SAS/IML
for
Best Linear Unbiased Prediction

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Preface

SAS/IML for Best Linear Unbiased Prediction (BLUP) was written for assisting graduate students to gain experiences in performing BLUP analysis. Nowadays, most tools for BLUP analysis are stressed on solving large numbers of mixed model equations. However, for comprehensive understanding BLUP process, good examples of small numbers are required. This text was structured into several parts: SAS/IML introduction, basic matrix operation, constructing selection indices, solving animal model, solving maternal effect model, solving multi-trait BLUP, constructing genetic relationship, and solving random regression.

I am thankful to colleges and friends at the University of Georgia who inspired me to write this material. I am indebted to my advisors who directly and indirectly contributed their experience to the development of this book.

Monchai Duangjinda
2007

Contents

Part A

1
Introduction to SAS/IML

IML (Interactive Matrix Language) is a part of SAS system to handle the matrix programing. SAS/IML has several statements and useful functions related to matrix operations. SAS/IML can handle data from SAS/BASE, SAS/STAT, etc. Useful technique for SAS/IML could be found in SAS publication lists. However, this chapter will discuss only basic statement and functions frequently use in BLUP analysis.

1. Creating IML data step

```
PROC IML;          : Start with PROC IML
  … …
  … …              } IML data step
  … …
QUIT;              : End with QUIT;
```

2. Printing matrix

```
PROC IML;
```

`PRINT A;`	: Printing matrix A
`PRINT A [format 8.1];`	: Printing matrix A with 8 char width for each column and has one decimal
`PRINT A B C;`	: Printing matrix A, B and C in the same paragraph
`PRINT A, B, C;`	: Printing matrix A, B and C in different paragraph

3. Setting up matrix

1) <u>Data entry into matrix</u>

```
PROC IML;
  A = {1  2  3  4,
       5  6  7  8,
       9 10 11 12};
  PRINT A;
QUIT;
```

: Creating matrix A

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

2) <u>Creating sub-matrix from original matrix</u>

```
i) A1 = A[3,];
```

: Creating vector A1 from row(3) of matrix A

$$A1 = \begin{bmatrix} 9 & 10 & 11 & 12 \end{bmatrix}$$

```
ii) A2 = A[,2];
```

: Creating vector A2 from column(2) of matrix A

$$A2 = \begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix}$$

`iii) A3 = A[3,2];` : Selecting value from row(2) and column (3) of matrix A and storing in variable name A3

$$A3 = 10$$

`iv) A4= A[{1 3},{2 4}];` : Creating matrix A4 by selecting the combination of row(1)-row(3) and column(2)-column(4) of matrix A

$$A4 = \begin{bmatrix} 2 & 4 \\ 10 & 12 \end{bmatrix}$$

3) <u>Modifying data in the matrix</u>

`i) A[1,1] = 100;` : Changing element a11 of matix A into 100

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \rightarrow \boldsymbol{A} = \begin{bmatrix} 100 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

`ii) A[1,] = {0 0 0 0};` : Changing all elements in row(1) of matrix A into zero (Note that row(1) has four values)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \rightarrow \boldsymbol{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}$$

`iii) A[,4] = {0, 0, 0};` : Changing all elements in column(1) of matrix A into zero (Note that column(1) has 3 values)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix} \rightarrow \boldsymbol{A} = \begin{bmatrix} 0 & 2 & 3 & 4 \\ 0 & 6 & 7 & 8 \\ 0 & 10 & 11 & 12 \end{bmatrix}$$

4) <u>Creating special matrix</u>

```
i) J-function
   B = J(2,3,4);
```

: Creating 2x3 matix with all elements equal 4.

$$\boldsymbol{B} = \begin{bmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \end{bmatrix}$$

```
ii) I-function
   B = I(3);
```

: Creating 3x3 identity matrix

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
iii) Diag function
   B = DIAG({1,2,3});
```

: Creating diagonal matrix with diagonal elements equal to 1, 2, and 3, respectively.

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

4. Matrix operations

1) `C = A+B`	: Matrix addition
2) `C = A–B`	: Matrix substraction
3) `C = A*B`	: Matrix multiplication
4) `C = A#4`	: Scalar multiplication
5) ``C = A` ``	: Matrix transpose
6) `C = A@B`	: Kronecor products
7) `C = A**2`	: Square of matrix

5. Useful functions

1) `C = T(A)`	: Matrix transpose
2) `C = INV(A)`	: Matrix inversion
3) `C = GINV(A)`	: Calculating Penrose generalized inversion
4) `C = EIGVAL(A)`	: Calculating eigen value
5) `C = EIGVEC(A)`	: Calculating eigen vector
6) `C = HALF(A)`	: Creating Cholesky decomposition matrix
7) `C = NCOL(A)`	: Counting column number of matrix
8) `C = NROW(A)`	: Counting row number of matrix
9) `C = TRACE(A)`	: Calculating trace

Note: SAS/IML dose not have function to calculate rank of matrix. Therefore, to find rank of matrix, function eigval might be used by counting the number of non-zero eigen values.

6. Creating matix from SAS dataset

`DATA one;`	: Creating SAS dataset with 3 variables

`INPUT x y z;`
`CARDS;`
`1 2 3`
`4 5 6`
`7 8 9`
`;`	

`PROC IML;`	: Starting IML language
`USE one;`	: Using SAS dataset
`READ ALL VAR{x y z} INTO A;`	: Reading all observations from variable x,y,z into matrix A
`READ ALL VAR{x y} INTO B;`	: Reading all observations from variable x,y into matrix B
`READ ALL VAR _ALL_ INTO B;`	: Reading all observations from all variables into matrix C
`READ ALL VAR{x y} INTO C WHERE (Z=3);`	: Reading all observations from variable x,z into matrix B and select only rows that have z equal 3
`PRINT A, B, C;`	: Printing matrix A, B, C
`QUIT;`	: End of IML Language

Part B

Basic matrix operation

▪ *Basic Operation*

SAS CODE

```
PROC IML;
/* Set up matrix */
   X = {1 2,
        3 4};

   Y = {5 6,
        7 8};

   Z = {1 2 3,
        2 4 5};

/* Matrix addtion */
   ANS1 = X+Y;

/* Matrix substraction */
   ANS2 = X-Y;

/* Matrix multiplication */
   ANS3 = X*Y;

/*  Matrix  transpose  using  Alt+96  for
transpose sign or transpose function */
   ANS4 = X`;
   ANS5 = T(X);

/* Print matrix */
   PRINT X, Y, Z;

/* Print matrix with labels */
   PRINT 'X+Y  =' ANS1;
   PRINT 'X-Y  =' ANS2;
   PRINT 'X*Y  =' ANS3;
   PRINT 'X`   =' ANS4;
   PRINT 'T(X) =' ANS5;
QUIT;
```

OUTPUT

```
X
1   2
3   4

Y
5   6
7   8

Z
1   2   3
2   4   5

                ANS1
X+Y  =             6    8
                  10   12

                ANS2
X-Y  =            -4   -4
                  -4   -4

                ANS3
X*Y  =            19   22
                  43   50

                ANS4
X`   =             1    3
                   2    4

                ANS5
T(X) =             1    3
                   2    4
```

▪ *Matrix and Vector Products*

SAS CODE

```
PROC IML;
   M = {1 0 2,
        0 1 4,
        2 4 1};

   G = {10 20,
         5 20};

/* Set up column vector */
   a = {1, 2, 5};
   b = {2, 3, 2};

/* Set up row vector */
   c = {4 5 6};
```

OUTPUT

```
M
1   0   2
0   1   4
2   4   1

G
10   20
5    20

A           B           C
1           2           4    5    6
2           3
5           2
```

<pre>/* Inner product */ ANS1 = a`*b;  /* Outer product */    ANS2 = a*b`; /* Cross product */ ANS3 = a#b; /* Cross product */ ANS3 = a#b; /* Kronekor product */ ANS4 = M@G; /* Scalar multiplication */ ANS5 = 3*M; /* Print matrix and vector*/ PRINT M, G, a, b, c; /* Print answer with labels */ PRINT 'Inner product =' ANS1; PRINT 'Outer product =' ANS2; PRINT 'Cross product =' ANS3; PRINT 'Kroneckor product =' ANS4; PRINT 'Scalar multiplication =' ANS5; QUIT;</pre>	<pre> ANS1 Inner product = 18 ANS2 Outer product = 2 3 2 4 6 4 10 15 10 ANS3 Cross product = 2 6 10 Kroneckor product = ANS4 10 20 0 0 20 40 5 20 0 0 10 40 0 0 10 20 40 80 0 0 5 20 20 80 20 40 40 80 10 20 10 40 20 80 5 20 Scalar multiplication = ANS5 3 0 6 0 3 12 6 12 3</pre>

- *Matrix Inversion*

SAS CODE	OUTPUT

<pre>PROC IML;
   A = {1 2,
        3 4};
 	
   B = {1 1 2,
        2 3 4,
        3 0 0};
 	
   D = {4 0 0,
        0 2 0,
        0 0 1};
 	
/* Matrix determinant */
   DetA = DET(A);
   DetB = DET(B);
   DetD = DET(D);
 	
/* Matrix Inversion */
   INVA = INV(A);
   INVB = INV(B);
   INVD = INV(D);
 	
/* Print matrix */
   PRINT A, B, D;
 	
/* Print solutions with format */
   PRINT DetA DetB DetD;
   PRINT INVA [FORMAT=8.1];
   PRINT INVB [FORMAT=8.2];
   PRINT INVD [FORMAT=8.1];
QUIT;</pre>
	<pre>A B
	1 2 1 1 2
	3 4 2 3 4
	                   3 0 0
	
	D
	4 0 0
	0 2 0
	0 0 1
	
	DETA DETB DETD
	  -2 -6 8
	
	INVA
	-2.0 1.0
	 1.5 -0.5
	
	INVB
	 0.00 0.00 0.33
	-2.00 1.00 0.00
	 1.50 -0.50 -0.17
	
	INVD
	 0.3 0.0 0.0
	 0.0 0.5 0.0
	 0.0 0.0 1.0</pre>

▪ *Matrix Partitions*

SAS CODE

```
PROC IML;
   X = {1  0  1  0,
        0  1  3 -1,
        0  1 -1  1};

   Y = {2 1 1 1,
        1 0 0 1,
        0 0 2 3};

   A = {2 1 0,
        3 4 1};

   B = {1  0,
        2  4,
        3 -1};

   ANS1 = X+Y;
   ANS2 = A*B;

/* Partitioning matrix X */
/* Extract row(1 to 2) and col(1 to 3) */
   X11 = X[1:2,1:3];

/* Extract row(1 to 2) and col(4) */
   X12 = X[1:2,4:4];

/* Extract row(3) and col(1 to 3) */
   X21 = X[3:3,1:3];

/* Extract row(3) and col(4) */
   X22 = X[3:3,4:4];

/* Partitioning matrix Y */
   Y11 = Y[1:2,1:3];
   Y12 = Y[1:2,4:4];
   Y21 = Y[3:3,1:3];
   Y22 = Y[3:3,4:4];

/* Addition of partitioned matrix */
   Z11 = X11+Y11;
   Z12 = X12+Y12;
   Z21 = X21+Y21;
   Z22 = X22+Y22;

/* Combine partitioned matrix   */
/* Use || to combine column and */
/* Use // to combine row        */
   Z   = (Z11||Z12)//
         (Z21||Z22);

   PRINT X11 X12, X21 X22,
         Y11 Y12, Y21 Y22,
         Z11 Z12, Z21 Z22;
   PRINT ANS1, Z;

/* Partition matrix A */
   A11 = A[1:1,1:2];
   A12 = A[1:1,3:3];
   A21 = A[2:2,1:2];
   A22 = A[2:2,3:3];
```

OUTPUT

```
X11                 X12
  1   0   1           0
  0   1   3          -1

X21                 X22
  0   1   -1          1

Y11                 Y12
  2   1    1          1
  1   0    0          1

Y21                 Y22
  0   0    2          3

Z11                 Z12
  3   1    2          1
  1   1    3          0

Z21                 Z22
  0   1   1           4

ANS1
  3   1   2   1
  1   1   3   0
  0   1   1   4

Z
  3   1   2   1
  1   1   3   0
  0   1   1   4

A11                 A12
  2   1               0

A21                 A22
  3   4               1

B11                 B12
  1                   0
  2                   4

B21                 B22
  3                  -1

Q11                 Q12
  4                  14

Q21                 Q22
 14                  15

ANS2
   4    4
  14   15

Q
   4    4
  14   15
```

```
/* Partition matrix B */
   B11 = B[1:2,1:1];
   B12 = B[1:2,2:2];
   B21 = B[3:3,1:1];
   B22 = B[3:3,2:2];

/* Multiplication of partitioned matrix */
   Q11 = A11*B11+A12*B21;
   Q12 = A11*B12+A12*B22;
   Q21 = A21*B11+A22*B21;
   Q22 = A21*B12+A22*B22;

/* Combine partitioned matrix   */
   Q   = (Q11||Q12)//
         (Q21||Q22);

   PRINT A11 A12, A21 A22,
         B11 B12, B21 B22,
         Q11 Q12, Q21 Q22;
   PRINT ANS2, Q;
QUIT;
```

- ***Eigenvalue and G-inverse***

SAS CODE

```
PROC IML;
   A = {2 2 6,
        2 3 8,
        6 8 10};

/* Calculate trace */
   TR = Trace(A);

/* Calculate eigenvalue and vector */
   L  = EIGVAL(A);
   X  = EIGVEC(A);

/* Print */
   PRINT tr, L[FORMAT=4.1],X[FORMAT=5.2];

/* Calculate G-inverse  */
/* Set last row and col to zero */
   G  = A;
   G[3,] = 0;
   G[,3] = 0;
   Gi = GINV(G);

/* Print matrix */
   PRINT A, G;

QUIT;
```

OUTPUT

```
  TR
  15

   L
17.6
 0.4
-3.1

   X
0.37  0.81  0.46
0.48 -0.59  0.65
0.79 -0.01 -0.61

  A
  2    2     6
  2    3     8
  6    8    10


  G
1.5    -1     0
 -1     1     0
  0     0     0
```

▪ *OLS Estimator*

SAS CODE

```
/* Example 3.1, 3.2 */
PROC IML;
   X  = {1 1 0 0,
         1 1 0 0,
         1 1 0 0,
         1 0 1 0,
         1 0 1 0,
         1 0 0 1,
         1 0 0 1,
         1 0 0 1};
   y  = {10,11,12,10,11,10,15,20};

   XPX = X`*X;
   XPy = X`*y;

/* Calculate G-inverse */
/* Set first row and col to zero */
   G   = XPX;
   G[1,] = 0;
   G[,1] =0;
   Gi = GINV(G);

/* Compute solutions */
   b = Gi*XPy;

   PRINT XPX, XPy;
   PRINT Gi[FORMAT=6.2], b[FORMAT=6.2];
QUIT;
```

OUTPUT

```
XPX
  8     3     2     3
  3     3     0     0
  2     0     2     0
  3     0     0     3

XPY
 99
 33
 21
 45

  GI
0.00   0.00   0.00   0.00
0.00   0.33   0.00   0.00
0.00   0.00   0.50   0.00
0.00   0.00   0.00   0.33

    B
 0.00
11.00
10.50
15.00
```

▪ *GLS, ML, and BLUE Estimator*

▪

SAS CODE

```
/* Example 3.3 */
PROC IML;
   X  = {1 1 0 0,
         1 1 0 0,
         1 1 0 0,
         1 0 1 0,
         1 0 1 0,
         1 0 0 1,
         1 0 0 1,
         1 0 0 1};

   y  = {10,11,12,10,11,10,15,20};
```

OUTPUT

```
 V
20   5   5   0   0   0   0  10
 5  20   5   0   0   0   0   0
 5   5  20   0   0   0   0   0
 0   0   0  30   5   0   0   0
 0   0   0   5  30   0   0   0
 0   0   0   0   0  20   1   1
 0   0   0   0   0   1  20   1
10   0   0   0   0   1   1  20
```

```
   V  = {20  5  5  0  0  0  0 10,
          5 20  5  0  0  0  0  0,
          5  5 20  0  0  0  0  0,
          0  0  0 30  5  0  0  0,
          0  0  0  5 30  0  0  0,
          0  0  0  0  0 20  1  1,
          0  0  0  0  0  1 20  1,
         10  0  0  0  0  1  1 20};

   Vi = INV(V);

   XVX = X`*Vi*X;
   XVy = X`*Vi*y;

/* Calculate G-inverse  */
/* Set first row and col to zero */
   G  = XVX;
   G[1,] = 0;
   G[,1] = 0;
   Gi = GINV(G);

/* Compute solutions */
   b = Gi*XVy;

   PRINT V[FORMAT=4.0], Vi[FORMAT=6.3];
   PRINT XVX[FORMAT=6.2], XVy[FORMAT=6.2];
   PRINT Gi[FORMAT=6.2], b[FORMAT=6.2];
QUIT;
```

```
  XVX
 0.28   0.09   0.06   0.13
 0.09   0.11   0.00  -0.02
 0.06   0.00   0.06   0.00
 0.13  -0.02   0.00   0.15

  XVY
 3.52
 0.73
 0.60
 2.19

   GI
 0.00   0.00   0.00   0.00
 0.00   9.54   0.00   1.32
 0.00   0.00  17.50   0.00
 0.00   1.32   0.00   6.75

    B
 0.00
 9.87
10.50
15.73
```

▪ *Best Linear Prediction (BLP)*

SAS CODE

```
/* Example 4.3 */
PROC IML;
   X  = {1 1 0 0,
         1 1 0 0,
         1 1 0 0,
         1 0 1 0,
         1 0 1 0,
         1 0 0 1,
         1 0 0 1,
         1 0 0 1};

   Z  = {1 0 0,
         0 1 0,
         0 0 1,
         1 0 0,
         0 1 0,
         1 0 0,
         0 1 0,
         0 0 1};

   y  = {10,11,12,10,11,10,15,20};
   b  = {12.375,-1.375,-1.875,2.625};

   G  = { 10  5 2.5,
           5 10   5,
         2.5  5  10};

   R  = 90*I(8);

   V  = Z*G*Z`+R;

   yadj = y-X*b;


/* Compute solutions */
   u  = G*Z`*INV(V)*yadj;

   PRINT y yadj[FORMAT=6.2];
   PRINT G[FORMAT=6.2],V[FORMAT=6.2];
   PRINT b[FORMAT=6.3],u[FORMAT=6.3];
QUIT;
```

OUTPUT

```
  Y      YADJ
 10        -1
 11         0
 12         1
 10      -0.5
 11       0.5
 10        -5
 15         0
 20         5

      G
 10.00   5.00   2.50
  5.00  10.00   5.00
  2.50   5.00  10.00

      B
12.375
-1.375
-1.875
 2.625

      U
-0.419
 0.036
 0.444
```

- *Best Linear Unbiased Prediction (BLUP)*

SAS CODE

```
/* Example 4.4 */
PROC IML;
   X  = {1 1 0 0,
         1 1 0 0,
         1 1 0 0,
         1 0 1 0,
         1 0 1 0,
         1 0 0 1,
         1 0 0 1,
         1 0 0 1};

   Z  = {1 0 0,
         0 1 0,
         0 0 1,
         1 0 0,
         0 1 0,
         1 0 0,
         0 1 0,
         0 0 1};

   y  = {10,11,12,10,11,10,15,20};

   G  = { 10  5 2.5,
           5 10   5,
         2.5  5  10};

   R  = 90*I(8);

   V  = Z*G*Z`+R;

/* Compute solutions */
   XPX = X`*X;
   XPy = X`*y;
   XVX = X`*INV(V)*X;
   XVy = X`*INV(V)*y;

   PRINT XPX,XPy;
   PRINT XVX[FORMAT=6.3],XVy[FORMAT=6.3];
   PRINT G[FORMAT=6.3],V[FORMAT=6.1];

   XVX[1,] = 0;
   XVX[,1] = 0;
   XVXi    = GINV(XVX);

   b  = XVXi*XVy;
   u  = G*Z`*INV(V)*(y-X*b);

   PRINT b[FORMAT=8.4],u[FORMAT=8.4];
QUIT;
```

OUTPUT

```
XPX
   8     3     2     3
   3     3     0     0
   2     0     2     0
   3     0     0     3

  XPY
   99
   33
   21
   45

   XVX
 0.057  0.022  0.014  0.022
 0.022  0.029 -0.003 -0.004
 0.014 -0.003  0.020 -0.003
 0.022 -0.004 -0.003  0.029

   XVY
 0.712
 0.221
 0.136
 0.355

      G
 10.000  5.000  2.500
  5.000 10.000  5.000
  2.500  5.000 10.000

       B
  0.0000
 10.9977
 10.7140
 14.9977

       U
 -0.4418
  0.0138
  0.4348
```

Part C

5 Selection index

- *Selection index from various sources of information*

SAS CODE

```
/* Example 5.4 */
PROC IML;
   G  = {2752 1376 1376,
         1376 1376    0,
         1376    0 2752};

   P  = {6400 1376 1376,
         1376 6400    0,
         1376    0 6400};

   X  = {900,
         800,
         450};

   mu = 720;
   Va = 2752;

   g1 = G[,1];

   b1  = INV(P)*g1;

   BV1  = b1`*(X-J(NROW(X),NCOL(X),Mu));
   ACC1 = SQRT(b1`*g1/Va);

   PRINT G, P;
   PRINT X, mu Va;
   PRINT g1 b1[FORMAT=8.4];
   PRINT BV1[FORMAT=8.4] ACC1[FORMAT=8.4];

QUIT;
```

OUTPUT

```
       G
2752      1376      1376
1376      1376         0
1376         0      2752

     P
6400      1376      1376
1376      6400         0
1376         0      6400

               X
             900
             800
             450

        MU         VA
       720       2752

        G1        B1
      2752    0.3719
      1376    0.1350
      1376    0.1350

       BV1      ACC1
   41.2919    0.7120
```

- *Multi-trait selection index*

SAS CODE

```
/* Example 5.6 */
PROC IML;
   P  = {6400   -57.60,
         -57.60  51.84};

   G  = {2752    62.06,
         62.06   15.55};

   v  = {1.5,
         0.5};
```

OUTPUT

```
     G
  2752     62.06
 62.06     15.55


     P
  6400     -57.6
 -57.6     51.84
```

```
   mu = {720,
         80};

   x  = {750,
         90};

   b  = INV(P)*G*v;

   BV  = b`*(X-Mu);
   ACC = SQRT(b`*G*b/(v`*G*v));

   PRINT G, P;
   PRINT X, mu;
   PRINT b[FORMAT=8.4];
   PRINT BV[FORMAT=8.4] ACC[FORMAT=8.4];

QUIT;
```

```
           X
         750
          90

          MU
         720
          80

          B
     0.6741
     2.6947

       BV      ACC
  47.1700   0.5026
```

- ***Selection index from sub-index***

SAS CODE

```
/* Example 5.7 */
PROC IML;
   P  = {6400   -57.60,
        -57.60  51.84};

   G  = {2752    62.06,
        62.06   15.55};

   g1  = G[,1];
   g2  = G[,2];

   v1  = 1.5;
   v2  = 0.5;

   b1  = INV(P)*g1;
   b2  = INV(P)*g2;

   b   = v1*b1+v2*b2;

   PRINT G, P;
   PRINT g1, g2;
   PRINT b1[FORMAT=8.4] b2[FORMAT=8.4];
   PRINT b[FORMAT=8.4];

QUIT;
```

OUTPUT

```
       G
    2752     62.06
   62.06     15.55

       P
    6400     -57.6
   -57.6     51.84

           G1
         2752
        62.06

           G2
        62.06
        15.55

        B1        B2
   0.4452    0.0125
   1.6918    0.3139

          B
     0.6741
     2.6947
```

▪ *Restricted selection index*

SAS CODE

```
/* Example 5.8 */
PROC IML;
   P  = {6400   -57.60,
         -57.60  51.84};

   G  = {2752    62.06,
         62.06   15.55};

   g2 = G[,2];
   zero = J(NROW(G),1,0);
   Pr = P;
   Gr = G;

   Pr = (Pr||g2)//(g2`||{0});
   Gr[,2]  =0;
   Gr = (Gr||zero)//(zero`||{0});

   v  = {1.5,
         0.5};
   vr = v;
   vr[2,] = 0;
   vr = vr//{0};

   mu = {720,
          80};

   x  = {750,
          90};

   br = INV(Pr)*Gr*vr;
   b  = br[1:NROW(x),];

   BV  = b`*(X-Mu);
   ACC = SQRT(b`*G*b/(v`*G*v));

   PRINT G, P;
   PRINT Gr, Pr;
   PRINT X, mu, v;
   PRINT br[FORMAT=8.4], b[FORMAT=8.4];
   PRINT BV[FORMAT=8.4] ACC[FORMAT=8.4];

QUIT;
```

OUTPUT

```
            G
         2752     62.06
        62.06     15.55

            P
         6400     -57.6
        -57.6     51.84

     GR
     2752         0         0
    62.06         0         0
        0         0         0

     PR
     6400     -57.6     62.06
    -57.6     51.84     15.55
    62.06     15.55         0

                  X
                750
                 90

                 MU
                720
                 80

                  V
                1.5
                0.5

                 BR
             0.4888
            -1.9507
            14.3002

                  B
             0.4888
            -1.9507

            BV      ACC
       -4.8438   0.3084
```

▪ ***Economic value by regression***

SAS CODE

```
/* Example 5.12 */
DATA;
  INPUT id HCW BF MB income;
CARDS;
1 65 1.2 30 50
2 72 1.0 35 65
3 50 2.1 38 58
4 67 1.5 25 42
5 68 0.8 40 68
;
PROC REG;
  MODEL income = HCW BF MB /NOINT;
RUN;
```

OUTPUT

Parameter Estimates

Variable	DF	Estimate	Std Error
HCW	1	0.154377	0.09444492
BF	1	-4.816773	2.58432808
MB	1	1.578920	0.20364426

```
/* ---------------------------------- */
/* Sire Model                         */
/* y = Xb + Zs + e                    */
/* Written by Monchai Duangjinda      */
/* ---------------------------------- */

*================*
 Data Description
*================*
Data file:
ID      sex     sire    wwt(kg)
4       M       1       245
5       F       2       229
6       F       1       239
7       M       3       235
8       M       2       250

Pedigree file:
sire    ss      ds
1       0       0
2       1       0
3       1       2
Where Vs = 10, Ve = 90, therefore alpha = Ve/Vs.
In case h2 is known, alpha = (1-.25*h2)/.25*h2;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
PROC IML;
  X = {1 0,
       0 1,
       0 1,
       1 0,
       1 0};

  Z = {1 0 0,
       0 1 0,
       1 0 0,
       0 0 1,
       0 1 0};

  y = {245,229,239,235,250};

  A = {1.00 0.50 0.25,
       0.50 1.00 0.50,
       0.25 0.50 1.00};

  Ai = inv(A);

  Vs = 10; Ve = 90;
  alpha = Ve/Vs;

  * --------- *
    MME Setup
  * --------- *;
    XPX  = X`*X;
    XPZ  = X`*Z;
    ZPZ  = Z`*Z;
    ZPZ2 = Z`*Z+alpha#Ai;

    lhs = (X`*X || X`*Z          )//
          (Z`*X || Z`*Z+alpha#Ai );
    rhs = X`*y // Z`*y;
    sol = GINV(lhs)*rhs;
```

```
  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b2';                          /* The levels for fix eff is 2 (sex) */
    label2 = 's1':'s3';                          /* The levels for sire eff is 3 */
    label  = label1 || label2;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));                    /* Select digonal of lhs inverse */
    PEV = Di#Ve;                                 /* Prediction error variance */
    I   = J(5,1,1);                              /* Set unit vector with number of b+s */
    Acc=J(5,1,.);                                /* Initialize accuracy */
    Acc[3:5,]   = SQRT(I[3:5,]-Di[3:5,]#alpha);  /* BV Accuracy */
    CC=INV(lhs);

  * ------ *
    Print
  * ------ *;

    PRINT 'Sire Model',,
          '[X`*X  X`*Z          ][b]   [X`*y]',
          '[Z`*X  Z`*Z+alpha#Ai][s] = [Z`*y]';
    PRINT X&f1, Z&f1;                            /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,ZPZ&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2;
    PRINT lhs&f2;                                /* Print LHS */
    PRINT rhs&f2;                                /* Print RHS */
    PRINT Vs Ve ALPHA;
    PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
    PRINT CC&f4;                                 /* Print inverse of LHS */
QUIT;
```

▪ *Sire model (OUTPUT)*

```
                    Sire Model

[X`*X  X`*Z         ][b]   [X`*y]
[Z`*X  Z`*Z+alpha#Ai][s] = [Z`*y]


        X
        1  0
        0  1
        0  1
        1  0
        1  0


      Z
      1  0  0
      0  1  0
      1  0  0
      0  0  1
      0  1  0


       XPX
        3  0
        0  2


     XPZ
      1  1  1
      1  1  0


     ZPZ
      2  0  0
      0  2  0
      0  0  1


   A
 1.00   0.50   0.25
 0.50   1.00   0.50
 0.25   0.50   1.00


  AI
 1.33  -0.67   0.00
-0.67   1.67  -0.67
 0.00  -0.67   1.33


 ZPZ2
14.00  -6.00   0.00
-6.00  17.00  -6.00
 0.00  -6.00  13.00


 LHS
3.00   0.00   1.00   1.00   1.00
0.00   2.00   1.00   1.00   0.00
1.00   1.00  14.00  -6.00   0.00
1.00   1.00  -6.00  17.00  -6.00
1.00   0.00   0.00  -6.00  13.00


            RHS
         730.00
         468.00
         484.00
         479.00
         235.00
```

VS	VE	ALPHA
10	90	9

SOL		DI	PEV	ACC
b1	243.317	0.401	36.103	.
b2	233.678	0.582	52.403	.
s1	0.543	0.107	9.586	0.203
s2	0.100	0.108	9.702	0.172
s3	-0.593	0.107	9.673	0.181

CC				
0.4011	0.0693	-0.0651	-0.0735	-0.0648
0.0693	0.5823	-0.0819	-0.0826	-0.0434
-0.0651	-0.0819	0.1065	0.0574	0.0315
-0.0735	-0.0826	0.0574	0.1078	0.0554
-0.0648	-0.0434	0.0315	0.0554	0.1075

Sire model with repeated records

```
/* ---------------------------------- */
/* Sire Model with Repeated records    */
/* y = Xb + Zs + Wp + e                */
/* Written by Monchai Duangjinda       */
/* ---------------------------------- */

*================*
 Data Description
*================*
Data file:
cow     herd    lact    sire    fat (%)
11      1       1       1       5
11      1       2       1       6
11      1       3       1       4
12      1       1       1       5
12      1       2       1       8
13      1       2       2       9
13      1       3       2       4
14      2       1       1       7
14      2       2       1       6
15      2       2       2       5
15      2       3       2       4
16      2       4       2       4
Pedigree file:
sire    ss      ds
1       0       0
2       1       0
3       2       0
Where Vs = 5, Vpe = 15, Ve = 40 therefore alpha = Ve/Vs, gamma = Ve/Vpe,
In case h2 and t are known, alpha = (1-t)/.25*h2, gamma=(1-t)/(t-.25*h2);

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
PROC IML;
  X = {1 0 1 0 0 0,
       1 0 0 1 0 0,
       1 0 0 0 1 0,
       1 0 1 0 0 0,
       1 0 0 1 0 0,
       1 0 0 1 0 0,
       1 0 0 0 1 0,
       0 1 1 0 0 0,
       0 1 0 1 0 0,
       0 1 0 1 0 0,
       0 1 0 0 1 0,
       0 1 0 0 0 1};

  Z = {1 0 0,
       1 0 0,
       1 0 0,
       1 0 0,
       1 0 0,
       0 1 0,
       0 1 0,
       1 0 0,
       1 0 0,
       0 1 0,
       0 1 0,
       0 1 0};
```

```
  W = {1 0 0 0 0 0,
       1 0 0 0 0 0,
       1 0 0 0 0 0,
       0 1 0 0 0 0,
       0 1 0 0 0 0,
       0 0 1 0 0 0,
       0 0 1 0 0 0,
       0 0 0 1 0 0,
       0 0 0 1 0 0,
       0 0 0 0 1 0,
       0 0 0 0 1 0,
       0 0 0 0 0 1};

  y = {5,6,4,5,8,9,4,7,6,5,4,4};
  A = { 1  .50 .25,
       .50  1  .50,
       .25 .50  1};
  Ai = inv(A);
  Vs = 5; Vpe = 15; Ve = 40;
  alpha = Ve/Vs; gamma = Ve/Vpe;

  * --------- *
    MME Setup
  * --------- *;
    XPX  = X`*X;
    XPZ  = X`*Z;
    XPW  = X`*W;
    ZPZ  = Z`*Z;
    ZPW  = Z`*W;
    WPW  = W`*W;
    ZPZ2 = Z`*Z+alpha#Ai;
    WPW2 = W`*W+gamma#I(6);
    lhs = (X`*X || X`*Z          || X`*W           )//
          (Z`*X || Z`*Z+alpha#Ai || Z`*W           )//
          (W`*X || W`*Z          || W`*W+gamma#I(6));
    rhs = X`*y // Z`*y // W`*y;
    sol = GINV(lhs)*rhs;

  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b6';                  /* The levels for fix eff is 6 (herd+lact) */
    label2 = 's1':'s3';                  /* The levels for sire eff is 3 */
    label3 = 'pe1':'pe6';                /* The levels for PE eff from cow is 6 */
    label  = label1 || label2 || label3;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));                /* Select digonal of lhs inverse */
    PEV = Di#Ve;                             /* Prediction error variance */
    I   = J(15,1,1);                         /* Set unit vector with number of b+s+pe */
    Acc =J(15,1,.);                          /* Initialize accuracy */
    Acc[7:9,]   = SQRT(I[7:9,]-Di[7:9,]#alpha);         /* BV Accuracy */
    Acc[10:15,] = SQRT(I[10:15,]-Di[10:15,]#gamma);     /* PE Accuracy */
    CC=GINV(lhs);
    C22=CC[7:9,7:9];
    C33=CC[10:15,10:15];

  * ------ *
    Print
  * ------ *;
    PRINT 'Repeatability Sire Model',,
          '[X`*X  X`*Z           X`*W        ][b ]   [X`*y]',
          '[Z`*X  Z`*Z+alpha#Ai  Z`*W        ][s ] = [Z`*y]',
          '[W`*X  W`*Z           W`*W+gamma#I][pe]   [W`*y]';
    PRINT X&f1, Z&f1, W&f1;                  /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,XPW&f1,ZPZ&f1,ZPW&f1,WPW&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2,WPW2&f2;
    PRINT lhs&f2;                            /* Print LHS */
    PRINT rhs&f2;                            /* Print RHS */
    PRINT Vs Vpe Ve alpha gamma;
    PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
    PRINT C22&f4;                            /* Print inverse of LHS */
    PRINT C33&f4;                            /* Print inverse of LHS */
QUIT;
```

▪ *Sire model with repeated records (OUTPUT)*

```
                 The SAS System
            Repeatability Sire Model

[X`*X  X`*Z           X`*W        ][b ]   [X`*y]
[Z`*X  Z`*Z+alpha#Ai  Z`*W        ][s ] = [Z`*y]
[W`*X  W`*Z           W`*W+gamma#I][pe]   [W`*y]

                X
                1  0  1  0  0  0
                1  0  0  1  0  0
                1  0  0  0  1  0
                1  0  1  0  0  0
                1  0  0  1  0  0
                1  0  0  1  0  0
                1  0  0  0  1  0
                0  1  1  0  0  0
                0  1  0  1  0  0
                0  1  0  1  0  0
                0  1  0  0  1  0
                0  1  0  0  0  1

                     Z
                     1  0  0
                     1  0  0
                     1  0  0
                     1  0  0
                     1  0  0
                     0  1  0
                     0  1  0
                     1  0  0
                     1  0  0
                     0  1  0
                     0  1  0
                     0  1  0

                W
                1  0  0  0  0  0
                1  0  0  0  0  0
                1  0  0  0  0  0
                0  1  0  0  0  0
                0  1  0  0  0  0
                0  0  1  0  0  0
                0  0  1  0  0  0
                0  0  0  1  0  0
                0  0  0  1  0  0
                0  0  0  0  1  0
                0  0  0  0  1  0
                0  0  0  0  0  1

               XPX
                7  0  2  3  2  0
                0  5  1  2  1  1
                2  1  3  0  0  0
                3  2  0  5  0  0
                2  1  0  0  3  0
                0  1  0  0  0  1

                    XPZ
                     5  2  0
                     2  3  0
                     3  0  0
                     3  2  0
                     1  2  0
                     0  1  0

               XPW
                3  2  2  0  0  0
                0  0  0  2  2  1
                1  1  0  1  0  0
                1  1  1  1  1  0
                1  0  1  0  1  0
                0  0  0  0  0  1
```

```
      ZPZ
       7  0  0
       0  5  0
       0  0  0

 ZPW
  3  2  0  2  0  0
  0  0  2  0  2  1
  0  0  0  0  0  0

 WPW
  3  0  0  0  0  0
  0  2  0  0  0  0
  0  0  2  0  0  0
  0  0  0  2  0  0
  0  0  0  0  2  0
  0  0  0  0  0  1

      A
  1.00   0.50   0.25
  0.50   1.00   0.50
  0.25   0.50   1.00

     AI
  1.33  -0.67   0.00
 -0.67   1.67  -0.67
  0.00  -0.67   1.33

  ZPZ2
 17.67  -5.33   0.00
 -5.33  18.33  -5.33
  0.00  -5.33  10.67

WPW2
5.67   0.00   0.00   0.00   0.00   0.00
0.00   4.67   0.00   0.00   0.00   0.00
0.00   0.00   4.67   0.00   0.00   0.00
0.00   0.00   0.00   4.67   0.00   0.00
0.00   0.00   0.00   0.00   4.67   0.00
0.00   0.00   0.00   0.00   0.00   3.67

     LHS
7.00 0.00 2.00 3.00 2.00 0.00 5.00 2.00 0.00 3.00 2.00 2.00 0.00 0.00 0.00
0.00 5.00 1.00 2.00 1.00 1.00 2.00 3.00 0.00 0.00 0.00 0.00 2.00 2.00 1.00
2.00 1.00 3.00 0.00 0.00 0.00 3.00 0.00 0.00 1.00 1.00 0.00 1.00 0.00 0.00
3.00 2.00 0.00 5.00 0.00 0.00 3.00 2.00 0.00 1.00 1.00 1.00 1.00 1.00 0.00
2.00 1.00 0.00 0.00 3.00 0.00 1.00 2.00 0.00 1.00 0.00 1.00 0.00 1.00 0.00
0.00 1.00 0.00 0.00 0.00 1.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
5.00 2.00 3.00 3.00 1.00 0.00 17.7 -5.3 0.00 3.00 2.00 0.00 2.00 0.00 0.00
2.00 3.00 0.00 2.00 2.00 1.00 -5.3 18.3 -5.3 0.00 0.00 2.00 0.00 2.00 1.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 -5.3 10.7 0.00 0.00 0.00 0.00 0.00 0.00
3.00 0.00 1.00 1.00 1.00 0.00 3.00 0.00 0.00 5.67 0.00 0.00 0.00 0.00 0.00
2.00 0.00 1.00 1.00 0.00 0.00 2.00 0.00 0.00 0.00 4.67 0.00 0.00 0.00 0.00
2.00 0.00 0.00 1.00 1.00 0.00 0.00 2.00 0.00 0.00 0.00 4.67 0.00 0.00 0.00
0.00 2.00 1.00 1.00 0.00 0.00 2.00 0.00 0.00 0.00 0.00 0.00 4.67 0.00 0.00
0.00 2.00 0.00 1.00 1.00 0.00 0.00 2.00 0.00 0.00 0.00 0.00 0.00 4.67 0.00
0.00 1.00 0.00 0.00 0.00 1.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 3.67

   RHS
 41.00
 26.00
 17.00
 34.00
 12.00
  4.00
 41.00
 26.00
  0.00
 15.00
 13.00
 13.00
 13.00
  9.00
  4.00
```

VS	VPE	VE	ALPHA	GAMMA
5	15	40	8	2.6666667

SOL		DI	PEV	ACC
b1	3.716	0.317	12.691	.
b2	3.160	0.286	11.460	.
b3	2.184	0.374	14.951	.
b4	3.309	0.237	9.476	.
b5	0.558	0.361	14.459	.
b6	0.824	1.187	47.480	.
s1	-0.016	0.121	4.848	0.174
s2	0.016	0.121	4.848	0.174
s3	0.008	0.124	4.962	0.087
pe1	-0.380	0.256	10.251	0.563
pe2	0.023	0.274	10.959	0.519
pe3	0.357	0.280	11.196	0.504
pe4	0.261	0.307	12.265	0.427
pe5	-0.261	0.307	12.265	0.427
pe6	0.000	0.375	15.000	0.000

C22		
0.1212	0.0663	0.0331
0.0663	0.1212	0.0606
0.0331	0.0606	0.1241

C33					
0.2563	0.0623	0.0564	0.0030	-0.0030	0.0000
0.0623	0.2740	0.0387	0.0105	-0.0105	-0.0000
0.0564	0.0387	0.2799	-0.0135	0.0135	0.0000
0.0030	0.0105	-0.0135	0.3066	0.0684	0.0000
-0.0030	-0.0105	0.0135	0.0684	0.3066	-0.0000
-0.0000	-0.0000	0.0000	0.0000	-0.0000	0.3750

Animal model

```
/* ---------------------------------- */
/* Animal Model                       */
/* y = Xb + Za + e                    */
/* Written by Monchai Duangjinda       */
/* ---------------------------------- */

*================*
 Data Description
*================*
Data file:
ID      sex     wt(kg)
4       M       2.5
5       F       2.9
6       F       3.1
7       M       2.5
8       M       2.9

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       1       0
5       3       2
6       1       2
7       4       5
8       3       6
Where Va = 0.1 Ve = 0.3, therefore alpha = Ve/Va.
In case h2 is known, alpha = (1-h2)/h2;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
PROC IML;
  X = {1 0,
       0 1,
       0 1,
       1 0,
       1 0};

  Z = {0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 1};

  y = {4.5,2.9,3.9,3.5,5.0};

  A = {1   0   0  .5    0   .5  .25 .25,
       0   1   0   0   .5   .5  .25 .25,
       0   0   1   0   .5    0  .25 .5,
      .5   0   0   1    0   .25 .5  .125,
       0  .5  .5   0    1   .25 .5  .375,
      .5  .5   0  .25  .25   1  .25 .5,
      .25 .25 .25 .5   .5   .25  1  .25,
      .25 .25 .5  .125 .375 .5  .25  1};

  Ai = inv(A);

  Va = 0.1; Ve = 0.3;
  alpha = Ve/Va;
```

```
  * --------- *
    MME Setup
  * --------- *;

    XPX  = X`*X;
    XPZ  = X`*Z;
    ZPZ  = Z`*Z;
    ZPZ2 = Z`*Z+alpha#Ai;



    lhs = (X`*X || X`*Z          )//
          (Z`*X || Z`*Z+alpha#Ai );
    rhs = X`*y // Z`*y;
    sol = GINV(lhs)*rhs;

  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b2';                           /* The levels for fix eff is 2 (sex) */
    label2 = 'a1':'a8';                           /* The levels for animal eff is 8 */
    label  = label1 || label2;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));                     /* Select digonal of lhs inverse */
    PEV = Di#Ve;                                  /* Prediction error variance */
    I   = J(10,1,1);                              /* Set unit vector with number of b+u */
    Acc=J(10,1,.);                                /* Initialize accuracy */
    Acc[3:10,]  = SQRT(I[3:10,]-Di[3:10,]#alpha); /* BV Accuracy */
    CC=INV(lhs);

  * ------ *
    Print
  * ------ *;

