$R = f(v_x, v_y, g)$: 4-5

$R : L \quad v_x : v_y : LT^{-1} \quad g : LT^{-2}$

$L \propto f\left((L_x T^{-1})^a (L_y T^{-1})^b (L_y T^{-2})^c\right)$

(6)

$L_x : 1 = a$

$L_y : b + c = 0 \Rightarrow b = a = 1, \; c = -1 \qquad R = f((v_x)(v_y)(g^{-1}))$

$[T]: -a - b + (-2c) = 0$

$\Rightarrow R = f\left(\frac{v_x v_y}{g}\right)$

$R = f\left(\frac{(v\cos\theta)(v\sin\theta)}{g}\right) = f\left(\frac{v^2 \sin 2\theta}{g}\right)$

پ س ۴-۱

$P = f(D, V, C, \omega, \rho, \mu)$

$P \doteq FLT^{-1}$, $D \doteq L$, $V \doteq LT^{-1}$, $C \doteq LT^{-1}$, $\omega \doteq T^{-1}$, $\rho \doteq FL^{-4}T^{2}$

$\mu \doteq FL^{-2}T$

$FLT^{-1} = f\left((L)^{a}(LT^{-1})^{b}(LT^{-1})^{c}(T^{-1})^{d}(FL^{-4}T^{2})^{e}(FL^{-2}T)^{f}\right)$

$F: 1 = e + f$

$L: 1 = a + b + c - 4e - 2f$

$T: -1 = -b - c - d + 2e + f$

$\Rightarrow \begin{cases} e = 1 - f \checkmark \\ a = 2 + d - f \checkmark \\ b = 3 - c - d - f \checkmark \end{cases}$

$P = f\left((D)^{2+d-f}(V)^{3-c-d-f}(C)^{c}(\omega)^{d}(\rho)^{1-f}(\mu)^{f}\right)$

$P = D^{2}V^{3}\rho\, f\left[\left(\frac{DV\rho}{\mu}\right)^{-f}\left(\frac{D\omega}{V}\right)^{d}\left(\frac{C}{V}\right)^{c}\right]$

(5)

$\frac{P}{\rho D^{2}V^{3}} = f\left(\frac{DV\rho}{\mu}, \frac{D\omega}{V}, \frac{V}{C}\right) \checkmark$

6-10 : یکی سری از معادلات داخل جعبه نیست اینها باشد :

(الف) $n=4,\ m=3 \Rightarrow n-m=1$, تکراری : μ, V, D

$\Pi_1 \doteq (FL^a)(LT^{-1})^b(FL^{-2}T)^c$: $a=b=c=-1$ $\Rightarrow$

$\Pi_1 = F_D D^{-1} V^{-1} \mu^{-1} \Rightarrow \dfrac{F_D}{D V \mu} = C$ ✓

(ب) $n=4,\ m=2 \Rightarrow n-m=2$, تکراری : D, γ

$\Pi_1 \doteq h D^a \gamma^b \Rightarrow a=-1,\ b=0 \Rightarrow \Pi_1 = h D^{-1} \gamma^0 \Rightarrow \dfrac{h}{D} = C$

$\Pi_2 \doteq \sigma D^c \gamma^d \Rightarrow c=-2,\ d=-1 \Rightarrow \Pi_2 = \sigma D^{-2} \gamma^{-1} \Rightarrow \dfrac{\sigma}{\gamma D^2} = C$

$\Pi_1 = f(\Pi_2) \Rightarrow \dfrac{h}{D} = f\left(\dfrac{\sigma}{\gamma D^2}\right)$ ✓

(6)

(ج) $n=6,\ m=2 \Rightarrow n-m=3$, تکراری : t, D, ρ

$\Pi_1 \doteq \Delta h\, D^{a_1} t^{b_1} \rho^{c_1}$ ($a_1=-1,\ b_1=0,\ c_1=0$)

$\Pi_2 \doteq \gamma D^{a_2} t^{b_2} \rho^{c_2}$ ($a_2=-1,\ b_2=2,\ c_2=-1$) $\Rightarrow$ دستگاه حل باید :

$\Pi_3 \doteq h D^{a_3} t^{b_3} \rho^{c_3}$ ($a_3=-1,\ b_3=0,\ c_3=0$)

