Error Propagation Questions

1) Bill, a Physics student at Durham University, decided to drop a ball off the top of Durham cathedral. The ball's mass was measured a number of times and was determined to be $m = (3.00 \pm 0.01) \times 10^{-3}$ kg.

a) Compute the force acting on the ball, assuming there are no resistive forces and that the acceleration due to gravity, g, is exactly 9.81 m s^{-2}. Show the error in your answer.
b) Ben, another Physics student, suggested that in fact g is not exactly 9.81 m s^{-2}. Ben found a reference where g was determined to be (9.81 ± 0.02) m s^{-2}. Compute the new error in the force calculation, including the error in g. Compare your answer to the error calculated in part a). Are they the same or different, to 1 significant figure? Explain your answer.
c) Bill decided to take into account a resistive force. He assumed that the force due to air resistance, F_{res}, was constant and had a value of $(2.0 \pm 0.2) \times 10^{-3}$ N. Calculate the error in the *net* force acting on the ball.

2) Consider two point charges with charge Q_1 and Q_2, separated by a distance r. The magnitude of the force of interaction between them is given by:

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

a) Compute F, given that:

$$Q_1 = (1.6 \pm 0.3)\ \mu\text{C}$$

$$Q_2 = (2.5 \pm 0.1)\ \mu\text{C}$$

$$r = (1.5 \pm 0.2)\ \text{m}$$

b) Compute r, given that:

$$Q_1 = (1.6 \pm 0.3)\ \mu\text{C}$$

$$Q_2 = (2.5 \pm 0.1)\ \mu\text{C}$$

$$F = (2.3 \pm 0.5)\ \text{N}$$

(Take ϵ_0 to be exactly 8.85×10^{-12} m^{-3} kg^{-1} s^4 A^2)

3) A group of Geographers created a model for the number of Uranium-238 atoms in a given region on the Earth's surface. Their model predicts that there should be $(1.6 \pm 0.2) \times 10^{28}$ atoms at one point in Earth's history in a given region of the Earth's surface. The number of radioactive nuclei remaining at a time *t* later is given by

$$N(t) = N_0 \exp(-\lambda t)$$

Where N_0 is the number of atoms at a time t = 0, given above, and λ is the decay rate. The half life, $t_{1/2}$, for uranium-238 is measured to be $(1.41 \pm 0.01) \times 10^{17}$ s.

a) Compute the decay rate λ.
b) Compute *N(t)* at a time *t* = 2.0 $\times 10^{18}$ s (you may assume there is no error in the time *t*).
c) A team of experimental Physicists set out to test the validity of the Geographers' model. They measured the number of uranium-238 atoms in identical regions and found that their best estimate of $N(t = 2.0 \times 10^{18}\text{ s})$ was $(8.9 \pm 0.3) \times 10^{23}$. Comment on the validity of the Geographers' model.

The following questions were taken from *"Measurements and their Uncertainties",* Ifan G. Hughes and Thomas P.A. Hase", page 52 questions 4.4 and 4.5 respectively.

4) The intensity reflect coefficient, *R*, for the component of the field parallel to the plane of incidence is

$$R = \frac{tan^2(\theta_i - \theta_t)}{tan^2(\theta_i + \theta_t)}$$

where θ_i and θ_r are the angles of incidence and transmission, respectively. Calculate *R* and its associated error if $\theta_i = (45.0 \pm 0.1)°$ and $\theta_t = (34.5 \pm 0.2)°$.

5) The angular dependence θ_r for a light ray in a medium of refractive index *n* which is incident from vacuum at an angle θ_i is obtained from Snell's law: $n \sin\theta_r = \sin\theta_i$. Calculate θ_r and its associated error if $\theta_i = (25.0 \pm 0.1)°$ and $n = 1.54 \pm 0.01$.