    PRINT 'Animal Model',,
          '[X`*X  X`*Z        ][b]   [X`*y]',
          '[Z`*X  Z`*Z+alpha#Ai][a] = [Z`*y]';
    PRINT X&f1, Z&f1;                             /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,ZPZ&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2;
    PRINT lhs&f2;                                 /* Print LHS */
    PRINT rhs&f2;                                 /* Print RHS */
    PRINT Va Ve ALPHA;
    PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
    PRINT CC&f4;                                  /* Print inverse of LHS */
QUIT;
```

▪ *Animal model (OUTPUT)*

```
                    The SAS System
                     Animal Model

        [X`*X  X`*Z         ][b]   [X`*y]
        [Z`*X  Z`*Z+alpha#Ai][a] = [Z`*y]

                       X
                       1  0
                       0  1
                       0  1
                       1  0
                       1  0

              Z
              0  0  0  1  0  0  0  0
              0  0  0  0  1  0  0  0
              0  0  0  0  0  1  0  0
              0  0  0  0  0  0  1  0
              0  0  0  0  0  0  0  1

                      XPX
                       3  0
                       0  2

             XPZ
              0  0  0  1  0  0  1  1
              0  0  0  0  1  1  0  0

             ZPZ
              0  0  0  0  0  0  0  0
              0  0  0  0  0  0  0  0
              0  0  0  0  0  0  0  0
              0  0  0  1  0  0  0  0
              0  0  0  0  1  0  0  0
              0  0  0  0  0  1  0  0
              0  0  0  0  0  0  1  0
              0  0  0  0  0  0  0  1

    A
 1.00   0.00   0.00   0.50   0.00   0.50   0.25   0.25
 0.00   1.00   0.00   0.00   0.50   0.50   0.25   0.25
 0.00   0.00   1.00   0.00   0.50   0.00   0.25   0.50
 0.50   0.00   0.00   1.00   0.00   0.25   0.50   0.13
 0.00   0.50   0.50   0.00   1.00   0.25   0.50   0.38
 0.50   0.50   0.00   0.25   0.25   1.00   0.25   0.50
 0.25   0.25   0.25   0.50   0.50   0.25   1.00   0.25
 0.25   0.25   0.50   0.13   0.38   0.50   0.25   1.00

   AI
 1.83   0.50   0.00  -0.67   0.00  -1.00   0.00   0.00
 0.50   2.00   0.50   0.00  -1.00  -1.00   0.00   0.00
 0.00   0.50   2.00   0.00  -1.00   0.50   0.00  -1.00
-0.67   0.00   0.00   1.83   0.50   0.00  -1.00   0.00
 0.00  -1.00  -1.00   0.50   2.50   0.00  -1.00   0.00
-1.00  -1.00   0.50   0.00   0.00   2.50   0.00  -1.00
 0.00   0.00   0.00  -1.00  -1.00   0.00   2.00   0.00
 0.00   0.00  -1.00   0.00   0.00  -1.00   0.00   2.00

 ZPZ2
 5.50   1.50   0.00  -2.00   0.00  -3.00   0.00   0.00
 1.50   6.00   1.50   0.00  -3.00  -3.00   0.00   0.00
 0.00   1.50   6.00   0.00  -3.00   1.50   0.00  -3.00
-2.00   0.00   0.00   6.50   1.50   0.00  -3.00   0.00
 0.00  -3.00  -3.00   1.50   8.50   0.00  -3.00   0.00
-3.00  -3.00   1.50   0.00   0.00   8.50   0.00  -3.00
 0.00   0.00   0.00  -3.00  -3.00   0.00   7.00   0.00
 0.00   0.00  -3.00   0.00   0.00  -3.00   0.00   7.00
```

LHS

3.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	1.00
0.00	2.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00
0.00	0.00	5.50	1.50	0.00	-2.00	0.00	-3.00	0.00	0.00
0.00	0.00	1.50	6.00	1.50	0.00	-3.00	-3.00	0.00	0.00
0.00	0.00	0.00	1.50	6.00	0.00	-3.00	1.50	0.00	-3.00
1.00	0.00	-2.00	0.00	0.00	6.50	1.50	0.00	-3.00	0.00
0.00	1.00	0.00	-3.00	-3.00	1.50	8.50	0.00	-3.00	0.00
0.00	1.00	-3.00	-3.00	1.50	0.00	0.00	8.50	0.00	-3.00
1.00	0.00	0.00	0.00	0.00	-3.00	-3.00	0.00	7.00	0.00
1.00	0.00	0.00	0.00	-3.00	0.00	0.00	-3.00	0.00	7.00

RHS
13.00
6.80
0.00
0.00
0.00
4.50
2.90
3.90
3.50
5.00

VA	VE	ALPHA
0.1	0.3	3

SOL		DI	PEV	ACC
b1	4.352	0.508	0.153	.
b2	3.402	0.703	0.211	.
a1	0.074	0.319	0.096	0.207
a2	-0.013	0.330	0.099	0.107
a3	-0.031	0.312	0.094	0.254
a4	-0.008	0.298	0.089	0.328
a5	-0.138	0.298	0.089	0.327
a6	0.134	0.305	0.092	0.291
a7	-0.184	0.305	0.091	0.292
a8	0.137	0.295	0.088	0.340

C22

0.3190	0.0035	0.0161	0.1516	0.0223	0.1523	0.0902	0.0879
0.0035	0.3295	-0.0052	0.0100	0.1604	0.1645	0.0810	0.0762
0.0161	-0.0052	0.3118	0.0239	0.1426	0.0111	0.0836	0.1507
0.1516	0.0100	0.0239	0.2976	0.0233	0.0845	0.1628	0.0718
0.0223	0.1604	0.1426	0.0233	0.2977	0.1055	0.1516	0.1204
0.1523	0.1645	0.0111	0.0845	0.1055	0.3050	0.0973	0.1513
0.0902	0.0810	0.0836	0.1628	0.1516	0.0973	0.3049	0.1048
0.0879	0.0762	0.1507	0.0718	0.1204	0.1513	0.1048	0.2947

Animal model with repeated records

```
/* --------------------------------- */
/* Animal Model with Repeated records  */
/* y = Xb + Za + Wp + e               */
/* Written by Monchai Duangjinda      */
/* --------------------------------- */

*================*
 Data Description
*================*
Data file:
cow     lact    hys     milk305 (kg)
4       1       1       4201
4       2       3       4280
5       1       1       3150
5       2       4       3200
6       1       2       2160
6       2       3       2190
7       1       1       4180
8       1       3       3250
8       2       2       4285
8       3       4       3300

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       3       2
5       1       2
6       3       0
7       1       4
8       3       5
Where Va = 120000 Vpe = 100000 Ve = 600000, therefore alpha = Ve/Va. gamma = Ve/Vpe,
In case h2 and t are known, alpha = (1-t)/h2, gamma = (1-t)/(t-h2);

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];

PROC IML;
  X = {1 0 0 1 0 0 0,
       0 1 0 0 0 1 0,
       1 0 0 1 0 0 0,
       0 1 0 0 0 0 1,
       1 0 0 0 1 0 0,
       0 1 0 0 0 1 0,
       1 0 0 1 0 0 0,
       1 0 0 0 0 1 0,
       0 1 0 0 1 0 0,
       0 0 1 0 0 0 1};

  Z = {0 0 0 1 0 0 0 0,
       0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 1,
       0 0 0 0 0 0 0 1,
       0 0 0 0 0 0 0 1};

  W = Z;
  y = {4201,4280,3150,3200,2160,2190,4180,3250,4285,3300};
```

```
 A = {1    0    0   0   .5    0   .5   .25,
      0    1    0  .5   .5    0   .25  .25,
      0    0    1  .5    0   .5   .25  .5,
      0   .5   .5   1   .25  .25  .5   .375,
     .5   .5    0  .25   1    0   .375 .5,
      0    0   .5  .25   0    1   .125 .25,
     .5   .25  .25 .5   .375 .125  1   .313,
     .25  .25  .5  .375 .5   .25  .313  1};

 Ai = inv(A);
 Va = 120000; Vpe = 100000; Ve = 600000;
 alpha = Ve/Va; gamma = Ve/Vpe;

 * --------- *
   MME Setup
 * --------- *;
   XPX  = X`*X;
   XPZ  = X`*Z;
   XPW  = X`*W;
   ZPZ  = Z`*Z;
   ZPW  = Z`*W;
   WPW  = W`*W;
   ZPZ2 = Z`*Z+alpha#Ai;
   WPW2 = W`*W+gamma#I(8);

   lhs = (X`*X || X`*Z          || X`*W           )//
         (Z`*X || Z`*Z+alpha#Ai || Z`*W           )//
         (W`*X || W`*Z          || W`*W+gamma#I(8));
   rhs = X`*y // Z`*y // W`*y;
   sol = GINV(lhs)*rhs;

 * --------------------- *
   Set Label for printing
 * --------------------- *;
   label1 = 'b1':'b7';                    /* The levels for fix eff is 7 (herd+lact) */
   label2 = 'a1':'a8';                    /* The levels for animal eff is 8 */
   label3 = 'pe1':'pe8';                  /* The levels for PE eff is 8 */
   label  = label1 || label2 || label3;



 * --------------- *
   Compute accuracy
 * --------------- *;
   Di  = vecdiag(GINV(lhs));              /* Select digonal of lhs inverse */
   PEV = Di#Ve;                           /* Prediction error variance */
   I   = J(23,1,1);                       /* Set unit vector with number of b+u+pe */
   Acc=J(23,1,.);                                    /* Initialize accuracy */
   Acc[8:15,]  = SQRT(I[8:15,]-Di[8:15,]#alpha);      /* BV Accuracy */
   Acc[16:23,] = SQRT(I[16:23,]-Di[16:23,]#gamma);    /* PE Accuracy */
   CC=GINV(lhs);
   C22=CC[8:15,8:15];
   C33=CC[16:23,16:23];

 * ------ *
   Print
 * ------ *;

   PRINT 'Repeatability Animal Model',,
         '[X`*X  X`*Z           X`*W        ][b ]   [X`*y]',
         '[Z`*X  Z`*Z+alpha#Ai  Z`*W        ][a ] = [Z`*y]',
         '[W`*X  W`*Z           W`*W+gamma#I][pe]   [W`*y]';
   PRINT X&f1, Z&f1, W&f1;                /* Print matrix using f1 format */
   PRINT XPX&f1,XPZ&f1,XPW&f1,ZPZ&f1,ZPW&f1,WPW&f1;
   PRINT A&f3,Ai&f3,ZPZ2&f2,WPW2&f2;
   PRINT lhs&f2;                                  /* Print LHS */
   PRINT rhs&f2;                                  /* Print RHS */
   PRINT Va Vpe Ve alpha gamma;
   PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
   PRINT C22&f4;                                  /* Print inverse of LHS */
   PRINT C33&f4;                                  /* Print inverse of LHS */

QUIT;
```

▪ *Animal model with repeated records (OUTPUT)*

```
                     The SAS System
               Repeatability Animal Model

[X`*X  X`*Z           X`*W        ][b ]   [X`*y]
[Z`*X  Z`*Z+alpha#Ai  Z`*W        ][a ] = [Z`*y]
[W`*X  W`*Z           W`*W+gamma#I][pe]   [W`*y]

                X
                1  0  0  1  0  0  0
                0  1  0  0  0  1  0
                1  0  0  1  0  0  0
                0  1  0  0  0  0  1
                1  0  0  0  1  0  0
                0  1  0  0  0  1  0
                1  0  0  1  0  0  0
                1  0  0  0  0  1  0
                0  1  0  0  1  0  0
                0  0  1  0  0  0  1


               Z
               0  0  0  1  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1
               0  0  0  0  0  0  0  1
               0  0  0  0  0  0  0  1


               W
               0  0  0  1  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1
               0  0  0  0  0  0  0  1
               0  0  0  0  0  0  0  1


               XPX
                5  0  0  3  1  1  0
                0  4  0  0  1  2  1
                0  0  1  0  0  0  1
                3  0  0  3  0  0  0
                1  1  0  0  2  0  0
                1  2  0  0  0  3  0
                0  1  1  0  0  0  2


              XPZ
               0  0  0  1  1  1  1  1
               0  0  0  1  1  1  0  1
               0  0  0  0  0  0  0  1
               0  0  0  1  1  0  1  0
               0  0  0  0  0  1  0  1
               0  0  0  1  0  1  0  1
               0  0  0  0  1  0  0  1

              XPW
               0  0  0  1  1  1  1  1
               0  0  0  1  1  1  0  1
               0  0  0  0  0  0  0  1
               0  0  0  1  1  0  1  0
               0  0  0  0  0  1  0  1
               0  0  0  1  0  1  0  1
               0  0  0  0  1  0  0  1
```

ZPZ

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0
0	0	0	0	2	0	0	0
0	0	0	0	0	2	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	3

ZPW

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0
0	0	0	0	2	0	0	0
0	0	0	0	0	2	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	3

WPW

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	2	0	0	0	0
0	0	0	0	2	0	0	0
0	0	0	0	0	2	0	0
0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	3

A

1.000	0.000	0.000	0.000	0.500	0.000	0.500	0.250
0.000	1.000	0.000	0.500	0.500	0.000	0.250	0.250
0.000	0.000	1.000	0.500	0.000	0.500	0.250	0.500
0.000	0.500	0.500	1.000	0.250	0.250	0.500	0.375
0.500	0.500	0.000	0.250	1.000	0.000	0.375	0.500
0.000	0.000	0.500	0.250	0.000	1.000	0.125	0.250
0.500	0.250	0.250	0.500	0.375	0.125	1.000	0.313
0.250	0.250	0.500	0.375	0.500	0.250	0.313	1.000

AI

2.000	0.500	-0.001	0.500	-1.001	0.000	-1.000	0.001
0.500	2.000	0.500	-1.000	-1.000	0.000	0.000	0.000
-0.001	0.500	2.333	-1.001	0.500	-0.667	0.001	-1.000
0.500	-1.000	-1.001	2.500	-0.001	0.000	-1.000	0.001
-1.001	-1.000	0.500	-0.001	2.500	0.000	0.001	-1.000
0.000	0.000	-0.667	0.000	0.000	1.333	0.000	0.000
-1.000	0.000	0.001	-1.000	0.001	0.000	2.000	-0.002
0.001	0.000	-1.000	0.001	-1.000	0.000	-0.002	2.000

ZPZ2

10.00	2.50	-0.00	2.50	-5.00	0.00	-5.00	0.01
2.50	10.00	2.50	-5.00	-5.00	0.00	0.00	0.00
-0.00	2.50	11.67	-5.00	2.50	-3.33	0.01	-5.00
2.50	-5.00	-5.00	14.50	-0.00	0.00	-5.00	0.01
-5.00	-5.00	2.50	-0.00	14.50	0.00	0.01	-5.00
0.00	0.00	-3.33	0.00	0.00	8.67	0.00	0.00
-5.00	0.00	0.01	-5.00	0.01	0.00	11.00	-0.01
0.01	0.00	-5.00	0.01	-5.00	0.00	-0.01	13.00

WPW2

6.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	6.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	6.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	8.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	8.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	8.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	9.00

RHS
16941
13955
3300.0
11531
6445.0
9720.0
6500.0
0.00
0.00
0.00
8481.0
6350.0
4350.0
4180.0
10835
0.00
0.00
0.00
8481.0
6350.0
4350.0
4180.0
10835

VA	VPE	VE	ALPHA	GAMMA
120000	100000	600000	5	6

SOL		DI	PEV	ACC
b1	1473.331	0.486	291630.4	.
b2	2275.783	0.328	196662.3	.
b3	2095.488	1.741	1044687	.
b4	2245.156	0.760	455980.3	.
b5	1425.901	0.642	385395.2	.
b6	1193.353	0.473	284064.9	.
b7	980.193	0.830	498225.2	.
a1	14.744	0.193	116022.7	0.182
a2	74.842	0.195	116896.5	0.161
a3	36.203	0.196	117680.5	0.139
a4	145.716	0.184	110460.7	0.282
a5	29.442	0.185	110753.9	0.278
a6	-170.583	0.183	109624.3	0.294
a7	110.398	0.191	114349.7	0.217
a8	104.652	0.185	110767.8	0.277
pe1	0.000	0.167	100000.0	0.000
pe2	0.000	0.167	100000.0	0.000
pe3	0.000	0.167	100000.0	0.000
pe4	125.243	0.143	85711.66	0.378
pe5	-85.418	0.152	91484.31	0.292
pe6	-209.650	0.147	88001.13	0.346
pe7	50.159	0.152	91204.65	0.297
pe8	119.666	0.146	87694.37	0.351

C22

0.1934	0.0003	0.0037	0.0062	0.0929	0.0058	0.0971	0.0451
0.0003	0.1948	0.0003	0.0945	0.0954	0.0071	0.0498	0.0465
0.0037	0.0003	0.1961	0.0949	0.0056	0.0979	0.0494	0.1004
0.0062	0.0945	0.0949	0.1841	0.0555	0.0540	0.0962	0.0762
0.0929	0.0954	0.0056	0.0555	0.1846	0.0120	0.0766	0.0896
0.0058	0.0071	0.0979	0.0540	0.0120	0.1827	0.0303	0.0622
0.0971	0.0498	0.0494	0.0962	0.0766	0.0303	0.1906	0.0621
0.0451	0.0465	0.1004	0.0762	0.0896	0.0622	0.0621	0.1846

C33

0.1667	-0.0000	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	0.1667	-0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.0000	-0.0000	0.1667	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.1429	0.0059	0.0057	0.0069	0.0054
0.0000	0.0000	0.0000	0.0059	0.1525	-0.0000	0.0071	0.0012
0.0000	0.0000	0.0000	0.0057	-0.0000	0.1467	0.0005	0.0138
0.0000	0.0000	0.0000	0.0069	0.0071	0.0005	0.1520	0.0001
0.0000	0.0000	0.0000	0.0054	0.0012	0.0138	0.0001	0.1462

Animal model with genetic groups

```
/* --------------------------------- */
/* Animal Model with Genetic Groups    */
/* y = Xb + Za + ZQg + e               */
/* Written by Monchai Duangjinda       */
/* --------------------------------- */
*================*
 Data Description
*================*
Data file:
ID      sex     adg(g)
4       M       845
5       F       629
6       F       639
7       M       735
8       M       850

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       1       0
5       3       2
6       1       2
7       4       5
8       3       6

Assign unknown sires and dam with unrelated phantom
parents.
an      s       d
1       p1      p2
2       p3      p4
3       p5      p6
4       1       p7
5       3       2
6       1       2
7       4       5
8       3       6

Let assign the simple grouping with the asumption that
unknown sire and dam are from different groups, therfore:
p1,p3,p5        are in group1
p2,p4,p6,p7     are in group2.

Where Va = 200 Ve = 400, therefore alpha = Ve/Va.
In case h2 is known, alpha = (1-h2)/h2;



*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];

PROC IML;
  X = {1 0,
       0 1,
       0 1,
       1 0,
       1 0};

  Z = {0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 1};
```

```
y = {845,629,639,735,850};

A = {1   0   0  .5    0   .5  .25 .25,
     0   1   0   0   .5   .5  .25 .25,
     0   0   1   0   .5    0  .25 .5,
    .5   0   0   1    0   .25 .5  .125,
     0  .5  .5   0    1   .25 .5  .375,
    .5  .5   0  .25  .25   1  .25 .5,
    .25 .25 .25 .5   .5   .25  1  .25,
    .25 .25 .5  .125 .375 .5  .25  1};

Ai = inv(A);

/* Q is defined from relating animal to genetic group */
Q  = {.5   .5,
      .5   .5,
      .5   .5,
      .25  .75,
      .5   .5,
      .5   .5,
      .375 .625,
      .5   .5};

Va = 200; Ve = 400;
alpha = Ve/Va;

* --------- *
  MME Setup
* --------- *;

  XPX    = X`*X;
  XPZ    = X`*Z;
  ZPZ    = Z`*Z;
  ZPZ2   = Z`*Z+alpha#Ai;
  XPZQ   = X`*Z*Q;
  ZPZQ   = Z`*Z*Q;
  QPZPZQ = Q`*Z`*Z*Q;

  lhs = (X`*X    || X`*Z          || X`*Z*Q    )//
        (Z`*X    || Z`*Z+alpha#Ai || Z`*Z*Q    )//
        (Q`*Z`*X || Q`*Z`*Z       || Q`*Z`*Z*Q);

  rhs = X`*y // Z`*y // Q`*Z`*y;

  lhs[1:12,11]=0;                      /* Set genetic group1 to zero */
  lhs[11,1:12]=0;
  rhs[11,]=0;
  sol = GINV(lhs)*rhs;

* --------------------- *
  Set Label for printing
* --------------------- *;
  label1 = 'b1':'b2';                  /* The levels for fix eff is 2 (sex) */
  label2 = 'a1':'a8';                  /* The levels for animal eff is 8 */
  label3 = 'g1':'g2';                  /* The levels for genetic group eff is 2 */
  label  = label1 || label2 || label3;

* --------------- *
  Compute accuracy
* --------------- *;
  Di  = vecdiag(GINV(lhs));            /* Select digonal of lhs inverse */
  PEV = Di#Ve;                         /* Prediction error variance */
  I   = J(12,1,1);                     /* Set unit vector with number of b+g+u */
  Acc = J(12,1,.);                     /* Initialize accuracy */
  Acc[3:10,]  = SQRT(I[3:10,]-Di[3:10,]#alpha);       /* BV Accuracy */
  CC=GINV(lhs);
  C22=CC[3:10,3:10];




* --------------------------- *
  Construct BV based on Groups
* --------------------------- *;
  BV  = sol[3:10,];                    /* Select BV for true animals */
  gr  = sol[11:12,];                   /* Select group estimates */
```

```
    BVG = BV+Q*gr;                               /* Compute BV based on groups */
    BVG = J(2,1,.)//BVG//J(2,1,.);               /* Add missing for fix and gr */

