تکلیف سری پنجم درس: پایداری هیدرودینامیکی مدرس: دکتر احمد صداقت

تاریخ تحویل: ۱۶ اردیبهشت ۱۳۹۳

5. The amplitude A of an instability wave satisfies the evolution equation:

$$\frac{\partial A}{\partial \tau} - \lambda \frac{\partial^2 A}{\partial \zeta^2} = \sigma_c A - \beta A^2 A^*$$

where $*$ denotes complex conjugate and λ, σ_c and β are all real positive constants. Show that the equation supports periodic solutions of the form

$$A = A_0 e^{i\mu_0 \zeta} \tag{1}$$

where A_0 can be taken to be real. Determine the linear evolution equation satisfied by $B(\zeta, \tau)$ a small perturbation to the above solution. Show that the equation has solutions of the form

$$B = a(\tau) e^{i\mu_1 \zeta} + b(\tau) e^{i\mu_2 \zeta}$$

with $\mu_1 + \mu_2 = 2\mu_0$ if a and b^* satisfy

$$a_\tau = (\sigma_1 - 2\sigma_0)a - \sigma_0 b^*$$
$$b^*_\tau = (\sigma_2 - 2\sigma_0)b^* - \sigma_0 a, \quad \sigma_j = \sigma_c - \lambda \mu_j^2, \quad j = 0, 1, 2.$$

Deduce that the solution (1) is stable for all μ_1 if

$$\sigma_1 - 2\sigma_0 + \sigma_2 - 2\sigma_0 < 0 \quad \text{and} \quad (\sigma_1 + \sigma_2) - 2\sigma_0 < \frac{\sigma_1 \sigma_2 - \sigma_0^2}{2\sigma_0} \tag{2}$$

Show that (2) is valid for all μ_1 if

$$\mu_0^2 < \frac{\sigma_c}{3\lambda}.$$