\,dr$$

$$\overline{V} = \frac{1}{r_0}\int_0^{r_0} 2V_{max}\left(\frac{r}{r_0} - \frac{r^2}{r_0^2}\right)dr = \frac{2V_{max}}{r_0}\left[\int_0^{r_0}\frac{r}{r_0}dr - \int_0^{r_0}\frac{r^2}{r_0^2}dr\right]$$

$$\overline{V} = \frac{2V_{max}}{r_0}\left[\frac{r_0^2}{2r_0} - \frac{r_0^3}{3r_0^2}\right] = \frac{2V_{max}}{r_0}\left[\frac{3r_0 - 2r_0}{6}\right] = \frac{2V_{max}}{r_0}\times\frac{r_0}{6} = \frac{V_{max}}{3}$$

گزینه ۲

④

$$V = \frac{10}{0.5} y = 20\, y \qquad (3-14)$$

$$Q = 2\int_{-0.5}^{0} 20\, y \times 0.8\, dy = 16\, y^2 \Big|_{0}^{0.5} = 4\ m^3/s \quad (5)$$

$$\vec{V} = Q/A = 4/0.8 = 5\ m/s \qquad \dot{m} = 1.23 \times 4 = 4.92\ kg/s$$

$$\int_{cs} \rho(v_r . n)\, dA = 0$$

$$-\dot{m}_f - \rho_1 v_{r_1} A_1 + \rho_2 v_{r_2} A_2 = 0$$

$$\dot{m}_f = \rho_2 v_{r_2} A_2 - \rho_1 v_{r_1} A_1$$

$$v = v_r + v_{cv} \Rightarrow \begin{cases} v_{r_1} = v_1 - v_{cv} = 0 - (-90) = 90 \text{ km/h} \\ v_{r_2} = v_2 - v_{cv} = 10\omega_0 - (-90) = 40 \text{ km/h} \end{cases}$$

$$\dot{m}_f = 0/010 \times 40 \times 10^3 \times 0/020 - 0/03 \times 90 \times 10^3 \times 0/1 = 90 \text{ kg/h}$$

3_19

جریان پایدار : $\int_{cs} \rho(\vec{V}\cdot\vec{n})\,dA = 0$

$$\Rightarrow \int_{ab} \rho(\vec{V}\cdot\vec{n})\,dA + \int_{ad} \rho(\vec{V}\cdot\vec{n})\,dA + \int_{dc} \rho(\vec{V}\cdot\vec{n})\,dA + \int_{bc} \rho(\vec{V}\cdot\vec{n})\,dA = 0$$

$$\int_{ab} \rho(V\cdot n)\,dA = -\rho V_{ab} A_{ab} = -1000 \times 3 \times (1{,}5\times10^{-3} \times 1{,}5) = -6{,}75 \ \text{kg/s}$$

$$\int_{ad} \rho(V\cdot n)\,dA = \rho V_{ad} A_{ad} = 1000 \times 0{,}0002 \times (2\times1{,}5) = 0{,}6 \ \text{kg/s}$$

$$\int_{dc} \rho(V\cdot n)\,dA = \rho b \int_{dc} v\,dy = \rho b \int_0^{\delta} V\left[3\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{1{,}5}\right] dy = \rho b V \left| \frac{3}{2}\frac{y^2}{\delta} - \frac{2\delta}{2{,}5}\left(\frac{y}{\delta}\right)^{2{,}5} \right|_0^{\delta} = \rho b V\left(\frac{3}{2}\delta - \frac{2\delta}{2{,}5}\right)$$

$$\int_{dc} \rho(\vec{V}\cdot\vec{n})\,dA = 0{,}7\rho b V \delta = 0{,}7 \times 1000 \times 1{,}5 \times 3 \times (1{,}5\times10^{-3}) = 4{,}73 \ \text{kg/s}$$

$$\int_{bc} \rho(\vec{V}\cdot\vec{n})\,dA = \dot{m}_{bc} = 6{,}75 - 0{,}6 - 4{,}73 = 1{,}42 \ \text{kg/s}$$