  * ------ *
    Print
  * ------ *;
    PRINT 'Animal Model with Genetic Groups',,
          '[X`X    X`Z           X`ZQ  ][b]   [X`y  ]',
          '[Z`X    Z`Z+alpha#Ai  Z`ZQ  ][a] = [Z`y  ]',
          '[Q`Z`X  Q`Z`Z         Q`Z`ZQ][g] = [Q`Z`y]';

    PRINT X&f1,Z&f1,Q;                           /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,ZPZ&f1,XPZQ&f1,QPZPZQ&f2;
    PRINT A&f3,Ai&f3,ZPZ2&f3;
    PRINT rhs&f2;
    PRINT Va Ve ALPHA;
    PRINT sol&f3 [rowname=label] BVG&f3 Di&f3 PEV&f3 ACC&f3;
    PRINT C22&f4;                                /* Print inverse of LHS */
QUIT;
```

▪ *Animal model with genetic groups (OUTPUT)*

```
                    The SAS System
             Animal Model with Genetic Groups
        [X`X    X`Z           X`ZQ  ][b]   [X`y  ]
        [Z`X    Z`Z+alpha#Ai  Z`ZQ  ][a] = [Z`y  ]
        [Q`Z`X  Q`Z`Z         Q`Z`ZQ][g] = [Q`Z`y]

                        X
                        1  0
                        0  1
                        0  1
                        1  0
                        1  0

               Z
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

                        Q
                      0.5       0.5
                      0.5       0.5
                      0.5       0.5
                     0.25      0.75
                      0.5       0.5
                      0.5       0.5
                    0.375     0.625
                      0.5       0.5

                       XPX
                        3  0
                        0  2

              XPZ
               0  0  0  1  0  0  1  1
               0  0  0  0  1  1  0  0

              ZPZ
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

                       XPZQ
                        1  2
                        1  1
                  ZPZQ
                 0.000    0.000
                 0.000    0.000
                 0.000    0.000
                 0.250    0.750
                 0.500    0.500
                 0.500    0.500
                 0.375    0.625
                 0.500    0.500

                QPZPZQ
                 0.953    1.172
                 1.172    1.703

    A
1.000    0.000    0.000    0.500    0.000    0.500    0.250    0.250
0.000    1.000    0.000    0.000    0.500    0.500    0.250    0.250
0.000    0.000    1.000    0.000    0.500    0.000    0.250    0.500
0.500    0.000    0.000    1.000    0.000    0.250    0.500    0.125
0.000    0.500    0.500    0.000    1.000    0.250    0.500    0.375
0.500    0.500    0.000    0.250    0.250    1.000    0.250    0.500
0.250    0.250    0.250    0.500    0.500    0.250    1.000    0.250
0.250    0.250    0.500    0.125    0.375    0.500    0.250    1.000
```

AI

1.833	0.500	0.000	-0.667	0.000	-1.000	0.000	0.000
0.500	2.000	0.500	0.000	-1.000	-1.000	0.000	0.000
0.000	0.500	2.000	0.000	-1.000	0.500	0.000	-1.000
-0.667	0.000	0.000	1.833	0.500	0.000	-1.000	0.000
0.000	-1.000	-1.000	0.500	2.500	0.000	-1.000	0.000
-1.000	-1.000	0.500	0.000	0.000	2.500	0.000	-1.000
0.000	0.000	0.000	-1.000	-1.000	0.000	2.000	0.000
0.000	0.000	-1.000	0.000	0.000	-1.000	0.000	2.000

ZPZ2

3.667	1.000	0.000	-1.333	0.000	-2.000	0.000	0.000
1.000	4.000	1.000	0.000	-2.000	-2.000	0.000	0.000
0.000	1.000	4.000	0.000	-2.000	1.000	0.000	-2.000
-1.333	0.000	0.000	4.667	1.000	0.000	-2.000	0.000
0.000	-2.000	-2.000	1.000	6.000	0.000	-2.000	0.000
-2.000	-2.000	1.000	0.000	0.000	6.000	0.000	-2.000
0.000	0.000	0.000	-2.000	-2.000	0.000	5.000	0.000
0.000	0.000	-2.000	0.000	0.000	-2.000	0.000	5.000

RHS
2430.0
1268.0
0.00
0.00
0.00
845.00
629.00
639.00
735.00
850.00
0.00
2152.1

VA	VE	ALPHA
200	400	2

SOL		BVG	DI	PEV	ACC
b1	806.339	.	18.309	732.359	.
b2	631.854	.	12.908	516.311	.
a1	3.429	8.347	0.476	19.052	0.218
a2	-3.696	1.222	0.498	19.917	0.064
a3	0.267	5.185	0.477	19.083	0.214
a4	1.715	9.092	0.494	19.763	0.109
a5	-8.840	-3.922	0.440	17.616	0.345
a6	3.296	8.214	0.448	17.932	0.322
a7	-18.347	-12.200	0.446	17.834	0.329
a8	9.174	14.092	0.486	19.459	0.165
g1	0.000	.	0.000	0.000	.
g2	9.836	.	45.834	1833.367	.

Animal model with genetic groups (QP-Transformation)

```
/* --------------------------------- */
/* Animal Model with Genetic Groups    */
/* (QP Transformation)                 */
/* y = Xb + Za + ZQg + e               */
/* Written by Monchai Duangjinda       */
/* --------------------------------- */

*================*
 Data Description
*================*
Data file:
ID      sex     adg(g)
4       M       845
5       F       629
6       F       639
7       M       735
8       M       850

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       1       0
5       3       2
6       1       2
7       4       5
8       3       6

Assign unknown sires and dam with unrelated phantom
parents.
an      s       d
1       p1      p2
2       p3      p4
3       p5      p6
4       1       p7
5       3       2
6       1       2
7       4       5
8       3       6

Let assign the simple grouping with the asumption that
unknown sire and dam are from different groups, therfore:
p1,p3,p5        are in group1
p2,p4,p6,p7     are in group2.

Where Va = 200 Ve = 400, therefore alpha = Ve/Va.
In case h2 is known, alpha = (1-h2)/h2;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];

PROC IML;
  X = {1 0,
       0 1,
       0 1,
       1 0,
       1 0};


  Z = {0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
```

```
      0 0 0 0 0 0 1 0,
      0 0 0 0 0 0 0 1};
 y = {845,629,639,735,850};
 A = {1   0   0  .5    0   .5  .25 .25,
      0   1   0   0   .5   .5  .25 .25,
      0   0   1   0   .5    0  .25 .5,
     .5   0   0   1    0   .25 .5  .125,
      0  .5  .5   0    1   .25 .5  .375,
     .5  .5   0  .25  .25   1  .25 .5,
     .25 .25 .25 .5   .5   .25  1  .25,
     .25 .25 .5  .125 .375 .5  .25  1};
 Ai = inv(A);

 /* Q is defined from relating animal to genetic group */
 Q  = {.5   .5,
       .5   .5,
       .5   .5,
       .25  .75,
       .5   .5,
       .5   .5,
       .375 .625,
       .5   .5};
 Ai11   = Ai;
 Ai12   = -Ai*Q;
 Ai21   = Ai12`;
 Ai22   = Q`*Ai*Q;
 Va = 200; Ve = 400;
 alpha = Ve/Va;

 * --------- *
   MME Setup
 * --------- *;
   XPX    = X`*X;
   XPZ    = X`*Z;
   ZPZ    = Z`*Z;
   ZPZ2   = Z`*Z+alpha#Ai11;
   ZERO   = J(2,2,0);           /* Create zero matrix for lhs relate to groups */
                                /* row=2 levels for fix, col= 2 levels for groups */

   lhs = (X`*X  || X`*Z            || ZERO      )//
         (Z`*X  || Z`*Z+alpha#Ai11 || alpha#Ai12)//
         (ZERO` || alpha#Ai21      || alpha#Ai22);

   lhs[11,1:12]=0;                      /* Set genetic group I mean to zero */
   lhs[1:12,11]=0;
   rhs = X`*y // Z`*y // J(2,1,0);      /* Add zero value for 2 groups for RHS */
   sol = GINV(lhs)*rhs;

 * --------------------- *
   Set Label for printing
 * --------------------- *;
   label1 = 'b1':'b2';                  /* The levels for fix eff is 2 (sex) */
   label2 = 'a1':'a8';                  /* The levels for animal eff is 8 */
   label3 = 'g1':'g2';                  /* The levels for genetic group eff is 2 */
   label  = label1 || label2 || label3;

 * ------ *
   Print
 * ------ *;
   PRINT 'Animal Model with Genetic Groups',
         '(Alternative Method)',,
         '[X`X    X`Z             0         ][b]   [X`y]',
         '[Z`X    Z`Z+alpha#Ai11  alpha#Ai12][a] = [Z`y]',
         '[0      alpha#Ai21      alpha#Ai22][g] = [0  ]',,
         'where Ai11=inv(A), Ai12=-Ai11*Q, Ai22=Q`*Ai11*Q';

   PRINT X&f1,Z&f1,Q;                   /* Print matrix using f1 format */
   PRINT XPX&f1,XPZ&f1,ZPZ&f1;
   PRINT A&f3,Ai&f3,Ai11&f3,Ai12&f3,Ai22&f3,ZPZ2&f3;
   PRINT rhs&f2,lhs&f2;
   PRINT Va Ve ALPHA;
   PRINT sol&f3 [rowname=label];
QUIT;
```

▪ *Genetic groups by QP transformation (OUTPUT)*

```
                              The SAS System

                   Animal Model with Genetic Groups
                         (Alternative Method)

            [X`X    X`Z             0         ][b]   [X`y]
            [Z`X    Z`Z+alpha#Ai11  alpha#Ai12][a] = [Z`y]
            [0      alpha#Ai21      alpha#Ai22][g] = [0  ]

            where Ai11=inv(A), Ai12=-Ai11*Q, Ai22=Q`*Ai11*Q

                              X
                              1  0
                              0  1
                              0  1
                              1  0
                              1  0

                     Z
                     0  0  0  1  0  0  0  0
                     0  0  0  0  1  0  0  0
                     0  0  0  0  0  1  0  0
                     0  0  0  0  0  0  1  0
                     0  0  0  0  0  0  0  1

                              Q
                            0.5       0.5
                            0.5       0.5
                            0.5       0.5
                           0.25      0.75
                            0.5       0.5
                            0.5       0.5
                          0.375     0.625
                            0.5       0.5

                             XPX
                              3  0
                              0  2

                    XPZ
                     0  0  0  1  0  0  1  1
                     0  0  0  0  1  1  0  0

                    ZPZ
                     0  0  0  0  0  0  0  0
                     0  0  0  0  0  0  0  0
                     0  0  0  0  0  0  0  0
                     0  0  0  1  0  0  0  0
                     0  0  0  0  1  0  0  0
                     0  0  0  0  0  1  0  0
                     0  0  0  0  0  0  1  0
                     0  0  0  0  0  0  0  1


    A
 1.000    0.000    0.000    0.500    0.000    0.500    0.250    0.250
 0.000    1.000    0.000    0.000    0.500    0.500    0.250    0.250
 0.000    0.000    1.000    0.000    0.500    0.000    0.250    0.500
 0.500    0.000    0.000    1.000    0.000    0.250    0.500    0.125
 0.000    0.500    0.500    0.000    1.000    0.250    0.500    0.375
 0.500    0.500    0.000    0.250    0.250    1.000    0.250    0.500
 0.250    0.250    0.250    0.500    0.500    0.250    1.000    0.250
 0.250    0.250    0.500    0.125    0.375    0.500    0.250    1.000

  AI11
 1.833    0.500    0.000   -0.667    0.000   -1.000    0.000    0.000
 0.500    2.000    0.500    0.000   -1.000   -1.000    0.000    0.000
 0.000    0.500    2.000    0.000   -1.000    0.500    0.000   -1.000
-0.667    0.000    0.000    1.833    0.500    0.000   -1.000    0.000
 0.000   -1.000   -1.000    0.500    2.500    0.000   -1.000    0.000
-1.000   -1.000    0.500    0.000    0.000    2.500    0.000   -1.000
 0.000    0.000    0.000   -1.000   -1.000    0.000    2.000    0.000
 0.000    0.000   -1.000    0.000    0.000   -1.000    0.000    2.000
```

AI12

-0.500	-0.167
-0.500	-0.500
-0.500	-0.500
0.000	-0.667
0.000	0.000
0.000	0.000
0.000	0.000
0.000	0.000

AI22

0.750	0.750
0.750	1.083

ZPZ2

3.667	1.000	0.000	-1.333	0.000	-2.000	0.000	0.000
1.000	4.000	1.000	0.000	-2.000	-2.000	0.000	0.000
0.000	1.000	4.000	0.000	-2.000	1.000	0.000	-2.000
-1.333	0.000	0.000	4.667	1.000	0.000	-2.000	0.000
0.000	-2.000	-2.000	1.000	6.000	0.000	-2.000	0.000
-2.000	-2.000	1.000	0.000	0.000	6.000	0.000	-2.000
0.000	0.000	0.000	-2.000	-2.000	0.000	5.000	0.000
0.000	0.000	-2.000	0.000	0.000	-2.000	0.000	5.000

RHS
2430.0
1268.0
0.00
0.00
0.00
845.00
629.00
639.00
735.00
850.00
0.00
0.00

LHS

3.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	1.00	1.00	0.00	0.00
0.00	2.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	3.67	1.00	0.00	-1.33	0.00	-2.00	0.00	0.00	0.00	-0.33
0.00	0.00	1.00	4.00	1.00	0.00	-2.00	-2.00	0.00	0.00	0.00	-1.00
0.00	0.00	0.00	1.00	4.00	0.00	-2.00	1.00	0.00	-2.00	0.00	-1.00
1.00	0.00	-1.33	0.00	0.00	4.67	1.00	0.00	-2.00	0.00	0.00	-1.33
0.00	1.00	0.00	-2.00	-2.00	1.00	6.00	0.00	-2.00	0.00	0.00	0.00
0.00	1.00	-2.00	-2.00	1.00	0.00	0.00	6.00	0.00	-2.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	-2.00	-2.00	0.00	5.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	-2.00	0.00	0.00	-2.00	0.00	5.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	-0.33	-1.00	-1.00	-1.33	0.00	0.00	0.00	0.00	0.00	2.17

VA	VE	ALPHA
200	400	2

SOL	
b1	806.339
b2	631.854
a1	8.347
a2	1.222
a3	5.185
a4	9.092
a5	-3.922
a6	8.214
a7	-12.200
a8	14.092
g1	0.000
g2	9.836

```
/* ------------------------------ */
/* Sire-Maternal Grandsire Model   */
/* y = Xb + Zs + Mmgs + e          */
/* Written by Monchai Duangjinda   */
/* ------------------------------ */

*================*
 Data Description
*================*
Data file:
calf    sex     parity  sire    dam     mgs     bw (kg)
11      1       1       1       4       0       35
12      2       2       1       4       0       20
13      2       1       1       5       0       25
14      1       1       2       6       1       40
15      2       2       3       6       2       42
16      2       3       3       6       2       22

Pedigree file:
anim    s       mgs (sire of dam)
1       0       0
2       1       0
3       2       1
11      1       0
12      1       0
13      1       0
14      2       1
15      3       2
16      3       2

Selected Pedigree file (Only related sire and mgs are required):
anim    s       mgs (sire of dam)
1       0       0
2       1       0
3       2       1

Where Vs = 5, Vmgs = 2,Cov(s,mgs) = 1, Vpe = 12 Ve = 40,
In case h2, c2, m2, are known, the model variance becomes:
Vs  = 1/4*h2, Vmgs = 1/16*h2 + 1/4*m2 + 1/4*Cov(a,m),
Vpe = 3/16*h2 + 3/4*m2 + 3/4*Cov(a,m) + c2,
Ve  = 1/2*h2 + e2
where Cov(a,m) and e2 are proportional to total variance;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
```

```
PROC IML;
  X = {1 0 1 0 0,
       0 1 0 1 0,
       0 1 1 0 0,
       1 0 1 0 0,
       0 1 0 1 0,
       0 1 0 0 1};

  Z = {1 0 0,
       1 0 0,
       1 0 0,
       0 1 0,
       0 0 1,
       0 0 1};

  M = {0 0 0,
       0 0 0,
       0 0 0,
       1 0 0,
       0 1 0,
       0 1 0};

  y = {35,20,25,40,42,22};

  A = {  1 0.5   0.5,
       0.5   1   0.625,
       0.5 0.625 1};

  Ai = inv(A);

  Vs = 4; Vmgs = 2; Vsmgs = 4; Ve = 40;
  G  = (Vs    || Vsmgs)//
       (Vsmgs || Vmgs );
  Gi = INV(G);
  alpha   = Ve#Gi;
  alpha11 = alpha[1,1];
  alpha12 = alpha[1,2];
  alpha21 = alpha12;
  alpha22 = alpha[2,2];

  * --------- *
    MME Setup
  * --------- *;

    XPX   = X`*X;
    XPZ   = X`*Z;
    XPM   = X`*M;
    ZPZ   = Z`*Z;
    ZPM   = Z`*M;
    MPM   = M`*M;

    ZPZ2 = Z`*Z+alpha11#Ai;
    ZPM2 = Z`*M+alpha12#Ai;
    MPM2 = M`*M+alpha22#Ai;

    lhs = (X`*X || X`*Z             || X`*M             )//
          (Z`*X || Z`*Z+alpha11#Ai || Z`*M+alpha12#Ai )//
          (M`*X || M`*Z+alpha21#Ai || M`*M+alpha22#Ai );

    rhs = X`*y // Z`*y // M`*y;
    sol = GINV(lhs)*rhs;
```

```
  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b5';                 /* The levels for fix eff is 5 (sex+parity) */
    label2 = 's1':'s3';                 /* The levels for sire eff is 3 */
    label3 = 'mgs1':'mgs3';             /* The levels for mgs eff is 3 */
    label  = label1 || label2 || label3;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));           /* Select digonal of lhs inverse */
    PEV = Di#Ve;                        /* Prediction error variance */
    I   = J(11,1,1);                    /* Set unit vector with number of b+u+m+pe */
    Acc = J(11,1,.);                                   /* Initialize accuracy */
    Acc[6:8,]   = SQRT(I[6:8,]-Di[6:8,]#(Ve/Vs));       /* BV Accuracy for sire*/
    Acc[9:11,]  = SQRT(I[9:11,]-Di[9:11,]#(Ve/Vmgs));   /* Maternal grandsire Accuracy */
    CC=GINV(lhs);

  * ------ *
    Print
  * ------ *;

    PRINT 'Sire-Maternal Grandsire Model',,
          '[X`*X  X`*Z             X`*M          ][b ]   [X`*y]',
          '[Z`*X  Z`*Z+alpha11#Ai  Z`*M+alpha12  ][u ] = [Z`*y]',
          '[M`*X  M`*Z+alpha21#Ai  M`*M+alpha22  ][m ]   [M`*y]';

    PRINT X&f1, Z&f1, M&f1;                            /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,XPM&f1,ZPZ&f1,ZPM&f1,MPM&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2,ZPM2&f2,MPM2&f2;
    PRINT rhs&f2;
    PRINT G Ve,
          Gi&f2 alpha&f2 alpha11&f2 alpha12&f2 alpha22&f2;
    PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
    PRINT CC&f4;                                       /* Print inverse of LHS */
QUIT;
```

▪ *Sire-MGS model (OUTPUT)*

```
                    The SAS System

              Sire-Maternal Grandsire Model
[X`*X  X`*Z             X`*M          ][b ]   [X`*y]
[Z`*X  Z`*Z+alpha11#Ai  Z`*M+alpha12  ][u ] = [Z`*y]
[M`*X  M`*Z+alpha21#Ai  M`*M+alpha22  ][m ]   [M`*y]

          X
          1  0  1  0  0
          0  1  0  1  0
          0  1  1  0  0
          1  0  1  0  0
          0  1  0  1  0
          0  1  0  0  1

             Z
             1  0  0
             1  0  0
             1  0  0
             0  1  0
             0  0  1
             0  0  1

             M
             0  0  0
             0  0  0
             0  0  0
             1  0  0
             0  1  0
             0  1  0

         XPX
          2  0  2  0  0
          0  4  1  2  1
          2  1  3  0  0
          0  2  0  2  0
          0  1  0  0  1

           XPZ
             1  1  0
             2  0  2
             2  1  0
             1  0  1
             0  0  1

            XPM
             1  0  0
             0  2  0
             1  0  0
             0  1  0
             0  1  0

            ZPZ
             3  0  0
             0  1  0
             0  0  2

            ZPM
             0  0  0
             1  0  0
             0  2  0

            MPM
             1  0  0
             0  2  0
             0  0  0


           A
        1.00   0.50   0.50
        0.50   1.00   0.63
        0.50   0.63   1.00

          AI
```

```
  1.44  -0.44  -0.44
 -0.44   1.78  -0.89
 -0.44  -0.89   1.78

  ZPZ2
-11.44   4.44   4.44
  4.44 -16.78   8.89
  4.44   8.89 -15.78

  ZPM2
 28.89  -8.89  -8.89
 -7.89  35.56 -17.78
 -8.89 -15.78  35.56

  MPM2
-27.89   8.89   8.89
  8.89 -33.56  17.78
  8.89  17.78 -35.56

          RHS
        75.00
       109.00
       100.00
        62.00
        22.00
        80.00
        40.00
        64.00
        40.00
        64.00
         0.00
```

```
   G                      VE
   4         4            40
   4         2
```

```
   GI            ALPHA             ALPHA11  ALPHA12  ALPHA22
-0.25   0.50   -10.00  20.00      -10.00    20.00   -20.00
 0.50  -0.50    20.00 -20.00
```

```
SOL                  DI       PEV       ACC
b1      24.569     0.839    33.546      .
b2      12.558     0.279    11.157      .
b3      12.342     0.470    18.805      .
b4      17.376     0.569    22.772      .
b5       7.410     0.947    37.895      .
s1       0.101     0.099     3.951    0.111
s2       1.330     0.090     3.589    0.321
s3       1.255     0.093     3.702    0.273
mgs1    -0.251     0.050     1.988    0.076
mgs2     0.778     0.046     1.825    0.295
mgs3     0.891     0.046     1.856    0.269
```