$\Pi_1 = \dfrac{\Delta h}{D}$

$\Pi_2 = \dfrac{\gamma t^2}{\rho D}$ $\Rightarrow$

$\Pi_3 = \dfrac{h}{D}$

$\Pi_1 = f(\Pi_2, \Pi_3) \Rightarrow \dfrac{\Delta h}{D} = f\left(\dfrac{\gamma t^2}{\rho D}, \dfrac{h}{D}\right)$ ✓

$\left\{ F, L, T \right.$ برای هر π

a, b, c و را پیدا می‌کنیم

(6-15

$n - j = 2 \quad f_K = T^{-1},\ V = LT^{-1},\ \rho = FL^{-4}T^{2},\ \mu = FL^{-2}T,\ D \doteq L$

$\Pi_1 = f_K D^{a_1} V^{b_1} \rho^{c_1}$

$\Pi_2 = \mu D^{a_2} V^{b_2} \rho^{c_2}$

$$\begin{cases} \pi_1 : \left\{ T^{-1},\ L^{a_1},\ (LT^{-1})^{b_1},\ (FL^{-4}T^{2})^{c_1} \right\} \\ \pi_2 : \left\{ FL^{-2}T,\ L^{a_2},\ (LT^{-1})^{b_2},\ (FL^{-4}T^{2})^{c_3} \right\} \end{cases}$$

$\pi_1 = \dfrac{f_K D}{V}$

$\pi_2 = \dfrac{\mu}{DV\rho}$

$\Pi_1 = f\,\pi_2 \qquad \dfrac{f_K D}{V} = f\left(\dfrac{\mu}{DV\rho}\right)$

(6-24

$f : \{K_s,\ D,\ \rho,\ V,\ \mu\}$ $\quad n - j = 2$

$L,\ FL^{-4}T^{2},\ LT^{-1},\ FL^{-2}T$

$\pi_1 = K_s \rho^{a_1} V^{b_1} D^{c_1}$

$\pi_2 = \mu \rho^{a_2} V^{b_2} D^{c_2}$

گزینه ۱) تمام متغیرهای داخل پرانتز به غیر از گزینه 2 $\left(\dfrac{VD}{\rho\mu}\right)$ بی بعد است.

س نیز متغیر ρV^2 بی بعد نمی‌شود بنابراین فقط گزینه ۱ این خاصیت را دارد.

$$\underbrace{V}_{LT^{-1}} = f\left(\underbrace{\rho}_{FL^{-4}T^{2}}, \underbrace{D}_{L}, \underbrace{W}_{F}, \underbrace{b}_{L}, \underbrace{d}_{L}\right)$$

$$F: \underbrace{V}_{LT^{-1}} = f\left(\underbrace{\rho/W}_{L^{-4}T^{2}}, \underbrace{D}_{L}, \underbrace{W/W}_{1}, \underbrace{b}_{L}, \underbrace{d}_{L}\right) \Rightarrow T: \underbrace{V\sqrt{\rho/W}}_{L^{-1}} = f\left(\underbrace{D}_{L}, \underbrace{b}_{L}, \underbrace{d}_{L}\right)$$

$$\Rightarrow L: VD\sqrt{\rho/W} = f\left(\underbrace{b/D}_{L^{0}}, \underbrace{d/D}_{L^{0}}\right) \Rightarrow VD\sqrt{\rho/W} = f(b/D, d/D)$$

(b)

(۴.-۶

$\Delta P_L = f(Q, D, \mu)$; $\Delta P_L \doteq FL^{-2}$, $Q \doteq L^3T^{-1}$, $D \doteq L$, $\mu \doteq FL^{-2}T$

$$\begin{cases} D \doteq L \\ Q \doteq L^3T^{-1} \doteq D^3T^{-1} \\ \mu \doteq FL^{-2}T \doteq F(D)^{-2}(D^3Q^{-1}) \doteq FDQ^{-1} \end{cases} , \begin{cases} L \doteq D \\ T \doteq D^2Q^{-1} \\ F \doteq \mu D^{-1}Q \end{cases}$$ (5.