5

سوال ۳۵: جریان ناپایدار است

$$\frac{\partial}{\partial t} V_w - Q_w = 0 \longrightarrow \frac{\partial}{\partial t}\left(\frac{1}{2}\frac{h}{\tan\theta_{\theta=45^\circ}}(h \times 1_0)\right) - 0,1 = 0$$

$$\longrightarrow \frac{d}{dt}(h^2 \times 1_0,44) = 0,1 \longrightarrow 22,22h \frac{dh}{dt} = 0,1 \longrightarrow \frac{dh}{dt} = \frac{0,0022}{h} \rightarrow 0,0022\, dt = h\, dh$$

$$\longrightarrow 0,0022 \int_{t_1}^{t_2} dt = \int_{1}^{1,2} h\, dh \longrightarrow \Delta t = \frac{\left(\frac{1,2^2 - 1^2}{2}\right)}{0,0022} = 81,48\,(s)$$

5

سوال ۳-۴۸

$$\sum \rho Q V_x = \sum F_x \rightarrow V_{x,1}(\dot{m}) + V_{x,2}(\dot{m}) = R_x$$

$$R_x = \dot{m}(V_{x,1} - V_{x,2}) = (1)[(10) - 10\cos 30^\circ) \rightarrow R_x = 1.34\,N$$

$$R_y = (1)(10\sin 30^\circ - 0) \rightarrow R_y = 5$$

$$W = mg = (1)(9.81) \rightarrow 9.8\,N > R_y \rightarrow \frac{F_f}{F_N} = \frac{F_f}{R_y + W}$$

$$0.1 = \frac{F_f}{5 + (9.8)} \rightarrow F_f = 1.48\,N > R_x \rightarrow$$ حرکتی رخ نمی دهد

-۲-۵۶

نیرو در راستای افق صفر است → $\int_{CS} \rho Q V_x = \sum_{CV} F_x = 0$

$\rightarrow -\rho Q_1 \times V \cos\theta + \rho Q_2 \times (-V_2) + \rho Q_3 \times V_3 = 0$

$\Rightarrow -V^2 A_1 \cos\theta - V^2 A_2 + V^2 A_3 = 0 \rightarrow -b_1 \cos\theta - d_2 + d_3 = 0$

$\rightarrow d_2 = d_3 - b_1 \cos\theta$

$Q_1 = Q_2 + Q_3 \rightarrow b_1 = d_2 + d_3 \Rightarrow b_1 = (d_3 - b_1 \cos\theta) + d_3$

5

$\rightarrow d_3 = \frac{b_1}{2}(1 + \cos\theta) \rightarrow$ گزینه ③

۲۵.۳- طبق اصل جمع پذیری جرم برای جریان تراکم ناپذیر

$Q_C = Q_A + Q_B$

طبق داده های مسئله داریم: $Q_A = 0.1t$ ، $Q_B = 0.05t^2$

$Q_C = 0.1t + 0.05t^2$

$v_C = \frac{Q_C}{A_C} \xrightarrow{A_C = 0.1} v_C = \frac{0.1t + 0.05t^2}{0.1} \Rightarrow v_C = t + 0.5t^2$

$\xrightarrow[t=1s]{} v_C = 1 + 0.5(1)^2 = 1 + 0.5 = 1.5\ m/s$

$a_C = \frac{dv_C}{dt} \Rightarrow a_C = \frac{d}{dt}(t + 0.5t^2) \Rightarrow a_C = 1 + t \xrightarrow{t=1} a_C = 2\ m/s^2$

(5)

سرعت متوسط در لوله C در $t=1s$: $1.5\ m/s$

شتاب جریان در لوله C در $t=1s$: $2\ m/s^2$

۳.۴۲- رابطه‌ی اندازه حرکت در جهت x جریان برای حجم کنترل نشان داده شده

$$\sum_{CS} \rho Q v_x = \sum_{CV} F_x = P_1 A_1 - P_2 A_2 + R_x$$

$$\Rightarrow (-\dot{m}_1)(+v_1) + (\dot{m}_2)(+v_2) = \frac{1}{2}(\gamma h_1)(h_1 B) - \frac{1}{2}(\gamma h_2)(h_2 B) - R_x$$

$$\Rightarrow R_x = \dot{m}_2 v_2 - \dot{m}_1 v_1 - \frac{1}{2} B \gamma h_1^2 + \frac{1}{2} B \gamma h_2^2$$

علامت دبی جرمی برای جریان ورودی منفی و جریان خروجی مثبت است.