```
 CC
 0.8386 -0.1969 -0.3249  0.4139  0.5527 -0.0782 -0.0958 -0.0822 -0.0535 -0.0737 -0.0649
-0.1969  0.2789  0.1749 -0.1060  0.0132 -0.0538 -0.0492 -0.0538 -0.0530 -0.0382 -0.0482
-0.3249  0.1749  0.4701 -0.2500 -0.3701 -0.0450  0.0015  0.0053 -0.0470 -0.0100 -0.0010
 0.4139 -0.1060 -0.2500  0.5693 -0.0114 -0.0440 -0.0483 -0.0453 -0.0355 -0.0373 -0.0377
 0.5527  0.0132 -0.3701 -0.0114  0.9474 -0.0430 -0.0982 -0.0959 -0.0240 -0.0646 -0.0744
-0.0782 -0.0538 -0.0450 -0.0440 -0.0430  0.0988  0.0477  0.0486  0.1000  0.0482  0.0492
-0.0958 -0.0492  0.0015 -0.0483 -0.0982  0.0477  0.0897  0.0540  0.0512  0.0934  0.0567
-0.0822 -0.0538  0.0053 -0.0453 -0.0959  0.0486  0.0540  0.0925  0.0513  0.0572  0.0948
-0.0535 -0.0530 -0.0470 -0.0355 -0.0240  0.1000  0.0512  0.0513  0.0497  0.0257  0.0259
-0.0737 -0.0382 -0.0100 -0.0373 -0.0646  0.0482  0.0934  0.0572  0.0257  0.0456  0.0277
-0.0649 -0.0482 -0.0010 -0.0377 -0.0744  0.0492  0.0567  0.0948  0.0259  0.0277  0.0464
```

```
/* ---------------------------------------- */
/* Animal Model with Maternal and PE effects */
/* y = Xb + Za + Mm + Wp + e                 */
/* Written by Monchai Duangjinda             */
/* ---------------------------------------- */

*================*
 Data Description
*================*
Data file:
calf    dam     hys     sex     ww205 (kg)
5       2       1       2       235
6       2       1       2       220
7       6       1       2       225
8       5       1       1       240
9       6       2       1       242
10      2       2       2       222

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       0       0
5       1       2
6       3       2
7       4       6
8       3       5
9       1       6
10      3       2

Where Va = 150 Vm = 90 Vam = -40 Vpe = 40 Ve = 350;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];

PROC IML;
  X = {1 0 0 1,
       1 0 0 1,
       1 0 0 1,
       1 0 1 0,
       0 1 1 0,
       0 1 0 1};

  Z = {0 0 0 0 1 0 0 0 0 0,
       0 0 0 0 0 1 0 0 0 0,
       0 0 0 0 0 0 1 0 0 0,
       0 0 0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 0 0 1};

  M = {0 1 0 0 0 0 0 0 0 0,
       0 1 0 0 0 0 0 0 0 0,
       0 0 0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0 0 0,
       0 0 0 0 0 1 0 0 0 0,
       0 1 0 0 0 0 0 0 0 0};
```

```
  W = {0 1 0 0 0 0 0 0 0 0,
       0 1 0 0 0 0 0 0 0 0,
       0 0 0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0 0 0,
       0 0 0 0 0 1 0 0 0 0,
       0 1 0 0 0 0 0 0 0 0};

  y = {235,220,225,240,242,222};


  A = {1   0   0   0 .5    0    0   .25  .5    0,
       0   1   0   0 .5   .5   .25  .25  .25  .5,
       0   0   1   0  0   .5   .25  .5   .25  .5,
       0   0   0   1  0    0   .5    0    0    0,
      .5  .5   0   0  1   .25  .125 .5   .375 .25,
       0  .5  .5   0 .25   1   .5   .375 .5   .5,
       0  .25 .25 .5 .125 .5    1   .188 .25  .25,
      .25 .25 .5   0 .5   .375 .188  1   .313 .375,
      .5  .25 .25  0 .375 .5   .25  .313  1   .25,
       0  .5  .5   0 .25  .5   .25  .375 .25   1};

  Ai = inv(A);

  Va = 150; Vm = 90; Vam = -40; Vpe = 40; Ve = 350;
  G  = (Va  || Vam)//
       (Vam || Vm);
  Gi = INV(G);
  alpha   = Ve#Gi;
  alpha11 = alpha[1,1];
  alpha12 = alpha[1,2];
  alpha21 = alpha12;
  alpha22 = alpha[2,2];
  gamma   = Ve/Vpe;

  * --------- *
    MME Setup
  * --------- *;

    XPX   = X`*X;
    XPZ   = X`*Z;
    XPM   = X`*M;
    XPW   = X`*W;
    ZPZ   = Z`*Z;
    ZPM   = Z`*M;
    ZPW   = Z`*W;
    MPM   = M`*M;
    MPW   = M`*W;
    WPW   = W`*W;

    ZPZ2 = Z`*Z+alpha11#Ai;
    ZPM2 = Z`*M+alpha12#Ai;
    MPM2 = M`*M+alpha22#Ai;
    WPW2 = W`*W+gamma#I(10);

    lhs = (X`*X || X`*Z            || X`*M            || X`*W            )//
          (Z`*X || Z`*Z+alpha11#Ai || Z`*M+alpha12#Ai || Z`*W            )//
          (M`*X || M`*Z+alpha21#Ai || M`*M+alpha22#Ai || M`*W            )//
          (W`*X || W`*Z            || W`*M            || W`*W+gamma#I(10));

    rhs = X`*y // Z`*y // M`*y // W`*y;
    sol = GINV(lhs)*rhs;

  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b4';                           /* The levels for fix eff is 5
(herd+sex) */
    label2 = 'a1':'a10';                          /* The levels for animal eff is 10 */
    label3 = 'm1':'m10';                          /* The levels for maternal eff is 10 */
    label4 = 'pe1':'pe10';                        /* The levels for PE eff is 8 */
    label  = label1 || label2 || label3 || label4;
```

```
  * ---------------- *
    Compute accuracy
  * ---------------- *;
    Di  = vecdiag(GINV(lhs));            /* Select digonal of lhs inverse */
    PEV = Di#Ve;                         /* Prediction error variance */
    I   = J(34,1,1);                     /* Set unit vector with number of b+u+m+pe */
    Acc = J(34,1,.);                                    /* Initialize accuracy */
    Acc[5:14,]  = SQRT(I[5:14,]-Di[5:14,]#(Ve/Va));     /* BV Accuracy */
    Acc[15:24,] = SQRT(I[15:24,]-Di[15:24,]#(Ve/Vm)) ;  /* Maternal Accuracy */
    Acc[25:34,] = SQRT(I[25:34,]-Di[25:34,]#gamma);     /* PE Accuracy */
    CC=GINV(lhs);

  * ------ *
    Print
  * ------ *;

    PRINT 'Animal Model with Maternal and PE Effect',,
          '[X`*X  X`*Z              X`*M          X`*W        ][b ]   [X`*y]',
          '[Z`*X  Z`*Z+alpha11#Ai  Z`*M+alpha12  Z`*W        ][a ] = [Z`*y]',
          '[M`*X  M`*Z+alpha21#Ai  M`*M+alpha22  M`*W        ][m ]   [M`*y]',
          '[W`*X  W`*Z              W`*M          W`*W+gamma#I][pe]   [W`*y]';
    PRINT X&f1, Z&f1, M&f1, W&f1;                       /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,XPM&f1,XPW&f1,ZPZ&f1,ZPM&f1,ZPW&f1,
          MPM&f1,MPW&f1,WPW&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2,ZPM2&f2,MPM2&f2,WPW2&f2;
    PRINT rhs&f2;
    PRINT G Vpe Ve,
          Gi&f2 alpha&f2 alpha11&f2 alpha12&f2 alpha22&f2 gamma&f2;
    PRINT sol&f3 [rowname=label] Di&f3 PEV&f3 ACC&f3;
QUIT;
```

▪ *Animal Model With Maternal Effect (OUTPUT)*

```
                    The SAS System

        Animal Model with Maternal and PE Effect

[X`*X  X`*Z              X`*M           X`*W         ][b ]   [X`*y]
[Z`*X  Z`*Z+alpha11#Ai  Z`*M+alpha12  Z`*W         ][a ] = [Z`*y]
[M`*X  M`*Z+alpha21#Ai  M`*M+alpha22  M`*W         ][m ]   [M`*y]
[W`*X  W`*Z              W`*M           W`*W+gamma#I][pe]   [W`*y]

                 X
                 1  0  0  1
                 1  0  0  1
                 1  0  0  1
                 1  0  1  0
                 0  1  1  0
                 0  1  0  1

        Z
        0  0  0  0  1  0  0  0  0  0
        0  0  0  0  0  1  0  0  0  0
        0  0  0  0  0  0  1  0  0  0
        0  0  0  0  0  0  0  1  0  0
        0  0  0  0  0  0  0  0  1  0
        0  0  0  0  0  0  0  0  0  1

        M
        0  1  0  0  0  0  0  0  0  0
        0  1  0  0  0  0  0  0  0  0
        0  0  0  0  0  1  0  0  0  0
        0  0  0  0  1  0  0  0  0  0
        0  0  0  0  0  1  0  0  0  0
        0  1  0  0  0  0  0  0  0  0

        W
        0  1  0  0  0  0  0  0  0  0
        0  1  0  0  0  0  0  0  0  0
        0  0  0  0  0  1  0  0  0  0
        0  0  0  0  1  0  0  0  0  0
        0  0  0  0  0  1  0  0  0  0
        0  1  0  0  0  0  0  0  0  0

                XPX
                 4  0  1  3
                 0  2  1  1
                 1  1  2  0
                 3  1  0  4

       XPZ
        0  0  0  0  1  1  1  1  0  0
        0  0  0  0  0  0  0  0  1  1
        0  0  0  0  0  0  0  1  1  0
        0  0  0  0  1  1  1  0  0  1

       XPM
        0  2  0  0  1  1  0  0  0  0
        0  1  0  0  0  1  0  0  0  0
        0  0  0  0  1  1  0  0  0  0
        0  3  0  0  0  1  0  0  0  0

       XPW
        0  2  0  0  1  1  0  0  0  0
        0  1  0  0  0  1  0  0  0  0
        0  1  0  0  1  1  0  0  0  0
        0  2  0  0  0  1  0  0  0  0
```

ZPZ

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	1

ZPM

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0	0

ZPW

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0	0

MPM

0	0	0	0	0	0	0	0	0	0
0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

MPW

0	0	0	0	0	0	0	0	0	0
0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

WPW

0	0	0	0	0	0	0	0	0	0
0	3	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

A

1.00	0.00	0.00	0.00	0.50	0.00	0.00	0.25	0.50	0.00
0.00	1.00	0.00	0.00	0.50	0.50	0.25	0.25	0.25	0.50
0.00	0.00	1.00	0.00	0.00	0.50	0.25	0.50	0.25	0.50
0.00	0.00	0.00	1.00	0.00	0.00	0.50	0.00	0.00	0.00
0.50	0.50	0.00	0.00	1.00	0.25	0.13	0.50	0.38	0.25
0.00	0.50	0.50	0.00	0.25	1.00	0.50	0.38	0.50	0.50
0.00	0.25	0.25	0.50	0.13	0.50	1.00	0.19	0.25	0.25
0.25	0.25	0.50	0.00	0.50	0.38	0.19	1.00	0.31	0.38
0.50	0.25	0.25	0.00	0.38	0.50	0.25	0.31	1.00	0.25
0.00	0.50	0.50	0.00	0.25	0.50	0.25	0.38	0.25	1.00

AI

2.00	0.50	-0.00	0.00	-1.00	0.50	-0.00	0.00	-1.00	0.00
0.50	2.50	1.00	-0.00	-1.00	-1.00	0.00	-0.00	0.00	-1.00
-0.00	1.00	2.50	-0.00	0.50	-1.00	0.00	-1.00	0.00	-1.00
0.00	0.00	-0.00	1.50	-0.00	0.50	-1.00	0.00	-0.00	0.00
-1.00	-1.00	0.50	-0.00	2.50	-0.00	0.00	-1.00	0.00	0.00
0.50	-1.00	-1.00	0.50	-0.00	3.00	-1.00	0.00	-1.00	0.00
-0.00	0.00	0.00	-1.00	0.00	-1.00	2.00	-0.00	0.00	0.00
0.00	0.00	-1.00	0.00	-1.00	0.00	-0.00	2.00	-0.00	0.00
-1.00	0.00	0.00	-0.00	0.00	-1.00	0.00	-0.00	2.00	0.00
0.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	2.00

ZPZ2

5.29	1.32	-0.00	0.00	-2.65	1.32	-0.00	0.00	-2.65	0.00
1.32	6.62	2.65	-0.00	-2.65	-2.65	0.00	-0.00	0.00	-2.65
-0.00	2.65	6.62	-0.00	1.32	-2.65	0.00	-2.65	0.00	-2.65
0.00	0.00	-0.00	3.97	-0.00	1.32	-2.65	0.00	-0.00	0.00
-2.65	-2.65	1.32	-0.00	7.62	-0.00	0.00	-2.65	0.00	0.00
1.32	-2.65	-2.65	1.32	-0.00	8.94	-2.65	0.01	-2.65	0.00
-0.00	0.00	0.00	-2.65	0.00	-2.65	6.29	-0.01	0.00	0.00
0.00	0.00	-2.65	0.00	-2.65	0.01	-0.01	6.29	-0.01	0.00
-2.65	0.00	0.00	-0.00	0.00	-2.65	0.00	-0.01	6.29	0.00
0.00	-2.65	-2.65	0.00	0.00	0.00	0.00	0.00	0.00	6.29

ZPM2

2.35	0.59	-0.00	0.00	-1.18	0.59	-0.00	0.00	-1.18	0.00
0.59	2.94	1.18	-0.00	-1.18	-1.18	0.00	-0.00	0.00	-1.18
-0.00	1.18	2.94	-0.00	0.59	-1.18	0.00	-1.18	0.00	-1.18
0.00	0.00	-0.00	1.76	-0.00	0.59	-1.18	0.00	-0.00	0.00
-1.18	-0.18	0.59	-0.00	2.94	-0.00	0.00	-1.18	0.00	0.00
0.59	-0.18	-1.18	0.59	-0.00	3.53	-1.18	0.00	-1.18	0.00
-0.00	0.00	0.00	-1.18	0.00	-0.18	2.35	-0.00	0.00	0.00
0.00	0.00	-1.18	0.00	-0.18	0.00	-0.00	2.35	-0.00	0.00
-1.18	0.00	0.00	-0.00	0.00	-0.18	0.00	-0.00	2.35	0.00
0.00	-0.18	-1.18	0.00	0.00	0.00	0.00	0.00	0.00	2.35

MPM2

8.82	2.21	-0.00	0.00	-4.41	2.21	-0.00	0.00	-4.41	0.00
2.21	14.03	4.41	-0.00	-4.41	-4.41	0.00	-0.00	0.00	-4.41
-0.00	4.41	11.03	-0.00	2.21	-4.42	0.00	-4.41	0.00	-4.41
0.00	0.00	-0.00	6.62	-0.00	2.21	-4.41	0.00	-0.00	0.00
-4.41	-4.41	2.21	-0.00	12.03	-0.00	0.00	-4.41	0.00	0.00
2.21	-4.41	-4.42	2.21	-0.00	15.24	-4.41	0.01	-4.41	0.00
-0.00	0.00	0.00	-4.41	0.00	-4.41	8.82	-0.01	0.00	0.00
0.00	0.00	-4.41	0.00	-4.41	0.01	-0.01	8.82	-0.01	0.00
-4.41	0.00	0.00	-0.00	0.00	-4.41	0.00	-0.01	8.82	0.00
0.00	-4.41	-4.41	0.00	0.00	0.00	0.00	0.00	0.00	8.82

WPW2

8.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	11.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	8.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	8.75	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	9.75	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	10.75	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	8.75	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.75	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.75	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.75

RHS
920.00
464.00
482.00
902.00
0.00
0.00
0.00
0.00
235.00
220.00
225.00
240.00
242.00
222.00
0.00
677.00
0.00
0.00
240.00
467.00
0.00
0.00
0.00
0.00
0.00
677.00
0.00
0.00
240.00
467.00
0.00
0.00
0.00
0.00

G		VPE	VE
150	-40	40	350
-40	90		

GI		ALPHA		ALPHA11	ALPHA12	ALPHA22	GAMMA
0.008	0.003	2.65	1.18	2.65	1.18	4.41	8.75
0.003	0.013	1.18	4.41				

	SOL	DI	PEV	ACC
b1	117.567	0.315	110.094	.
b2	115.471	0.459	160.771	.
b3	124.457	0.472	165.261	.
b4	108.581	0.379	132.644	.
a1	1.602	0.395	138.351	0.279
a2	0.036	0.428	149.630	0.050
a3	-1.485	0.396	138.513	0.277
a4	-0.152	0.409	143.203	0.213
a5	2.159	0.372	130.229	0.363
a6	-1.688	0.383	134.161	0.325
a7	-1.072	0.390	136.329	0.302
a8	0.076	0.407	142.451	0.224
a9	0.218	0.399	139.502	0.265
a10	-0.986	0.399	139.633	0.263

m1	-0.565	0.249	87.289	0.174
m2	0.071	0.253	88.520	0.128
m3	0.454	0.241	84.375	0.250
m4	0.041	0.256	89.517	0.073
m5	-0.743	0.245	85.799	0.216
m6	0.577	0.239	83.771	0.263
m7	0.349	0.255	89.320	0.087
m8	-0.075	0.255	89.282	0.089
m9	-0.064	0.253	88.659	0.122
m10	0.332	0.252	88.322	0.137
pe1	0.000	0.114	40.000	0.000
pe2	0.081	0.109	38.067	0.220
pe3	0.000	0.114	40.000	0.000
pe4	0.000	0.114	40.000	0.000
pe5	-0.139	0.112	39.109	0.149
pe6	0.058	0.106	36.942	0.276
pe7	0.000	0.114	40.000	0.000
pe8	0.000	0.114	40.000	0.000
pe9	0.000	0.114	40.000	0.000
pe10	0.000	0.114	40.000	0.000

Animal model with dominance effects

```
/* --------------------------------- */
/* Animal Model with Dominance Effects */
/* y = Xb + Za + Wd + e              */
/* Written by Monchai Duangjinda      */
/* --------------------------------- */

*================*
 Data Description
*================*
Data file:
ID      sex     bw(kg)
4       1       1.7
5       2       2.0
6       2       1.8
7       2       1.4
8       1       1.5

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       1       0
4       1       0
5       2       0
6       4       5
7       4       5
8       2       3

Where Va = 20 Vd = 10, Ve = 70;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
PROC IML;
  X = {1 0,
       0 1,
       0 1,
       0 1,
       1 0};

  Z = {0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 1};

  W = Z;

  y = {1.7,2.0,1.8,1.4,1.5};

  A = {1   0  .5   .5    0  .25  .25  .25,
       0   1   0    0   .5  .25  .25  .5,
      .5   0   1   .25   0  .125 .125 .5,
      .5   0  .25   1    0  .5   .5   .125,
       0  .5   0    0    1  .5   .5   .25,
      .25 .25 .125 .5   .5   1   .5   .187,
      .25 .25 .125 .5   .5  .5    1   .187,
      .25 .5  .5   .125 .25 .187 .187  1};

  /* Dominance relationship is calculated from additive relationship
     Using Cockerham formula which is
     d(x,y) = 1/4[a(sx,sy)*a(dx,dy)+a(sx,dy)*a(sy,dx)] */
```

```
  D = {1   0   0   0   0   0     0     0,
       0   1   0   0   0   0     0     0,
       0   0   1   0   0   0     0     0,
       0   0   0   1   0   0     0     0,
       0   0   0   0   1   0     0     0,
       0   0   0   0   0   1    .25   .0313,
       0   0   0   0   0  .25    1    .0313,
       0   0   0   0   0  .0313 .0313  1};

  Ai = inv(A);
  Di = inv(D);
  Dii = Di;
  Va = 20; Vd = 10; Ve = 70;
  alpha = Ve/Va;
  delta = Ve/Vd;

  * --------- *
    MME Setup
  * --------- *;
    XPX  = X`*X;
    XPZ  = X`*Z;
    XPW  = X`*W;
    ZPZ  = Z`*Z;
    ZPW  = Z`*W;
    WPW  = W`*W;
    ZPZ2 = Z`*Z+alpha#Ai;
    WPW2 = W`*W+delta#Di;
    lhs = (X`*X || X`*Z          || X`*W)//
          (Z`*X || Z`*Z+alpha#Ai || Z`*W)//
          (W`*X || W`*Z          || W`*W+delta#Di);
    rhs = X`*y // Z`*y // W`*y;
    sol = GINV(lhs)*rhs;

  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b2';                 /* The levels for fix eff is 2 (sex) */
    label2 = 'a1':'a8';                 /* The levels for animal eff is 8 */
    label3 = 'd1':'d8';                 /* The levels for dominance eff is 8 */
    label  = label1 || label2 || label3;
    labela = 'id1':'id8';

  * --------------- *
    Select solutions
  * --------------- *;
    EBV   = sol[3:10];                  /* The levels for EBV start from 3 */
    DOM   = sol[11:18];                 /* The levels for DOM start from 10 */
    TOTAL = EBV+DOM;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));                   /* Select digonal of lhs inverse */
    PEV = Di#Ve;                                /* Prediction error variance */
    I   = J(18,1,1);                            /* Set unit vector with number of b+u */
    Acc=J(18,1,.);                                     /* Initialize accuracy */
    Acc[3:10,]  = SQRT(I[3:10,]-Di[3:10,]#alpha);      /* BV Accuracy */
    Acc[14:18,]  = SQRT(I[14:18,]-Di[14:18,]#delta);   /* Dom Accuracy */
    CC=INV(lhs);

  * ------ *
    Print
  * ------ *;
    PRINT 'Animal Model with Dominance Effects',,
          '[X`X  X`Z          X`W         ][b]   [X`y]',
          '[Z`X  Z`Z+alpha#Ai Z`W         ][a] = [Z`y]',
          '[W`X  W`Z          W`W+delta#Di][d] = [W`y]';
    PRINT X&f1, Z&f1, W&f1;                            /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,XPW&f1,ZPZ&f1,ZPW&f1,WPW&f1;
    PRINT A&f3,Ai&f2,D&f3,Dii&f2,ZPZ2&f2,WPW2&f2;
    PRINT rhs&f2;
    PRINT Va Vd Ve ALPHA DELTA;
    PRINT sol&f3 [rowname=label] Di&f3 Acc&f3;
    PRINT EBV&f3 [rowname=labela] DOM&f3 TOTAL&f3;
    PRINT CC&f4;                                       /* Print inverse of LHS */
QUIT;
```

▪ *Animal Model With Dominance Effects (OUTPUT)*

```
                  The SAS System
        Animal Model with Dominance Effects
    [X`X  X`Z          X`W         ][b]   [X`y]
    [Z`X  Z`Z+alpha#Ai Z`W         ][a] = [Z`y]
    [W`X  W`Z          W`W+delta#Di][d] = [W`y]


                        X
                        1  0
                        0  1
                        0  1
                        0  1
                        1  0

               Z
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

               W
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

                       XPX
                        2  0
                        0  3

              XPZ
               0  0  0  1  0  0  0  1
               0  0  0  0  1  1  1  0

              XPW
               0  0  0  1  0  0  0  1
               0  0  0  0  1  1  1  0

              ZPZ
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

              ZPW
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1



              WPW
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  0  0  0  0  0
               0  0  0  1  0  0  0  0
               0  0  0  0  1  0  0  0
               0  0  0  0  0  1  0  0
               0  0  0  0  0  0  1  0
               0  0  0  0  0  0  0  1

A
```

1.000	0.000	0.500	0.500	0.000	0.250	0.250	0.250
0.000	1.000	0.000	0.000	0.500	0.250	0.250	0.500
0.500	0.000	1.000	0.250	0.000	0.125	0.125	0.500
0.500	0.000	0.250	1.000	0.000	0.500	0.500	0.125
0.000	0.500	0.000	0.000	1.000	0.500	0.500	0.250
0.250	0.250	0.125	0.500	0.500	1.000	0.500	0.187
0.250	0.250	0.125	0.500	0.500	0.500	1.000	0.187
0.250	0.500	0.500	0.125	0.250	0.187	0.187	1.000

AI

1.67	0.00	-0.67	-0.67	0.00	-0.00	-0.00	0.00
0.00	1.83	0.50	0.00	-0.67	-0.00	-0.00	-1.00
-0.67	0.50	1.83	0.00	0.00	-0.00	-0.00	-1.00
-0.67	0.00	0.00	2.33	1.00	-1.00	-1.00	-0.00
0.00	-0.67	0.00	1.00	2.33	-1.00	-1.00	-0.00
0.00	-0.00	-0.00	-1.00	-1.00	2.00	0.00	0.00
0.00	-0.00	-0.00	-1.00	-1.00	0.00	2.00	0.00
0.00	-1.00	-1.00	-0.00	-0.00	0.00	0.00	2.00

D

1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	1.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	1.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	1.000	0.250	0.031
0.000	0.000	0.000	0.000	0.000	0.250	1.000	0.031
0.000	0.000	0.000	0.000	0.000	0.031	0.031	1.000

DII

1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	1.07	-0.27	-0.03
0.00	0.00	0.00	0.00	0.00	-0.27	1.07	-0.03
0.00	0.00	0.00	0.00	0.00	-0.03	-0.03	1.00

ZPZ2

5.83	0.00	-2.33	-2.33	0.00	-0.00	-0.00	0.00
0.00	6.42	1.75	0.00	-2.33	-0.00	-0.00	-3.50
-2.33	1.75	6.42	0.00	0.00	-0.00	-0.00	-3.50
-2.33	0.00	0.00	9.17	3.50	-3.50	-3.50	-0.01
0.00	-2.33	0.00	3.50	9.17	-3.50	-3.50	-0.01
0.00	-0.00	-0.00	-3.50	-3.50	8.00	0.00	0.01
0.00	-0.00	-0.00	-3.50	-3.50	0.00	8.00	0.01
0.00	-3.50	-3.50	-0.01	-0.01	0.01	0.01	8.00

WPW2

7.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	8.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	8.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	8.47	-1.86	-0.18
0.00	0.00	0.00	0.00	0.00	-1.86	8.47	-0.18
0.00	0.00	0.00	0.00	0.00	-0.18	-0.18	8.01

RHS
3.20
5.20
0.00
0.00
0.00
1.70
2.00
1.80
1.40
1.50
0.00

0.00
0.00
1.70
2.00
1.80
1.40
1.50

VA	VD	VE	ALPHA	DELTA
20	10	70	3.5	7

SOL		DI	ACC
b1	1.614	0.730	.
b2	1.733	0.579	.
a1	-0.009	0.282	0.121
a2	0.003	0.276	0.181
a3	-0.014	0.283	0.096
a4	-0.009	0.255	0.329
a5	0.024	0.274	0.203
a6	0.015	0.272	0.216
a7	-0.031	0.272	0.216
a8	-0.017	0.263	0.280
d1	0.000	0.143	.
d2	0.000	0.143	.
d3	0.000	0.143	.
d4	0.012	0.135	0.227
d5	0.030	0.132	0.272
d6	-0.002	0.134	0.247
d7	-0.036	0.134	0.247
d8	-0.013	0.135	0.228

EBV		DOM	TOTAL
id1	-0.009	0.000	-0.009
id2	0.003	0.000	0.003
id3	-0.014	0.000	-0.014
id4	-0.009	0.012	0.003
id5	0.024	0.030	0.055
id6	0.015	-0.002	0.013
id7	-0.031	-0.036	-0.067
id8	-0.017	-0.013	-0.030

Animal model with parental subclass effects

```
/* ------------------------------------------ */
/* Animal Model with Parental Subclass Effects */
/* y = Xb + Za + Sf + e                        */
/* Written by Monchai Duangjinda               */
/* ------------------------------------------ */
*================*
 Data Description
*================*
Data file:
ID      sex     bw(kg)
4       1       1.7
5       2       2.0
6       2       1.8
7       2       1.4
8       1       1.5

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       1       0
4       1       0
5       2       0
6       4       5
7       4       5
8       2       3

Where Va = 20 Vd = 10, Ve = 70;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=8.4];
PROC IML;
  X = {1 0,
       0 1,
       0 1,
       0 1,
       1 0};

  Z = {0 0 0 1 0 0 0 0,
       0 0 0 0 1 0 0 0,
       0 0 0 0 0 1 0 0,
       0 0 0 0 0 0 1 0,
       0 0 0 0 0 0 0 1};

  S = {0 0 0 0 0,
       0 0 0 0 0,
       1 0 0 0 0,
       1 0 0 0 0,
       0 1 0 0 0};

  y = {1.7,2.0,1.8,1.4,1.5};
```

```
A = {1   0  .5   .5    0  .25  .25  .25,
     0   1   0    0   .5  .25  .25  .5,
    .5   0   1   .25   0  .125 .125 .5,
    .5   0  .25   1    0  .5   .5   .125,
     0  .5   0    0    1  .5   .5   .25,
    .25 .25 .125 .5   .5   1   .5   .187,
    .25 .25 .125 .5   .5  .5    1   .187,
    .25 .5  .5   .125 .25 .187 .187  1};

F = {1.000 0.124   0   0   0,
     0.124 1.000   0   0   0,
     0         0   0   0   0,
     0         0   0   0   0,
     0         0   0   0   0};

Ai = inv(A);
Fi = ginv(F);
Va = 20; Vf = 2.5; Ve = 77.5;
alpha = Ve/Va;
lamda = Ve/Vf;

* --------- *
  MME Setup
* --------- *;
  XPX  = X`*X;
  XPZ  = X`*Z;
  XPS  = X`*S;
  ZPZ  = Z`*Z;
  ZPS  = Z`*S;
  SPS  = S`*S;
  ZPZ2 = Z`*Z+alpha#Ai;
  SPS2 = S`*S+lamda#Fi;
  lhs = (X`*X || X`*Z          || X`*S)//
        (Z`*X || Z`*Z+alpha#Ai || Z`*S)//
        (S`*X || S`*Z          || S`*S+lamda#Fi);
  rhs = X`*y // Z`*y // S`*y;
  sol = GINV(lhs)*rhs;

* --------------------- *
  Set Label for printing
* --------------------- *;
  label1 = 'b1':'b2';                  /* The levels for fix eff is 2 (sex) */
  label2 = 'a1':'a8';                  /* The levels for animal eff is 8 */
  label3 = 'f1':'f5';                  /* The levels for fullsib eff is 5 */
  label  = label1 || label2 || label3;
  labela = 'id1':'id8';

* --------------- *
  Select solutions
* --------------- *;
  EBV   = sol[3:10];                   /* The levels for EBV start from 3 */
  FS    = sol[11:15];                  /* The levels for FS start from 10 */

* --------------- *
  Compute accuracy
* --------------- *;
  Di  = vecdiag(GINV(lhs));                     /* Select digonal of lhs inverse */
  PEV = Di#Ve;                                  /* Prediction error variance */
  I   = J(15,1,1);                              /* Set unit vector with number of b+u */

  Acc=J(15,1,.);                                             /* Initialize accuracy */
  Acc[3:10,]  = SQRT(I[3:10,]-Di[3:10,]#alpha);              /* BV Accuracy */
  Acc[11:15,] = SQRT(I[11:15,]-Di[11:15,]#lamda);            /* FS Accuracy */
```

```
    Acc[13:15,] =.;                                  /* Set Accuracy for Fs=0 */
    CC=GINV(lhs);

  * ------ *
    Print
  * ------ *;
    PRINT 'Animal Model with fullsib subclass Effects',,
          '[X`X  X`Z          X`S         ][b]   [X`y]',
          '[Z`X  Z`Z+alpha#Ai Z`S         ][a] = [Z`y]',
          '[S`X  S`Z          S`S+lamda#Fi][f] = [S`y]';

    PRINT X&f1, Z&f1, S&f1;                         /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,XPS&f1,ZPZ&f1,ZPS&f1,SPS&f1;
    PRINT A&f3,Ai&f2,F&f3,Fi&f2,ZPZ2&f2,SPS2&f2;
    PRINT rhs&f2;
    PRINT Va Vf Ve ALPHA LAMDA;
    PRINT sol&f3 [rowname=label] Acc&f3;
    PRINT CC&f4;                                        /* Print inverse of LHS */
QUIT;
```

▪ *Animal Model With Dominance Effects (OUTPUT)*

```
Animal Model with fullsib subclass Effects

[X`X  X`Z          X`S         ][b]   [X`y]
[Z`X  Z`Z+alpha#Ai Z`S         ][a] = [Z`y]
[S`X  S`Z          S`S+lamda#Fi][f] = [S`y]

                    X
                    1  0
                    0  1
                    0  1
                    0  1
                    1  0

           Z
           0  0  0  1  0  0  0  0
           0  0  0  0  1  0  0  0
           0  0  0  0  0  1  0  0
           0  0  0  0  0  0  1  0
           0  0  0  0  0  0  0  1

               S
               0  0  0  0  0
               0  0  0  0  0
               1  0  0  0  0
               1  0  0  0  0
               0  1  0  0  0

                   XPX
                    2  0
                    0  3

          XPZ
           0  0  0  1  0  0  0  1
           0  0  0  0  1  1  1  0

              XPS
               0  1  0  0  0
               2  0  0  0  0

          ZPZ
           0  0  0  0  0  0  0  0
           0  0  0  0  0  0  0  0
           0  0  0  0  0  0  0  0
           0  0  0  1  0  0  0  0
           0  0  0  0  1  0  0  0
           0  0  0  0  0  1  0  0
           0  0  0  0  0  0  1  0
           0  0  0  0  0  0  0  1

              ZPS
               0  0  0  0  0
               0  0  0  0  0
               0  0  0  0  0
               0  0  0  0  0
               0  0  0  0  0
               1  0  0  0  0
               1  0  0  0  0
               0  1  0  0  0
```

SPS

2	0	0	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

A

1.000	0.000	0.500	0.500	0.000	0.250	0.250	0.250
0.000	1.000	0.000	0.000	0.500	0.250	0.250	0.500
0.500	0.000	1.000	0.250	0.000	0.125	0.125	0.500
0.500	0.000	0.250	1.000	0.000	0.500	0.500	0.125
0.000	0.500	0.000	0.000	1.000	0.500	0.500	0.250
0.250	0.250	0.125	0.500	0.500	1.000	0.500	0.187
0.250	0.250	0.125	0.500	0.500	0.500	1.000	0.187
0.250	0.500	0.500	0.125	0.250	0.187	0.187	1.000

AI

1.67	0.00	-0.67	-0.67	0.00	-0.00	-0.00	0.00
0.00	1.83	0.50	0.00	-0.67	-0.00	-0.00	-1.00
-0.67	0.50	1.83	0.00	0.00	-0.00	-0.00	-1.00
-0.67	0.00	0.00	2.33	1.00	-1.00	-1.00	-0.00
0.00	-0.67	0.00	1.00	2.33	-1.00	-1.00	-0.00
0.00	-0.00	-0.00	-1.00	-1.00	2.00	0.00	0.00
0.00	-0.00	-0.00	-1.00	-1.00	0.00	2.00	0.00
0.00	-1.00	-1.00	-0.00	-0.00	0.00	0.00	2.00

F

1.000	0.124	0.000	0.000	0.000
0.124	1.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

FI

1.02	-0.13	0.00	0.00	0.00
-0.13	1.02	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00

ZPZ2

6.46	0.00	-2.58	-2.58	0.00	-0.00	-0.00	0.00
0.00	7.10	1.94	0.00	-2.58	-0.00	-0.00	-3.88
-2.58	1.94	7.10	0.00	0.00	-0.00	-0.00	-3.88
-2.58	0.00	0.00	10.04	3.88	-3.88	-3.88	-0.01
0.00	-2.58	0.00	3.88	10.04	-3.88	-3.88	-0.01
0.00	-0.00	-0.00	-3.88	-3.88	8.75	0.00	0.01
0.00	-0.00	-0.00	-3.88	-3.88	0.00	8.75	0.01
0.00	-3.88	-3.88	-0.01	-0.01	0.01	0.01	8.75

SPS2

33.48	-3.90	0.00	0.00	0.00
-3.90	32.48	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00

RHS
3.20
5.20
0.00
0.00
0.00
1.70
2.00
1.80
1.40
1.50
3.20
1.50
0.00
0.00
0.00

VA	VF	VE	ALPHA	LAMDA
20	2.5	77.5	3.875	31

SOL		ACC
b1	1.616	.
b2	1.736	.
a1	-0.009	0.122
a2	0.003	0.183
a3	-0.014	0.097
a4	-0.010	0.332
a5	0.025	0.205
a6	0.015	0.216
a7	-0.031	0.216
a8	-0.017	0.282
f1	-0.008	0.139
f2	-0.004	0.117
f3	0.000	.
f4	0.000	.
f5	0.000	.

Part D

```
/* ----------------------------------------- */
/* Finding relationship using PROC INBREED     */
/* Written by Monchai Duangjinda               */
/* ----------------------------------------- */
DATA one;
  INPUT id s d;
CARDS;
1      .      .
2      .      .
3      1      2
4      1      .
5      4      3
6      5      2
;
PROC INBREED COVAR MATRIX OUTCOV=three NOPRINT;
  VAR id s d;
RUN;
/* ----------------------------------------- */
/* Finding inverse relationship form PROC IML  */
/* Written by Monchai Duangjinda               */
/* ----------------------------------------- */
%LET N = 6;                     /* Number of animal in pedigree */
%LET f4 = [FORMAT=8.4];
PROC IML;
  USE three;
  READ ALL VAR('COL1':'COL6') INTO A;
  Ai = INV(A);
  PRINT 'Finding A inverse using PROC IML';
  PRINT A&f4, Ai&f4;
QUIT;

/* ----------------------------------------- */
/* Finding A inverse (Ignore inbreeding)       */
/* Using decomposition                         */
/* Ignore inbreeding                           */
/* Written by Monchai Duangjinda               */
/* ----------------------------------------- */
PROC IML;
  Ped = {1 0 0,
         2 0 0,
         3 1 2,
         4 1 0,
         5 4 3,
         6 5 2};
  Ti = J(&N,&N,0);                /* create matrix 6x6 */
  Di = J(&N,&N,0);                /* create matrix 6x6 */
  DO i = 1 TO &N;
    s  = Ped[i,2];
    d  = Ped[i,3];
    Ti[i,i] = 1.0;
    IF s > 0 THEN Ti[i,s] = -0.5;
    IF d > 0 THEN Ti[i,d] = -0.5;

    IF s = 0 & d = 0 THEN Di[i,i] = 1;
    IF s > 0 & d = 0 THEN Di[i,i] = 4/3;
    IF s = 0 & d > 0 THEN Di[i,i] = 4/3;
    IF s > 0 & d > 0 THEN Di[i,i] = 2;
  END;
```

```
  Ai = Ti`*Di*Ti;
  PRINT 'Finding A inverse (Ignore inbreeing)';
  PRINT 'Using decomposition';
  PRINT 'Ai = Ti`*Di*Ti';
  PRINT Ped, Ti&f4, Di&f4, Ai&f4;
QUIT;

/* ------------------------------------------ */
/* Finding A inverse (Ignore inbreeding)      */
/* Using rapid method                         */
/* Written by Monchai Duangjinda              */
/* ------------------------------------------ */
PROC IML;
  Ped = {1 0 0,
         2 0 0,
         3 1 2,
         4 1 0,
         5 4 3,
         6 5 2};

  Ai = J(&N,&N,0);                  /* create matrix 6x6 */
  DO i = 1 TO &N;
    s  = Ped[i,2];
    d  = Ped[i,3];
    IF s = 0 & d = 0 THEN Ai[i,i] = 1;
    IF s > 0 & d = 0 THEN
       DO;
         Ai[i,i] = Ai[i,i]+4/3;
         Ai[i,s] = Ai[i,s]-2/3;
         Ai[s,i] = Ai[s,i]-2/3;
         Ai[s,s] = Ai[s,s]+1/3;
       END;
    IF s = 0 & d > 0 THEN
       DO;
         Ai[i,i] = Ai[i,i]+4/3;
         Ai[i,d] = Ai[i,d]-2/3;
         Ai[d,i] = Ai[d,i]-2/3;
         Ai[d,d] = Ai[d,d]+1/3;
       END;
    IF s > 0 & d > 0 THEN
       DO;
         Ai[i,i] = Ai[i,i]+2;
         Ai[i,s] = Ai[i,s]-1;
         Ai[s,i] = Ai[s,i]-1;
         Ai[i,d] = Ai[i,d]-1;
         Ai[d,i] = Ai[d,i]-1;
         Ai[s,s] = Ai[s,s]+1/2;
         Ai[d,d] = Ai[d,d]+1/2;
         Ai[s,d] = Ai[s,d]+1/2;
         Ai[d,s] = Ai[d,s]+1/2;
       END;
   END;

  PRINT 'Finding A inverse (Ignore inbreeding)';
  PRINT 'Using Rapid method';
  PRINT Ai&f4;
QUIT;
```

```
/* -------------------------------------------------- */
/* Finding dominance relationship using PROC INBREED/IML */
/* Written by Monchai Duangjinda                         */
/* -------------------------------------------------- */
DATA one;
  INPUT id s d;
CARDS;
1      .      .
2      .      .
3      .      .
4      1      2
5      1      3
6      1      3
7      4      3
8      4      3
;
PROC INBREED COVAR MATRIX OUTCOV=three NOPRINT;
  VAR id s d;
RUN;
%LET N = 8;                     /* Number of animal in pedigree */
%LET f4 = [FORMAT=8.4];
PROC IML;
  USE three;
  READ ALL VAR('COL1':'COL8') INTO A;
  Ped = {1 0 0,
         2 0 0,
         3 0 0,
         4 1 2,
         5 1 3,
         6 1 3,
         7 4 3,
         8 4 3};
  Dom = J(&N,&N,0);                 /* create matrix 8x8 */
  DO i = 1 TO &N;
    DO j = 1 TO &N;
      si  = Ped[i,2];
      di  = Ped[i,3];
      sj  = Ped[j,2];
      dj  = Ped[j,3];

      IF si > 0 & di >0 & sj > 0 & dj >0 THEN
        DO;
          Dom[i,j] = 1/4*(A[si,sj]*A[di,dj]+A[si,dj]*A[sj,di]);
        END;
      iF i = j THEN Dom[i,j] = 1;
    END;
  END;
  Domi = GINV(Dom);
  PRINT 'Finding Dominance relationship';
  PRINT 'From additive relationship';
  PRINT Ped, Dom&f4, Domi&f4;
QUIT;
```

▪ *Genetic Additive Relationship (OUTPUT)*

Finding A inverse using PROC IML

A

1.0000	0.0000	0.5000	0.5000	0.5000	0.2500
0.0000	1.0000	0.5000	0.0000	0.2500	0.6250
0.5000	0.5000	1.0000	0.2500	0.6250	0.5625
0.5000	0.0000	0.2500	1.0000	0.6250	0.3125
0.5000	0.2500	0.6250	0.6250	1.1250	0.6875
0.2500	0.6250	0.5625	0.3125	0.6875	1.1250

AI

1.8333	0.5000	-1.0000	-0.6667	0.0000	0.0000
0.5000	2.0333	-1.0000	0.0000	0.5333	-1.0667
-1.0000	-1.0000	2.5000	0.5000	-1.0000	0.0000
-0.6667	0.0000	0.5000	1.8333	-1.0000	0.0000
0.0000	0.5333	-1.0000	-1.0000	2.5333	-1.0667
0.0000	-1.0667	0.0000	0.0000	-1.0667	2.1333

Finding A inverse (Ignore inbreeing)
Using decomposition
Ai = Ti`*Di*Ti

PED

1	0	0
2	0	0
3	1	2
4	1	0
5	4	3
6	5	2

TI

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
-0.5000	-0.5000	1.0000	0.0000	0.0000	0.0000
-0.5000	0.0000	0.0000	1.0000	0.0000	0.0000
0.0000	0.0000	-0.5000	-0.5000	1.0000	0.0000
0.0000	-0.5000	0.0000	0.0000	-0.5000	1.0000

DI

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	2.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.3333	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	2.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	2.0000

AI

1.8333	0.5000	-1.0000	-0.6667	0.0000	0.0000
0.5000	2.0000	-1.0000	0.0000	0.5000	-1.0000
-1.0000	-1.0000	2.5000	0.5000	-1.0000	0.0000
-0.6667	0.0000	0.5000	1.8333	-1.0000	0.0000
0.0000	0.5000	-1.0000	-1.0000	2.5000	-1.0000
0.0000	-1.0000	0.0000	0.0000	-1.0000	2.0000

Finding A inverse (Ignore inbreeding)
Using Rapid method

AI

1.8333	0.5000	-1.0000	-0.6667	0.0000	0.0000
0.5000	2.0000	-1.0000	0.0000	0.5000	-1.0000
-1.0000	-1.0000	2.5000	0.5000	-1.0000	0.0000
-0.6667	0.0000	0.5000	1.8333	-1.0000	0.0000
0.0000	0.5000	-1.0000	-1.0000	2.5000	-1.0000
0.0000	-1.0000	0.0000	0.0000	-1.0000	2.0000

▪ *Genetic Dominance Relationship (OUTPUT)*

Finding Dominance relationship
From additive relationship

PED

1	0	0
2	0	0
3	0	0
4	1	2
5	1	3
6	1	3
7	4	3
8	4	3

DOM

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0000	0.2500	0.1250	0.1250
0.0000	0.0000	0.0000	0.0000	0.2500	1.0000	0.1250	0.1250
0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	1.0000	0.2500
0.0000	0.0000	0.0000	0.0000	0.1250	0.1250	0.2500	1.0000

DOMI

1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	1.0833	-0.2500	-0.0833	-0.0833
0.0000	0.0000	0.0000	0.0000	-0.2500	1.0833	-0.0833	-0.0833
0.0000	0.0000	0.0000	0.0000	-0.0833	-0.0833	1.0833	-0.2500
0.0000	0.0000	0.0000	0.0000	-0.0833	-0.0833	-0.2500	1.0833

Part E

Multi-trait Animal Model (Similar Model)

```
/* --------------------------------------- */
/* Multiple Trait Animal Model              */
/* (Similar Model)                          */
/* y1 = X1b1 + Z1u1 + e1                    */
/* y2 = X1b2 + Z1u2 + e2                    */
/* Written by Monchai Duangjinda            */
/* --------------------------------------- */

*================*
 Data Description
*================*
Data file:
id     sex     bw(kg)  adg(kg)
2      1       40      0.9
3      2       50      0.8
4      2       45      1.0
5      1       50      1.2

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       2       1
4       2       1
5       0       4

Where Va1 =  5 Va2 = 0.2 Cov(a1,a2) = 2
      Ve1 = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=4.2];
%LET f5 = [FORMAT=8.4];

PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        1 0};
  /* Use same sex as cg for both trait */
  X2 = X1;

  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 1};

  Z2 = Z1;

  y1 = {40,50,45,50};
  y2 = {0.9,0.8,1.0,1.2};
```

```
A = { 1   0  .5  .5 .25,
      0   1  .5  .5 .25,
     .5  .5   1  .5 .25,
     .5  .5  .5   1 .5,
     .25 .25 .25 .5  1};

Ai = inv(A);

Va1 =  5; Va2 = 0.2; Va12 = 2;
Ve1 = 10; Ve2 = 0.5; Ve12 = -0.5;

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);

Gi = INV(G);
Ri = INV(R);

G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];

R11 = Ri[1,1];
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2];

* --------- *
  MME Setup
* --------- *;

  X1PX1 = X1`*R11*X1;
  X1PX2 = X1`*R12*X2;
  X1PZ1 = X1`*R11*Z1;
  X1PZ2 = X1`*R12*Z2;
  X2PX2 = X2`*R22*X2;
  X2PZ1 = X2`*R21*Z1;
  X2PZ2 = X2`*R22*Z2;
  Z1PZ1 = Z1`*R11*Z1;
  Z1PZ2 = Z1`*R12*Z2;
  Z2PZ2 = Z2`*R22*Z2;


  lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2       )//
        (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2       )//
        (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
        (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);

  rhs = (X1`*R11*y1+X1`*R12*y2)//
        (X2`*R21*y1+X2`*R22*y2)//
        (Z1`*R11*y1+Z1`*R12*y2)//
        (Z2`*R21*y1+Z2`*R22*y2);

  sol = GINV(lhs)*rhs;

* ---------------- *
  Compute accuracy
* ---------------- *;
  Di  = vecdiag(GINV(lhs));                           /* Select digonal of lhs inverse */
  PEV = Di;                                           /* Prediction error variance */
```

```
    I   = J(14,1,1);                            /* Set unit vector with number of b+u */
    Acc=J(14,1,.);                                      /* Initialize accuracy */
    Acc[5:9,]   = SQRT((I[5:9,]#Va1-Di[5:9,])/Va1);     /* BV Accuracy for t1*/
    Acc[10:14,] = SQRT((I[10:14,]#Va2-Di[10:14,])/Va2); /* BV Accuracy for t2*/
    CC=INV(lhs);

  * ---------------------- *
    Set Label for printing
  * ---------------------- *;
    label1 = 'b1':'b2';             /* The levels for fix eff is 2 (sex) */
    label2 = 'u1':'u5';             /* The levels for animal eff is 5 */
    label  = label1 || label1 || label2 || label2;

    labelt = REPEAT('t1',2)//REPEAT('t2',2)//
             REPEAT('t1',5)//REPEAT('t2',5);
    /* The levels for trait 1 and 2 for 2 sex */
    /* The levels for trait 1 and 2 for 5 animals */

  * ------ *
    Print
  * ------ *;

    PRINT 'Multiple Trait Animal Model',
          '(Similar Model)',,
          'LHS',
          '[r11*X1`*X1 r12*X1`*X2 r11*X1`*Z1        r12*X1`*Z2       ][b1]   ',
          '[r21*X2`*X1 r22*X2`*X2 r21*X2`*Z1        r22*X2`*Z2       ][b2] = ',
          '[r11*Z1`*X1 r12*Z1`*X2 r11*Z1`*Z1+g11*Ai r12*Z1`*Z2+g12*Ai][u1]   ',
          '[r21*Z2`*X1 r22*Z2`*X2 r21*Z2`*Z1+g21*Ai r22*Z2`*Z2+g22*Ai][u2]   ',,

          'RHS',
          '[r11*X1`*y1+r12*X1`*y2]',
          '[r21*X2`*y1+r22*X2`*y2]',
          '[r11*Z1`*y1+r12*Z1`*y2]',
          '[r21*Z2`*y1+r22*Z2`*y2]';

    PRINT X1&f1,X2&f1,Z1&f1,Z2&f1;                      /* Print matrix using f1 format */
    PRINT X1PX1&f4,X1PX2&f4,X1PZ1&f4,X1PZ2&f4,
          X2PX2&f4,X2PZ1&f4,X2PZ2&f4,
          Z1PZ1&f4,Z1PZ2&f4,Z2PZ2&f4;
    PRINT A&f2,Ai&f2;
    PRINT rhs&f2;
    PRINT G R,
          Gi&f3 Ri&f3;
    PRINT labelt sol&f3 [rowname=label] Di&f3 ACC&f3;
QUIT;
```

▪ *Multi-trait Animal Model-Similar Model (OUTPUT)*

```
                      Multiple Trait Animal Model
                            (Similar Model)

                                  LHS
[r11*X1`*X1 r12*X1`*X2 r11*X1`*Z1        r12*X1`*Z2       ][b1]
[r21*X2`*X1 r22*X2`*X2 r21*X2`*Z1        r22*X2`*Z2       ][b2] =
[r11*Z1`*X1 r12*Z1`*X2 r11*Z1`*Z1+g11*Ai r12*Z1`*Z2+g12*Ai][u1]
[r21*Z2`*X1 r22*Z2`*X2 r21*Z2`*Z1+g21*Ai r22*Z2`*Z2+g22*Ai][u2]

                                  RHS
                      [r11*X1`*y1+r12*X1`*y2]
                      [r21*X2`*y1+r22*X2`*y2]
                      [r11*Z1`*y1+r12*Z1`*y2]
                      [r21*Z2`*y1+r22*Z2`*y2]


                                 X1
                                  1  0
                                  0  1
                                  0  1
                                  1  0



                                 X2
                                  1  0
                                  0  1
                                  0  1
                                  1  0



                            Z1
                             0  1  0  0  0
                             0  0  1  0  0
                             0  0  0  1  0
                             0  0  0  0  1



                            Z2
                             0  1  0  0  0
                             0  0  1  0  0
                             0  0  0  1  0
                             0  0  0  0  1


                               X1PX1
                               0.21 0.00
                               0.00 0.21



                               X1PX2
                               0.21 0.00
                               0.00 0.21
```

X1PZ1

0.00	0.11	0.00	0.00	0.11
0.00	0.00	0.11	0.11	0.00

X1PZ2

0.00	0.11	0.00	0.00	0.11
0.00	0.00	0.11	0.11	0.00

X2PX2

4.21	0.00
0.00	4.21

X2PZ1

0.00	0.11	0.00	0.00	0.11
0.00	0.00	0.11	0.11	0.00

X2PZ2

0.00	2.11	0.00	0.00	2.11
0.00	0.00	2.11	2.11	0.00

Z1PZ1

0.00	0.00	0.00	0.00	0.00
0.00	0.11	0.00	0.00	0.00
0.00	0.00	0.11	0.00	0.00
0.00	0.00	0.00	0.11	0.00
0.00	0.00	0.00	0.00	0.11

Z1PZ2

0.00	0.00	0.00	0.00	0.00
0.00	0.11	0.00	0.00	0.00
0.00	0.00	0.11	0.00	0.00
0.00	0.00	0.00	0.11	0.00
0.00	0.00	0.00	0.00	0.11

Z2PZ2

0.00	0.00	0.00	0.00	0.00
0.00	2.11	0.00	0.00	0.00
0.00	0.00	2.11	0.00	0.00
0.00	0.00	0.00	2.11	0.00
0.00	0.00	0.00	0.00	2.11

A

1.00	0.00	0.50	0.50	0.25
0.00	1.00	0.50	0.50	0.25
0.50	0.50	1.00	0.50	0.25
0.50	0.50	0.50	1.00	0.50
0.25	0.25	0.25	0.50	1.00

AI				
2.00	1.00	-1.00	-1.00	0.00
1.00	2.00	-1.00	-1.00	0.00
-1.00	-1.00	2.00	0.00	0.00
-1.00	-1.00	0.00	2.33	-0.67
0.00	0.00	0.00	-0.67	1.33

RHS
9.69
10.19
13.89
13.79
0.00
4.31
5.35
4.84
5.39
0.00
6.11
6.95
6.84
7.79

G		R	
5	2	10	-0.5
2	0.2	-0.5	0.5

GI		RI	
-0.067	0.667	0.105	0.105
0.667	-1.667	0.105	2.105

LABELT	SOL		DI	ACC
t1	b1	45.088	8.065	.
t1	b2	47.671	8.699	.
t2	b1	1.100	0.372	.
t2	b2	0.991	0.398	.
t1	u1	0.342	4.796	0.202
t1	u2	-1.027	3.166	0.606
t1	u3	0.011	3.910	0.467
t1	u4	-0.354	4.042	0.438
t1	u5	0.850	3.034	0.627
t2	u1	0.181	0.191	0.216
t2	u2	-0.543	0.116	0.648
t2	u3	0.021	0.150	0.499
t2	u4	-0.202	0.156	0.467
t2	u5	0.442	0.110	0.672

Multi-trait Animal Model (Different Model)

```
/* --------------------------------------- */
/* Multiple Trait Animal Model              */
/* (Different Model)                        */
/* y1 = Xb1 + Zu1 + Wc1 + e1               */
/* y2 = Xb2 + Zu2 + e2                      */
/* Written by Monchai Duangjinda            */
/* --------------------------------------- */
*================*
 Data Description
*================*
Data file:
id     sex    season    dam      bw(kg)   adg(kg)
2      1        1       0        40       0.9
3      2        1       1        50       0.8
4      2        2       3        45       1.0
5      1        3       0        50       1.2

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       2       1
4       2       1
5       0       4

Where Va1  = 5 Va2 = 0.2 Cov(a1,a2) = 2
      Vc1 = 2
      Ve1  = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=4.2];

PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        1 0};
  X2 = {1 0 0,
        1 0 0,
        0 1 0,
        0 0 1};
  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 1};
  Z2 = Z1;
  W1 = {0 0 0 0 0,
        1 0 0 0 0,
        0 0 1 0 0,
        0 0 0 0 0};
  y1 = {40,50,45,50};
  y2 = {0.9,0.8,1.0,1.2};
```

```
    A = { 1   0  .5  .5 .25,
          0   1  .5  .5 .25,
         .5  .5   1  .5 .25,
         .5  .5  .5   1 .5,
         .25 .25 .25 .5  1};

    Ai = inv(A);

    Va1  = 5; Va2  = 0.2; Va12  = 2;
    Vc1 = 2;
    Ve1  = 10; Ve2  = 0.5; Ve12  = -0.5;

    G  = (Va1  || Va12)//
         (Va12 || Va2);

    R  = (Ve1  || Ve12)//
         (Ve12 || Ve2);

    Gi = INV(G);
    Ri = INV(R);

    G11 = Gi[1,1];
    G12 = Gi[1,2];
    G21 = G12;
    G22 = Gi[2,2];

    R11 = Ri[1,1];
    R12 = Ri[1,2];
    R21 = R12;
    R22 = Ri[2,2];

* --------- *
  MME Setup
* --------- *;

     X1PX1 = X1`*R11*X1;
     X1PX2 = X1`*R12*X2;
     X1PZ1 = X1`*R11*Z1;
     X1PZ2 = X1`*R12*Z2;
     X1PW1 = X1`*R11*W1;

     X2PX2 = X2`*R22*X2;
     X2PZ1 = X2`*R21*Z1;
     X2PZ2 = X2`*R22*Z2;
     X2PW1 = X2`*R21*W1;

     Z1PZ1 = Z1`*R11*Z1;
     Z1PZ2 = Z1`*R12*Z2;
     Z1PW1 = Z1`*R11*W1;
     Z2PZ2 = Z2`*R22*Z2;
     Z2PW1 = Z2`*R21*W1;
     W1PW1 = W1`*R11*W1;


lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2         || X1`*R11*W1)//
      (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2         || X2`*R21*W1)//
      (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai  || Z1`*R11*W1)//
      (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai  || Z2`*R21*W1)//
      (W1`*R11*X1 || W1`*R12*X2 || W1`*R11*Z1        || W1`*R12*Z2         ||
       W1`*R11*W1+1/Vc1#I(5));
```

```
    rhs = (X1`*R11*y1+X1`*R12*y2)//
          (X2`*R21*y1+X2`*R22*y2)//
          (Z1`*R11*y1+Z1`*R12*y2)//
          (Z2`*R21*y1+Z2`*R22*y2)//
          (W1`*R11*y1+W1`*R12*y2);

    sol = GINV(lhs)*rhs;

  * ---------------- *
    Compute accuracy
  * ---------------- *;
    Di  = vecdiag(GINV(lhs));                    /* Select digonal of lhs inverse */
    PEV = Di;                                    /* Prediction error variance */
    I   = J(20,1,1);                             /* Set unit vector with number of b+u */
    Acc=J(20,1,.);                                     /* Initialize accuracy */
    Acc[6:10,]   = SQRT((I[6:10,]#Va1-Di[6:10,])/Va1);  /* BV Accuracy for t1*/
    Acc[11:15,] = SQRT((I[11:15,]#Va2-Di[11:15,])/Va2); /* BV Accuracy for t2*/
    CC=INV(lhs);

  * ---------------------- *
    Set Label for printing
  * ---------------------- *;
    label1 = 'b1':'b2';                                /* The levels for cg1 is 2 */
    label2 = 'b1':'b3';                                /* The levels for cg2 is 3 */
    label3 = 'u1':'u5';                                /* The levels for animal eff is 5 */
    label4 = 'c1':'c5';
    label  = label1 || label2 || label3 || label3 || label4;

    labelt = REPEAT('t1',2)//REPEAT('t2',3)//
             REPEAT('t1',5)//REPEAT('t2',5)//
             REPEAT('t1',5);

  * ------ *
    Print
  * ------ *;

  PRINT 'Multiple Trait Animal Model',
        '(Different Model)',,
        'LHS',
        '[r11*X1`X1 r12*X1`X2 r11*X1`Z1        r12*X1`Z2        r11*X1`W1       ][b1 ]   ',
        '[r21*X2`X1 r22*X2`X2 r21*X2`Z1        r22*X2`Z2        r21*X2`W1       ][b2 ] = ',
        '[r11*Z1`X1 r12*Z1`X2 r11*Z1`Z1+G11*Ai r12*Z1`Z2+G12*Ai r11*Z1`W1       ][u1 ]   ',
        '[r21*Z2`X1 r22*Z2`X2 r21*Z2`Z1+G21*Ai r22*Z2`Z2+G22*Ai r21*Z2`W1       ][u2 ]   ',
        '[r11*W1`X1 r12*W1`X2 r11*W1`Z1        r12*W1`Z2        r11*W1`W1+1/Vc*I][c1]   ',,

        'RHS',
        '[r11*X1`y1+r12*X1`y2]',
        '[r21*X2`y1+r22*X2`y2]',
        '[r11*Z1`y1+r12*Z1`y2]',
        '[r21*Z2`y1+r22*Z2`y2]',
        '[r11*W1`y1+r12*W1`y2]';

    PRINT X1&f1,X2&f1,Z1&f1,Z2&f1,W1&f1;                /* Print matrix using f1 format */
    PRINT X1PX1&f4,X1PX2&f4,X1PW1&f4,X1PZ1&f4,X1PZ2&f4,X1PW1&f4,
          X2PX2&f4,X2PZ1&f4,X2PW1&f4,X2PZ2&f4,X2PW1&f4,
          Z1PZ1&f4,Z1PZ2&f4,Z1PW1&f4,Z2PZ2&f4,Z2PW1&f4,W1PW1;
    PRINT A&f2,Ai&f2;
    PRINT rhs&f2;
    PRINT G R, Vc1,
          Gi&f3 Ri&f3;
    PRINT labelt sol&f3 [rowname=label] Di&f3 ACC&f3;
QUIT;
```

▪ *Multi-trait Animal Model-Different Model (OUTPUT)*

```
                     Multiple Trait Animal Model
                          (Different Model)

                               LHS
[r11*X1`X1 r12*X1`X2 r11*X1`Z1        r12*X1`Z2        r11*X1`W1        ][b1]
[r21*X2`X1 r22*X2`X2 r21*X2`Z1        r22*X2`Z2        r21*X2`W1        ][b2] =
[r11*Z1`X1 r12*Z1`X2 r11*Z1`Z1+G11*Ai r12*Z1`Z2+G12*Ai r11*Z1`W1        ][u1]
[r21*Z2`X1 r22*Z2`X2 r21*Z2`Z1+G21*Ai r22*Z2`Z2+G22*Ai r21*Z2`W1        ][u2]
[r11*W1`X1 r12*W1`X2 r11*W1`Z1        r12*W1`Z2        r11*W1`W1+1/Vc*I][pe1]

                               RHS
                      [r11*X1`y1+r12*X1`y2]
                      [r21*X2`y1+r22*X2`y2]
                      [r11*Z1`y1+r12*Z1`y2]
                      [r21*Z2`y1+r22*Z2`y2]
                      [r11*W1`y1+r12*W1`y2]

                                X1
                                 1  0
                                 0  1
                                 0  1
                                 1  0

                              X2
                               1  0  0
                               1  0  0
                               0  1  0
                               0  0  1

                           Z1
                            0  1  0  0  0
                            0  0  1  0  0
                            0  0  0  1  0
                            0  0  0  0  1

                           Z2
                            0  1  0  0  0
                            0  0  1  0  0
                            0  0  0  1  0
                            0  0  0  0  1

                           W1
                            0  0  0  0  0
                            1  0  0  0  0
                            0  0  1  0  0
                            0  0  0  0  0

                              X1PX1
                              0.21 0.00
                              0.00 0.21

                           X1PX2
                           0.11 0.00 0.11
                           0.11 0.11 0.00

                      X1PW1
                      0.00 0.00 0.00 0.00 0.00
                      0.11 0.00 0.11 0.00 0.00
```

```
X1PZ1
0.00 0.11 0.00 0.00 0.11
0.00 0.00 0.11 0.11 0.00

X1PZ2
0.00 0.11 0.00 0.00 0.11
0.00 0.00 0.11 0.11 0.00

X1PW1
0.00 0.00 0.00 0.00 0.00
0.11 0.00 0.11 0.00 0.00

     X2PX2
     4.21 0.00 0.00
     0.00 2.11 0.00
     0.00 0.00 2.11

X2PZ1
0.00 0.11 0.11 0.00 0.00
0.00 0.00 0.00 0.11 0.00
0.00 0.00 0.00 0.00 0.11

X2PW1
0.11 0.00 0.00 0.00 0.00
0.00 0.00 0.11 0.00 0.00
0.00 0.00 0.00 0.00 0.00

X2PZ2
0.00 2.11 2.11 0.00 0.00
0.00 0.00 0.00 2.11 0.00
0.00 0.00 0.00 0.00 2.11

X2PW1
0.11 0.00 0.00 0.00 0.00
0.00 0.00 0.11 0.00 0.00
0.00 0.00 0.00 0.00 0.00

Z1PZ1
0.00 0.00 0.00 0.00 0.00
0.00 0.11 0.00 0.00 0.00
0.00 0.00 0.11 0.00 0.00
0.00 0.00 0.00 0.11 0.00
0.00 0.00 0.00 0.00 0.11

Z1PZ2
0.00 0.00 0.00 0.00 0.00
0.00 0.11 0.00 0.00 0.00
0.00 0.00 0.11 0.00 0.00
0.00 0.00 0.00 0.11 0.00
0.00 0.00 0.00 0.00 0.11

Z1PW1
0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00
0.11 0.00 0.00 0.00 0.00
0.00 0.00 0.11 0.00 0.00
0.00 0.00 0.00 0.00 0.00
```

Z2PZ2

0.00	0.00	0.00	0.00	0.00
0.00	2.11	0.00	0.00	0.00
0.00	0.00	2.11	0.00	0.00
0.00	0.00	0.00	2.11	0.00
0.00	0.00	0.00	0.00	2.11

Z2PW1

0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00
0.11	0.00	0.00	0.00	0.00
0.00	0.00	0.11	0.00	0.00
0.00	0.00	0.00	0.00	0.00

W1PW1

0.1052632	0	0	0	0
0	0	0	0	0
0	0	0.1052632	0	0
0	0	0	0	0
0	0	0	0	0

A

1.00	0.00	0.50	0.50	0.25
0.00	1.00	0.50	0.50	0.25
0.50	0.50	1.00	0.50	0.25
0.50	0.50	0.50	1.00	0.50
0.25	0.25	0.25	0.50	1.00

AI

2.00	1.00	-1.00	-1.00	0.00
1.00	2.00	-1.00	-1.00	0.00
-1.00	-1.00	2.00	0.00	0.00
-1.00	-1.00	0.00	2.33	-0.67
0.00	0.00	0.00	-0.67	1.33

RHS
9.69
10.19
13.05
6.84
7.79
0.00
4.31
5.35
4.84
5.39
0.00
6.11
6.95
6.84
7.79
5.35
0.00
4.84
0.00
0.00

G		R	
5	2	10	-0.5
2	0.2	-0.5	0.5

VC1
2

GI		RI	
-0.067	0.667	0.105	0.105
0.667	-1.667	0.105	2.105

LABELT	SOL		DI	ACC
t1	b1	45.041	8.063	.
t1	b2	47.817	9.678	.
t2	b1	1.085	0.378	.
t2	b2	1.097	0.691	.
t2	b3	0.919	0.656	.
t1	u1	0.151	4.136	0.416
t1	u2	-1.097	3.758	0.498
t1	u3	-0.309	3.926	0.463
t1	u4	-0.486	4.782	0.209
t1	u5	1.176	4.446	0.333
t2	u1	0.157	0.184	0.283
t2	u2	-0.535	0.114	0.657
t2	u3	-0.027	0.151	0.496
t2	u4	-0.194	0.165	0.417
t2	u5	0.470	0.111	0.666
t1	c1	0.389	1.861	.
t1	c2	0.000	2.000	.
t1	c3	-0.389	1.861	.
t1	c4	0.000	2.000	.
t1	c5	0.000	2.000	.

