HW Set 3, Theory of Elasticity

1) Examine the significance of the stress function $C\Theta$ where C is a constant. Apply it to a ring $a \le r \le b$ and to an infinite plate.
A ring is fixed at $r=a$ and subjected to a uniform circumferential shear at $r=b$ forming a couple M. Find an expression for the circumferential displacement v at $r=b$.

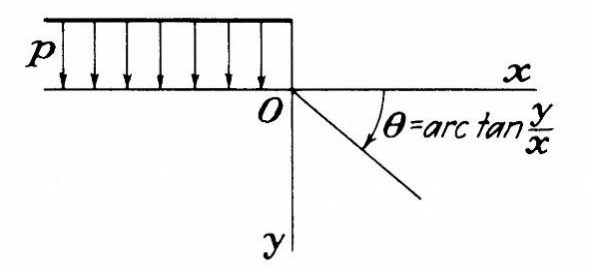
2) Drive the stress distribution

$$\sigma_{xx} = \frac{p}{\pi}\left(arctan\frac{y}{x} + \frac{xy}{x^2+y^2}\right), \qquad \sigma_{xy} = -\frac{p}{\pi}\frac{y^2}{x^2+y^2},$$

$$\sigma_{yy} = -\frac{p}{\pi}\left(arctan\frac{y}{x} + \frac{xy}{x^2+y^2}\right)$$

from the stress function $\emptyset = -\frac{p}{2\pi}\left((x^2+y^2)\ arctan\frac{y}{x} - xy\right)$

And show that it satisfies the conditions on the edge $y=0$ of the semi-infinite plate indicated in below figure, with axes as shown. The load extends indefinitely to the left. Examine the value of σ_{xy}, (a) approaching O along the boundary Ox, (b) approaching O along the y axis (the discrepancy is due to the discontinuity of loading at O).



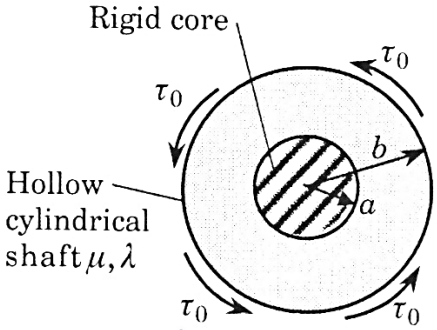
3) By supersession, using the results of Prob. 2, obtain σ_{xx}, σ_{yy}, σ_{xy} for pressure p on a segment $-a < x < a$ of the straight edge of the semi-infinite plat. Shown that the shear stress is

$$\sigma_{xy} = -\frac{p}{\pi}\frac{4axy^2}{[(x-a)^2+y^2][(x+a)^2+y^2]}$$

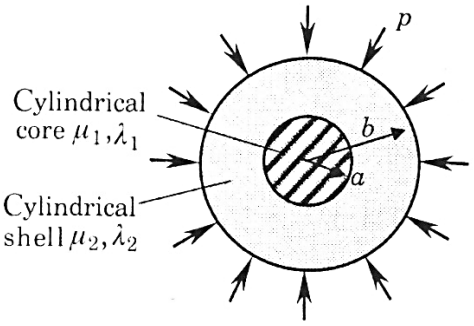
And examine the behavior of this stress as the point $x=a$, $y=0$ is approached (a) along the boundary, (b) along the line $x=a$.

4) Find by superposition the stresses in the infinite plate with a hole when the undistributed stress at infinity is uniform tension S in both x and y directions. The results should correspond with results of pressurized circular cylinders with special case $b/a \to \infty,\ Pi = o\ and\ Po = -S$

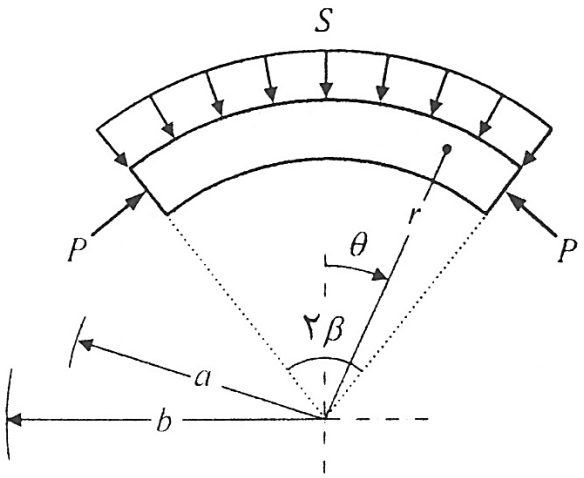
5) Determine the boundary conditions for loadings shown in below.



(a)



(b)



(c)