

$$\begin{aligned}
 g''(t) + y' + \Delta y &= 0 \implies r^2 + r + \Delta = 0 \implies r = -1 \pm \sqrt{1 - \Delta} \\
 g(t) &= e^{-t} \cos t + e^{-t} \sin t + \frac{1}{2} t e^{-t} + \frac{1}{2} t e^{-t} \cos t \\
 g'(t) &= \frac{1}{2} t e^{-t} \implies y_{pr} = (A_0 + A_1 t) e^{-t} \\
 g''(t) &= \frac{1}{2} t e^{-t} \cos t \implies y_{pr} = t e^{-t} [(B_0 + B_1 t) \cos t + (C_0 + C_1 t) \sin t] \\
 \implies y_p &= y_{p1} + y_{p2} + y_{p3} = \frac{\Delta}{2} + (A_0 + A_1 t) e^{-t} + t e^{-t} [(B_0 + B_1 t) \cos t + (C_0 + C_1 t) \sin t]
 \end{aligned}$$

جواب سوال ۳ قسمت ب:

$$\begin{aligned}
 x = \ln t \implies y''(x) + \Delta y(x) &= 0, x \in \mathbb{R} \\
 y(x) = e^{rx} &\implies r^2 + \Delta = 0 \implies r = \pm \sqrt{-\Delta} \\
 \implies e^{rx} &= e^{(\pm i)x} \implies e^{\pm i x} \cos x \pm i e^{\pm i x} \sin x \\
 \implies y_1 &= e^{ix} \cos x = e^{i \ln t} \cos(\ln t), y_2 = e^{ix} \sin x = e^{i \ln t} \sin(\ln t) \\
 \implies y &= c_1 y_1 + c_2 y_2 = t^i \cos(\ln t) + c_2 t^i \sin(\ln t)
 \end{aligned}$$

جواب سوال ۳ قسمت الف:

$$\begin{aligned}
 \implies \ln(y) &= \int \frac{1}{y} dy = e^{\int \frac{1}{y} dy} = e^{\ln y} = y \\
 M &= \frac{\sin y}{y} - \frac{x}{y}, N = \frac{\cos y}{y} - \frac{1}{y} \ln x \\
 R(y) &= \frac{M_y - N_x}{y} = \frac{\frac{\cos y}{y} - \frac{1}{y} \ln x - \frac{1}{y} \ln x - \frac{\sin y}{y}}{\frac{1}{y}} \\
 &= \frac{\cos y - \sin y - 1 - \ln x}{1}
 \end{aligned}$$

جواب سوال ۲ و ۳ بندی آن: