Stresses and Displacements in Polar Coordinates for Biharmonic Functions

$\kappa = 3 - 4\nu$ for plane strain

$= (3 - \nu)/(1 + \nu)$ for plane stress

Rigid body displacements

$u_r = C_1\cos\theta + C_2\sin\theta$

$u_\theta = -C_1\sin\theta + C_2\cos\theta + C_3 r$

ϕ	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$	$2\mu u_r$	$2\mu u_\theta$
r^2	2	0	2	$(\kappa-1)r$	0
$\log r$	$1/r^2$	0	$-1/r^2$	$-1/r$	0
θ	0	$1/r^2$	0	0	$-1/r$
$r^2\log r$	$2\log r+1$	0	$2\log r+3$	$(\kappa-1)r\log r-r$	$(\kappa+1)r\theta$
$r^2\theta$	2θ	-1	2θ	$(\kappa-1)r\theta$	$-(\kappa+1)r\log r$
$r^3\cos\theta$	$2r\cos\theta$	$2r\sin\theta$	$6r\cos\theta$	$(\kappa-2)r^2\cos\theta$	$(\kappa+2)r^2\sin\theta$
$r^3\sin\theta$	$2r\sin\theta$	$-2r\cos\theta$	$6r\sin\theta$	$(\kappa-2)r^2\sin\theta$	$-(\kappa+2)r^2\cos\theta$
$r\theta\sin\theta$	$2\cos\theta/r$	0	0	$\frac{1}{2}[(\kappa-1)\theta\sin\theta+(\kappa+1)\log r\cos\theta-\cos\theta]$	$\frac{1}{2}[(\kappa-1)\theta\cos\theta-(\kappa+1)\log r\sin\theta-\sin\theta]$
$r\theta\cos\theta$	$-2\sin\theta/r$	0	0	$\frac{1}{2}[(\kappa-1)\theta\cos\theta-(\kappa+1)\log r\sin\theta+\sin\theta]$	$\frac{1}{2}[-(\kappa-1)\theta\sin\theta-(\kappa+1)\log r\cos\theta-\cos\theta]$
$r\log r\cos\theta$	$\cos\theta/r$	$\sin\theta/r$	$\cos\theta/r$	$\frac{1}{2}[(\kappa+1)\theta\sin\theta+(\kappa-1)\log r\cos\theta-\cos\theta]$	$\frac{1}{2}[(\kappa+1)\theta\cos\theta-(\kappa-1)\log r\sin\theta-\sin\theta]$
$r\log r\sin\theta$	$\sin\theta/r$	$-\cos\theta/r$	$\sin\theta/r$	$\frac{1}{2}[-(\kappa+1)\theta\cos\theta+(\kappa-1)\log r\sin\theta-\sin\theta]$	$\frac{1}{2}[(\kappa+1)\theta\sin\theta+(\kappa-1)\log r\cos\theta+\cos\theta]$
$\cos\theta/r$	$-2\cos\theta/r^3$	$-2\sin\theta/r^3$	$2\cos\theta/r^3$	$\cos\theta/r^2$	$\sin\theta/r^2$
$\sin\theta/r$	$-2\sin\theta/r^3$	$2\cos\theta/r^3$	$2\sin\theta/r^3$	$\sin\theta/r^2$	$-\cos\theta/r^2$

ϕ	σ_{rr}	$\sigma_{r\theta}$	$\sigma_{\theta\theta}$	$2\mu u_r$	$2\mu u_\theta$
$r^2\cos2\theta$	$-2\cos2\theta$	$2\sin2\theta$	$2\cos2\theta$	$-2r\cos2\theta$	$2r\sin2\theta$
$r^2\sin2\theta$	$-2\sin2\theta$	$-2\cos2\theta$	$2\sin2\theta$	$-2r\sin2\theta$	$-2r\cos2\theta$
$r^4\cos2\theta$	0	$6r^2\sin2\theta$	$12r^2\cos2\theta$	$-(3-\kappa)r^3\cos2\theta$	$(3+\kappa)r^3\sin2\theta$
$r^4\sin2\theta$	0	$-6r^2\cos2\theta$	$12r^2\sin2\theta$	$-(3-\kappa)r^3\sin2\theta$	$-(3+\kappa)r^3\cos2\theta$
$\cos2\theta/r^2$	$-6\cos2\theta/r^4$	$-6\sin2\theta/r^4$	$6\cos2\theta/r^4$	$2\cos2\theta/r^3$	$2\sin2\theta/r^3$
$\sin2\theta/r^2$	$-6\sin2\theta/r^4$	$6\cos2\theta/r^4$	$6\sin2\theta/r^4$	$2\sin2\theta/r^3$	$-2\cos2\theta/r^3$
$\cos2\theta$	$-4\cos2\theta/r^2$	$-2\sin2\theta/r^2$	0	$(\kappa+1)\cos2\theta/r$	$-(\kappa-1)\sin2\theta/r$
$\sin2\theta$	$-4\sin2\theta/r^2$	$2\cos2\theta/r^2$	0	$(\kappa+1)\sin2\theta/r$	$(\kappa-1)\cos2\theta/r$
$r^n\cos n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$	$n(n-1)r^{n-2}\cos n\theta$	$-nr^{n-1}\cos n\theta$	$nr^{n-1}\sin n\theta$
$r^n\sin n\theta$	$-n(n-1)r^{n-2}\sin n\theta$	$-n(n-1)r^{n-2}\cos n\theta$	$n(n-1)r^{n-2}\sin n\theta$	$-nr^{n-1}\sin n\theta$	$-nr^{n-1}\cos n\theta$
$r^{n+2}\cos n\theta$	$-(n+1)(n-2)r^n\cos n\theta$	$(n+1)nr^n\sin n\theta$	$(n+2)(n+1)r^n\cos n\theta$	$-(n+1-\kappa)r^{n+1}\cos n\theta$	$(n+1+\kappa)r^{n+1}\sin n\theta$
$r^{n+2}\sin n\theta$	$-(n+1)(n-2)r^n\sin n\theta$	$-(n+1)nr^n\cos n\theta$	$(n+2)(n+1)r^n\sin n\theta$	$-(n+1-\kappa)r^{n+1}\sin n\theta$	$-(n+1+\kappa)r^{n+1}\cos n\theta$
$\cos n\theta/r^n$	$-(n+1)n\cos n\theta/r^{n+2}$	$-(n+1)n\sin n\theta/r^{n+2}$	$(n+1)in\cos n\theta/r^{n+2}$	$n\cos n\theta/r^{n+1}$	$n\sin n\theta/r^{n+1}$
$\sin n\theta/r^n$	$-(n+1)n\sin n\theta/r^{n+2}$	$(n+1)n\cos n\theta/r^{n+2}$	$(n+1)n\sin n\theta/r^{n+2}$	$n\sin n\theta/r^{n+1}$	$-n\cos n\theta/r^{n+1}$
$\cos\theta/r^{n-2}$	$-(n+2)(n-1)\cos n\theta/r^n$	$-n(n-1)\sin n\theta/r^n$	$(n-1)(n-2)\cos n\theta/r^n$	$(n-1+\kappa)\cos n\theta/r^{n-1}$	$(n-1-\kappa)\sin n\theta/r^{n-1}$
$\sin n\theta/r^{n-2}$	$-(n+2)(n-1)\sin n\theta/r^n$	$n(n-1)\cos n\theta/r^n$	$(n-1)(n-2)\sin n\theta/r^n$	$(n-1+\kappa)\sin n\theta/r^{n-1}$	$-(n-1-\kappa)\cos n\theta/r^{n-1}$