Homwork set #1

1- The stress field in a continues body is given by:

$$[\sigma_{ij}] = \begin{bmatrix} 1 & 0 & 2x \\ 0 & 1 & -2y \\ 2x & -2y & x \end{bmatrix}$$

(a) Find the stress vector at a point M: $(1, \frac{1}{\sqrt{2}}, 0)$ on a plane with unit normal $n = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$.

(b) What are the normal and tangential stresses acting on this plane?

(c) What are the principal stresses and the principal direction that corresponds to the smallest principal value at this point?

2- The displacement field of a body is:

$$u_x = c_1 - x, \qquad u_y = c_2 - y, \qquad u_z = c_3 z$$

(a) Find the components ε_{ij} of the strain matrix, and the value of the three invariants of the state of strain if the constants c_1, c_2 and c_3 are so small that their squares and products are negligible.

(b) What is the value of the volumetric strain ε_v?

3- The stress field in a continuous body is given by:

$$[\sigma_{ij}] = 10^3 \begin{bmatrix} 1 & 0 & 2y \\ 0 & 1 & 4x \\ 2y & 4x & 1 \end{bmatrix}$$

Find the stress vector $\bar{T}$ at a point M (l, 1, 1), acting on a plane x-y-z = -l

4- Given the displacement field:

$$u_1 = cx_1(x_2 + x_3), \qquad u_2 = cx_2(x_3 + x_1), \qquad u_3 = cx_3(x_1 + x_2),$$

Where *c* is *a* small constant:

(a) Find the components of the liner strain ε_{ij}.

(b) Find the components of the rotation ω_{ij}.

5- Are the following states of strain possible?

(a) $\varepsilon_{xx} = c(x^2 + y^2)$,
$\varepsilon_{yy} = cy^2$,
$\varepsilon_{xy} = 2cxy$,
$\varepsilon_{zz} = \varepsilon_{xz} = \varepsilon_{yz} = 0$

(b) $\varepsilon_{xx} = cz(x^2 + y^2)$,
$\varepsilon_{yy} = cy^2z$,
$\varepsilon_{xy} = 2cxyz$,
$\varepsilon_{zz} = \varepsilon_{xz} = \varepsilon_{yz} = 0$

Where *c* is *a* constant.

6- A stress field is given by:

$\sigma_{xx} = 2x^3 + y^2$,
$\sigma_{yy} = 3x^3 + 20$,
$\sigma_{zz} = 3y^2 + 3z^3$,
$\sigma_{xy} = z$
$\sigma_{xz} = y$
$\sigma_{yz} = x^3$

What are the components of the body force required to insure equilibrium?

7- Derive the equations of equilibrium in terms of displacements.