$$v_1 A_1 = v_2 A_2 \Rightarrow 1\,m/s \times (3 \times 2) = v_2 \times [1 \times 3] \Rightarrow v_2 = 2\,m/s$$

$$\Rightarrow R_x = \frac{1}{2} \times B \gamma (h_1^2 - h_2^2) - \dot{m}(v_2 - v_1) = \frac{1}{2} B \gamma (h_1^2 - h_2^2) - \rho v_1 A_1 (v_2 - v_1)$$

$$\Rightarrow R_x = \frac{1}{2} \times 2 \times 10000 \times [2^2 - 1^2] - (1000) \times 1 \times [2 \times 3] \times [2 - 1] = 24000\,N$$

گزینه ۴

۷۲_۳

$$T_{shaft} = \dot{m}_{in} r_{in} V_{\theta in} + \dot{m}_{out} r_{out} V_{\theta out}$$

$$\dot{m}_{in} = \dot{m}_{out} = \rho V_{in} \frac{\pi}{4} D^2 = \pi \rho V_{in} D^2$$

(5)

$$V_{\theta in} = V_{in} = V_{out} \qquad V_{\theta out} = V_{in} \cos 45^\circ$$

$$\rightarrow T_{shaft} = \pi \times 1.2 \times 0.02^3 \times 1000 \times 50^2 (1 + \cos 45^\circ) = 1302 \ N.m$$

$$Q = \int (v.n)\, dA = \int_1^{r_1 a} \frac{r_1 a - r}{r_1 a} v_{max} (h\, dr) = 1,\text{۲}\text{۵} h v_{max}$$

$$(\text{۰,۰۲}) = 1,\text{۲}\text{۵}(\text{۰,۰۰۹})\, v_{max} \Rightarrow v_{max} = \text{۴ m/s}$$

$$T_{MM} = \int (r \times v)\, \rho (v.n)\, dA = -\int_{cs} r v^2 \rho\, dA = -\int_1^{r_1 a} r \left(\frac{r_1 a - r}{r_1 a} \times v_{max} \right)^2 \rho (h\, dr)$$

$$\Rightarrow -\rho h \left[\text{۱۵,۹۸}\, r^2 + \text{۰,۹۶}\, r^4 - \text{۵,۹}\, r^5 \right]_1^{-r_1 a} \Rightarrow M_m = -1000 \times \text{۰,۰۰۹} \times \text{۱۹۷۲,۸}$$

$$\Rightarrow M_m = -\text{۱۲۱ N.m}$$

$$\beta = \frac{\int_A \vec{V}\rho(\vec{V}.\hat{n})\,dA}{\dot{m}\vec{V}}$$

$$\int_A \vec{V}\rho(\vec{V}.\hat{n})\,dA = 2\int_A (y-r)\rho(y-r)w\,dy = 2\rho w\int_A (y-r)^2 dy$$

$$= 2\rho w\int_A (y^2+r^2-2yr)\,dy = 2\rho w\left(\frac{r^3}{3}+r^2-r^3\right) = 2\rho w\frac{r^3}{3}$$

$$\left.\begin{aligned}\dot{m} &= \rho w r^2\\ \vec{V} &= \frac{\rho w r^2}{2\rho w r}\end{aligned}\right\} \quad \dot{m}\vec{V} = \rho w\frac{r^2}{2}$$

$$\Rightarrow \beta = \frac{2\rho w\frac{r^3}{3}}{\rho w\frac{r^2}{2}} = \frac{4}{3}$$

★ سوال (۹۱)

$$\sum_{CS} \rho Q V_2 = \sum_{CV} F_2 \longrightarrow (-\rho Q)(-V) = -W_w - W_R + R_2$$

وزن مخزن

$$R_2 = \rho Q V + W_w + W_R = 1000 \times 0.04 \times 7.7 + 9810(1)(0.2) + 800$$

$$R_2 = 4081\ N$$

(5)

* سوال (۵۳)

$$\sum_{CS} \rho V_n Q = \sum_{CV} F_n \longrightarrow (-\rho Q)(+V_0) + (\rho \frac{Q}{2})(-V_0)$$

$$+ (\rho \frac{Q}{2})(-V_0) = -R_n \longrightarrow R_n = 2\rho Q V_0$$

گزینه سوم