$\Delta P_f \doteq FL^{-3} \doteq (\mu D^{-1}Q)(D)^{-3} = D^{-4}Q\mu$, $\pi_1 = \dfrac{\Delta P_L D^4}{Q\mu} = C$ ✓

$\dfrac{\Delta P_L D^4}{Q\mu} = ۴۰۱۴$, $\Delta P_L = ۴۰۱۴ \dfrac{Q\mu}{D^4}$ ✓

۴۰۱۸۲ | ۴۰۱۵۱ | ۴۰۱۲۱ | ۴۰۱۸۲ | ۴۰۱۷۴ | $\frac{\Delta P D}{L}$

$$D_m = \sqrt[p_p^x]{\frac{v_p}{v_m} \times \frac{\mu_m}{\mu_p} \times \frac{\rho_p}{\rho_m}} \Rightarrow D_m = 150 \times \frac{4}{2} \times \frac{0.000018}{1.5} \times \frac{1260}{1.23} = 3.7 \text{ m/s} \quad (0.3)$$

55 6

B

$V = f(\Delta P, d, l, \mu)$

LT^{-1}		FL^{-2}	L	L	$FL^{-2}T$

$n = 6$

$j = 3$

$L \equiv D \rightarrow \underline{D \equiv L}$

$\Delta P = F L^{-2} \equiv F D^{-2} \Rightarrow F = \underline{\Delta P D^2}$

$\mu = FL^{-2}T \Rightarrow \mu \equiv \Delta P D^2 \cdot D^{-2} T$

$\underline{T = \mu \Delta P^{-1}}$ (5)

$V \equiv LT^{-1} \Rightarrow \underline{D d^{-1} \Delta P}$

$\Pi_1 = V D^{-1} \mu \Delta P^{-1} = \underline{\dfrac{V\mu}{D\Delta P}}$

$\Pi_2 = \dfrac{l}{d}$

$\Rightarrow \dfrac{V\mu}{D\Delta P} = f'\left(\dfrac{l}{d}\right)$

$\left(\dfrac{V\mu}{D\Delta P}\right)_m = \left(\dfrac{V\mu}{d\Delta P}\right)_p \Rightarrow \dfrac{V_m}{V_p} = \dfrac{\mu_p}{\mu_m} \dfrac{dm}{dp} \dfrac{\Delta P_m}{\Delta P_p} = \dfrac{1}{F} \times \dfrac{1}{\Sigma} \times \dfrac{1}{1} = \dfrac{1}{\Lambda}$

$\underline{\dfrac{V_m}{V_p} = \dfrac{1}{\Lambda}}$

$$\frac{V_m}{V_p} = \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{D_p}{D_m}\right) \quad D_m = \left(\frac{V_p}{V_m}\right)\left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right) D_p$$

(55-6

6

$$D_m = \frac{4\ \pi/s \times 10^{-3}\ kg\ m^{-1}\ s^{-1}}{2\ \pi/s \times 1.5\ kg\ m^{-1}\ s^{-1}} \times \frac{1260\ kg/m^3}{823\ kg/m^3}\ (150\ mm), \quad D_m = 3{,}7\ mm$$

سوال ۶ـ۴۲ : گزینه ۴

$$\frac{L_m}{L_p} = \frac{1}{20} \qquad \frac{FD_p}{FD_m} = \left(\frac{P_p}{P_m}\right)\left(\frac{V_p}{V_m}\right)^2\left(\frac{l_p}{l_m}\right)^2$$

$$\frac{V_m}{V_p} = \frac{l_p}{l_m} = 20 \rightarrow \quad 20 \times 480 = 192000$$

سوال ۶ـ۴۳ : گزینه ۴

$$\frac{V}{\sqrt{gL}} = \frac{V^2}{gL} \qquad \left(\frac{l_m}{l_p}\right) = \left(\frac{V_m}{V_p}\right)^2 \times \left(\frac{g_m}{g_p}\right) \rightarrow \sqrt{\frac{l_m}{l_p}}$$

سوال ۶ـ۴۴ : گزینه ۴

$$\sqrt{\frac{l_m}{l_p}} = \frac{V_m \times A}{V_p \times A}\left(L_r^{\frac{1}{2}}\right)^2 \qquad \frac{Q_m}{Q_p} = L_r^{\frac{5}{2}} \rightarrow \sqrt{(0.00549)^2} = 0.049 \simeq 0.05$$