```
/* ---------------------------------------- */
/* Multiple Trait Animal Model              */
/* (Similar Model with Missing Traits)      */
/* y1 = Xb1 + Zu1 + e1                      */
/* y2 = Xb2 + Zu2 + e2                      */
/* Written by Monchai Duangjinda            */
/* ---------------------------------------- */

*================*
 Data Description
*================*
Data file:
id     sex     bw(kg)   yeargain (kg)
2      1       40      1.9
3      2       50      1.8
4      2       45       .
5      1       .       1.2

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       2       1
4       2       1
5       0       4

Where Va1 =  5 Va2 = 0.2 Cov(a1,a2) = 2
      Ve1 = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=4.3];

PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        0 0};

  X2 = {1 0,
        0 1,
        0 0,
        1 0};

  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 0};

  Z2 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 0 0,
        0 0 0 0 1};
```

```
y1 = {40,50,45,0};
y2 = {1.9,1.8,0,1.2};

A = { 1   0  .5  .5 .25,
      0   1  .5  .5 .25,
     .5  .5   1  .5 .25,
     .5  .5  .5   1 .5,
     .25 .25 .25 .5  1};

Ai = inv(A);
Va1 =  5; Va2 = 0.2; Va12 = 2;
Ve1 = 10; Ve2 = 0.5; Ve12 = -0.5;

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);

Gi = INV(G);
Ri = INV(R);
G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];

/* Create Residual variance structure for 4 records */
R11 = Ri[1,1]*I(4);
R12 = Ri[1,2]*I(4);
R21 = R12;
R22 = Ri[2,2]*I(4);

/* Change some residual variance with missing trait residual variance */
/* Inverse of Residual variance for missing traits*/
Ri0 = I(2);
Ri0[1,1] = 1/Ve1;
Ri0[2,2] = 1/Ve2;

/* records 3 of trait 1 missing trait 2 */
R11[3,3] = 1/Ve1;
/* records 4 of trait 2 missing trait 1 */
R22[4,4] = 1/Ve2;
/* records 4 of trait 1 is missing */
R11[4,4] = 0;
R12[4,4] = 0;
/* records 3 of trait 2 is missing */
R22[3,3] = 0;
R12[3,3] = 0;

* --------- *
  MME Setup
* --------- *;
  X1PX1 = X1`*R11*X1;
  X1PX2 = X1`*R12*X2;
  X1PZ1 = X1`*R11*Z1;
  X1PZ2 = X1`*R12*Z2;
  X2PX2 = X2`*R22*X2;
  X2PZ1 = X2`*R21*Z1;
  X2PZ2 = X2`*R22*Z2;
  Z1PZ1 = Z1`*R11*Z1;
  Z1PZ2 = Z1`*R12*Z2;
  Z2PZ2 = Z2`*R22*Z2;
```

```
    lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2       )//
          (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2       )//
          (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
          (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);

    rhs = (X1`*R11*y1+X1`*R12*y2)//
          (X2`*R21*y1+X2`*R22*y2)//
          (Z1`*R11*y1+Z1`*R12*y2)//
          (Z2`*R21*y1+Z2`*R22*y2);

    sol = GINV(lhs)*rhs;

  * ---------------- *
    Compute accuracy
  * ---------------- *;
    Di  = vecdiag(GINV(lhs));                   /* Select digonal of lhs inverse */
    PEV = Di;                                   /* Prediction error variance */
    I   = J(14,1,1);                            /* Set unit vector with number of b+u */
    Acc=J(14,1,.);                                      /* Initialize accuracy */
    Acc[5:9,]   = SQRT((I[5:9,]#Va1-Di[5:9,])/Va1);     /* BV Accuracy for t1*/
    Acc[10:14,] = SQRT((I[10:14,]#Va2-Di[10:14,])/Va2); /* BV Accuracy for t2*/
    CC=INV(lhs);

  * ---------------------- *
    Set Label for printing
  * ---------------------- *;
    label1 = 'b1':'b2';                         /* The levels for fix eff is 2 (sex) */
    label2 = 'u1':'u5';                         /* The levels for animal eff is 5 */
    label  = label1 || label1 || label2 || label2;

    labelt = REPEAT('t1',2)//REPEAT('t2',2)//   /* The levels for trait 1 and 2 for sex */
             REPEAT('t1',5)//REPEAT('t2',5);    /* The levels for trait 1 and 2 for animals*/

  * ------ *
    Print
  * ------ *;

    PRINT 'Multiple Trait Animal Model',
          '(Similar Model with Missing Trait)',,
          'LHS',
          '[X1`*R11*X1 X1`*R12*X2 X1`*R11*Z1        X1`*R12*Z2       ][b1]   ',
          '[X2`*R21*X1 X2`*R22*X2 X2`*R21*Z1        X2`*R22*Z2       ][b2] = ',
          '[Z1`*R11*X1 Z1`*R12*X2 Z1`*R11*Z1+G11*Ai Z1`*R12*Z2+G12*Ai][u1]   ',
          '[Z2`*R21*X1 Z2`*R22*X2 Z2`*R21*Z1+G21*Ai Z2`*R22*Z2+G22*Ai][u2]   ',,

          'RHS',
          '[X1`*R11*y1+X1`*R12*y2]',
          '[X2`*R21*y1+X2`*R22*y2]',
          '[Z1`*R11*y1+Z1`*R12*y2]',
          '[Z2`*R21*y1+Z2`*R22*y2]';

    PRINT X1&f1,X2&f1,Z1&f1,Z2&f1;              /* Print matrix using f1 format */
    PRINT R11&f4,R12&f4,R22&f4;
    PRINT X1PX1&f4,X1PX2&f4,X1PZ1&f4,X1PZ2&f4,
          X2PX2&f4,X2PZ1&f4,X2PZ2&f4,
          Z1PZ1&f4,Z1PZ2&f4,Z2PZ2&f4;
    PRINT A&f2,Ai&f2;
    PRINT rhs&f2;
    PRINT G R,
          Gi&f3 Ri&f3 Ri0&f3;
    PRINT labelt sol&f3 [rowname=label] Di&f3 ACC&f3;
QUIT;
```

▪ *Multi-trait Animal Model-Missing Trait Model (OUTPUT)*

```
                         The SAS System
                   Multiple Trait Animal Model
              (Similar Model with Missing Trait)

                               LHS
[X1`*R11*X1 X1`*R12*X2 X1`*R11*Z1        X1`*R12*Z2       ][b1]
[X2`*R21*X1 X2`*R22*X2 X2`*R21*Z1        X2`*R22*Z2       ][b2] =
[Z1`*R11*X1 Z1`*R12*X2 Z1`*R11*Z1+G11*Ai Z1`*R12*Z2+G12*Ai][u1]
[Z2`*R21*X1 Z2`*R22*X2 Z2`*R21*Z1+G21*Ai Z2`*R22*Z2+G22*Ai][u2]

                               RHS
                      [X1`*R11*y1+X1`*R12*y2]
                      [X2`*R21*y1+X2`*R22*y2]
                      [Z1`*R11*y1+Z1`*R12*y2]
                      [Z2`*R21*y1+Z2`*R22*y2]

                              X1
                               1  0
                               0  1
                               0  1
                               0  0

                              X2
                               1  0
                               0  1
                               0  0
                               1  0

                         Z1
                          0  1  0  0  0
                          0  0  1  0  0
                          0  0  0  1  0
                          0  0  0  0  0

                         Z2
                          0  1  0  0  0
                          0  0  1  0  0
                          0  0  0  0  0
                          0  0  0  0  1

                        R11
                       .105 .000 .000 .000
                       .000 .105 .000 .000
                       .000 .000 .100 .000
                       .000 .000 .000 .000

                        R12
                       .105 .000 .000 .000
                       .000 .105 .000 .000
                       .000 .000 .000 .000
                       .000 .000 .000 .000

                        R22
                       2.11 .000 .000 .000
                       .000 2.11 .000 .000
                       .000 .000 .000 .000
                       .000 .000 .000 2.00
```

X1PX1

.105	.000
.000	.205

X1PX2

.105	.000
.000	.105

X1PZ1

.000	.105	.000	.000	.000
.000	.000	.105	.100	.000

X1PZ2

.000	.105	.000	.000	.000
.000	.000	.105	.000	.000

X2PX2

4.11	.000
.000	2.11

X2PZ1

.000	.105	.000	.000	.000
.000	.000	.105	.000	.000

X2PZ2

.000	2.11	.000	.000	2.00
.000	.000	2.11	.000	.000

Z1PZ1

.000	.000	.000	.000	.000
.000	.105	.000	.000	.000
.000	.000	.105	.000	.000
.000	.000	.000	.100	.000
.000	.000	.000	.000	.000

Z1PZ2

.000	.000	.000	.000	.000
.000	.105	.000	.000	.000
.000	.000	.105	.000	.000
.000	.000	.000	.000	.000
.000	.000	.000	.000	.000

Z2PZ2

.000	.000	.000	.000	.000
.000	2.11	.000	.000	.000
.000	.000	2.11	.000	.000
.000	.000	.000	.000	.000
.000	.000	.000	.000	2.00

A

1.00	0.00	0.50	0.50	0.25
0.00	1.00	0.50	0.50	0.25
0.50	0.50	1.00	0.50	0.25
0.50	0.50	0.50	1.00	0.50
0.25	0.25	0.25	0.50	1.00

AI

2.00	1.00	-1.00	-1.00	0.00
1.00	2.00	-1.00	-1.00	0.00
-1.00	-1.00	2.00	0.00	0.00
-1.00	-1.00	0.00	2.33	-0.67
0.00	0.00	0.00	-0.67	1.33

RHS
4.41
9.95
10.61
9.05
0.00
4.41
5.45
4.50
0.00
0.00
8.21
9.05
0.00
2.40

G		R	
5	2	10	-0.5
2	0.2	-0.5	0.5

GI		RI		RI0	
-0.067	0.667	0.105	0.105	0.100	0.000
0.667	-1.667	0.105	2.105	0.000	2.000

LABELT	SOL		DI	ACC
t1	b1	39.535	14.225	.
t1	b2	47.384	8.702	.
t2	b1	1.598	0.372	.
t2	b2	1.681	0.689	.
t1	u1	-0.233	4.806	0.197
t1	u2	0.698	3.256	0.591
t1	u3	0.709	4.593	0.285
t1	u4	-0.477	4.748	0.224
t1	u5	-0.936	3.251	0.591
t2	u1	-0.023	0.198	0.098
t2	u2	0.070	0.183	0.295
t2	u3	0.214	0.159	0.451
t2	u4	-0.191	0.160	0.449
t2	u5	-0.165	0.175	0.355

Multi-trait Animal Model (Sex-limited Model)

```
/* --------------------------------------- */
/* Multiple Trait Animal Model             */
/* (Sex limit Model)                       */
/* y1 = Xb1 + Zu1 + e1                     */
/* y2 = Xb2 + Zu2 + e2                     */
/* Written by Monchai Duangjinda           */
/* --------------------------------------- */
*================*
 Data Description
*================*
Data file: (Dual purpose records)
id      sex     herd    ww(kg) milk(kg)
3       m       1       350       -
4       m       2       250       -
5       m       2       300       -
6       m       2       450       -
7       f       1       -         20
8       f       1       -         16
9       f       2       -         15
10      f       2       -         25

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       0       0
4       1       0
5       1       2
6       3       2
7       4       0
8       4       0
9       0       7
10      6       7

Where Va1 = 40 Va2 = 20 Cov(a1,a2) =  5
      Ve1 = 80 Ve2 = 50 Cov(e1,e2) =  0;
/* Cov(e1,e2) need to set to zero since data are from different environment */

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=4.2];

PROC IML;
  /* Use herd as fixed effects */
  X1 = {1 0,
        0 1,
        0 1,
        0 1,
        0 0,
        0 0,
        0 0,
        0 0};
```

```
X2 = {0 0,
      0 0,
      0 0,
      0 0,
      1 0,
      1 0,
      0 1,
      0 1};

Z1 = {0 0 1 0 0 0 0 0 0 0,
      0 0 0 1 0 0 0 0 0 0,
      0 0 0 0 1 0 0 0 0 0,
      0 0 0 0 0 1 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0};

Z2 = {0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 0 0 0 0,
      0 0 0 0 0 0 1 0 0 0,
      0 0 0 0 0 0 0 1 0 0,
      0 0 0 0 0 0 0 0 1 0,
      0 0 0 0 0 0 0 0 0 1};
y1 = {350,250,300,450,0,0,0,0};
y2 = {0,0,0,0,20,16,15,25};
A = {1    0    0  .5   .5    0   .25  .25   .125 .125,
     0    1    0   0   .5   .5    0    0     0   .25,
     0    0    1   0    0   .5    0    0     0   .25,
    .5    0    0   1   .25   0   .5   .5    .25  .25,
    .5   .5    0  .25   1   .25  .125 .125  .063 .188,
     0   .5   .5   0   .25   1    0    0     0   .5,
    .25   0    0  .5   .125  0    1   .25   .5   .5,
    .25   0    0  .5   .125  0   .25   1    .125 .125,
    .125  0    0  .25  .063  0   .5   .125   1   .25,
    .125 .25  .25 .25  .188 .5   .5   .125  .25   1};

Ai = inv(A);
Va1 = 40; Va2 = 20; Va12 = 5;
Ve1 = 80; Ve2 = 50; Ve12 = 0;
G  = (Va1  || Va12)//
     (Va12 || Va2);
R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);
Gi = INV(G);
Ri = INV(R);
G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];
R11 = Ri[1,1];
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2];
* --------- *
  MME Setup
* --------- *;
  X1PX1 = X1`*R11*X1;
  X1PX2 = X1`*R12*X2;
  X1PZ1 = X1`*R11*Z1;
  X1PZ2 = X1`*R12*Z2;
```

```
    X2PX2 = X2`*R22*X2;
    X2PZ1 = X2`*R21*Z1;
    X2PZ2 = X2`*R22*Z2;
    Z1PZ1 = Z1`*R11*Z1;
    Z1PZ2 = Z1`*R12*Z2;
    Z2PZ2 = Z2`*R22*Z2;
    lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2       )//
          (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2       )//
          (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
          (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);
    rhs = (X1`*R11*y1+X1`*R12*y2)//
          (X2`*R21*y1+X2`*R22*y2)//
          (Z1`*R11*y1+Z1`*R12*y2)//
          (Z2`*R21*y1+Z2`*R22*y2);
    sol = GINV(lhs)*rhs;

  * --------------- *
    Compute accuracy
  * --------------- *;
    Di  = vecdiag(GINV(lhs));                   /* Select digonal of lhs inverse */
    PEV = Di;                                   /* Prediction error variance */
    I   = J(24,1,1);                            /* Set unit vector with number of b+u */
    Acc=J(24,1,.);                                      /* Initialize accuracy */
    Acc[5:14,]  = SQRT((I[5:14,]#Va1-Di[5:14,])/Va1);   /* BV Accuracy for t1*/
    Acc[15:24,] = SQRT((I[15:24,]#Va2-Di[15:24,])/Va2); /* BV Accuracy for t2*/
    CC=INV(lhs);

  * --------------------- *
    Set Label for printing
  * --------------------- *;
    label1 = 'b1':'b2';                                 /* The levels for sex is 2 */
    label2 = 'h1':'h2';                                 /* The levels for herd is 2 */
    label3 = 'u1':'u10';                                /* The levels for animal is 5 */
    label  = label1 || label2 || label3 || label3;
    labelt = REPEAT('t1',2)//REPEAT('t2',2)//
             REPEAT('t1',10)//REPEAT('t2',10);

  * ------ *
    Print
  * ------ *;
    PRINT 'Multiple Trait Animal Model',
          '(Sex-limit Model)',,
          'LHS',
          '[X1`*R11*X1      0     X1`*R11*Z1             0           ][b1]   ',
          '[     0     X2`*R22*X2      0           X2`*R22*Z2       ][b2] = ',
          '[Z1`*R11*X1      0     Z1`*R11*Z1+G11*Ai     G12*Ai       ][u1]   ',
          '[     0     Z2`*R22*X2     G21*Ai        Z2`*R22*Z2+G22*Ai][u2]   ',,
          'RHS',
          '[X1`*R11*y1]',
          '[X2`*R22*y2]',
          '[Z1`*R11*y1]',
          '[Z2`*R22*y2]';
    PRINT X1&f1,X2&f1,Z1&f1,Z2&f1;                      /* Print matrix using f1 format */
    PRINT X1PX1&f4,X1PX2&f4,X1PZ1&f4,X1PZ2&f4,
          X2PX2&f4,X2PZ1&f4,X2PZ2&f4,
          Z1PZ1&f4,Z1PZ2&f4,Z2PZ2&f4;
    PRINT A&f2,Ai&f2;
    PRINT rhs&f2;
    PRINT G R,
          Gi&f3 Ri&f3;
    PRINT labelt sol&f3 [rowname=label] Di&f3 ACC&f3;
QUIT;
```

▪ *Multi-trait Animal Model-Missing Trait Model (OUTPUT)*

```
                          The SAS System
                   Multiple Trait Animal Model
                        (Sex-limit Model)

                               LHS
[X1`*R11*X1      0     X1`*R11*Z1             0           ][b1]
[     0     X2`*R22*X2      0            X2`*R22*Z2       ][b2] =
[Z1`*R11*X1      0     Z1`*R11*Z1+G11*Ai     G12*Ai       ][u1]
[     0     Z2`*R22*X2     G21*Ai        Z2`*R22*Z2+G22*Ai][u2]

                               RHS
                          [X1`*R11*y1]
                          [X2`*R22*y2]
                          [Z1`*R11*y1]
                          [Z2`*R22*y2]

                              X1
                               1  0
                               0  1
                               0  1
                               0  1
                               0  0
                               0  0
                               0  0
                               0  0

                              X2
                               0  0
                               0  0
                               0  0
                               0  0
                               1  0
                               1  0
                               0  1
                               0  1

                  Z1
                   0  0  1  0  0  0  0  0  0  0
                   0  0  0  1  0  0  0  0  0  0
                   0  0  0  0  1  0  0  0  0  0
                   0  0  0  0  0  1  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0

                  Z2
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  0  0  0  0
                   0  0  0  0  0  0  1  0  0  0
                   0  0  0  0  0  0  0  1  0  0
                   0  0  0  0  0  0  0  0  1  0
                   0  0  0  0  0  0  0  0  0  1

                            X1PX1
                            0.01 0.00
                            0.00 0.04
```

X1PX2

0.00	0.00
0.00	0.00

X1PZ1

0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.01	0.01	0.01	0.00	0.00	0.00	0.00

X1PZ2

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

X2PX2

0.04	0.00
0.00	0.04

X2PZ1

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

X2PZ2

0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02

Z1PZ1

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Z1PZ2

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Z2PZ2

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02

A

1.00	0.00	0.00	0.50	0.50	0.00	0.25	0.25	0.13	0.13
0.00	1.00	0.00	0.00	0.50	0.50	0.00	0.00	0.00	0.25
0.00	0.00	1.00	0.00	0.00	0.50	0.00	0.00	0.00	0.25
0.50	0.00	0.00	1.00	0.25	0.00	0.50	0.50	0.25	0.25
0.50	0.50	0.00	0.25	1.00	0.25	0.13	0.13	0.06	0.19
0.00	0.50	0.50	0.00	0.25	1.00	0.00	0.00	0.00	0.50
0.25	0.00	0.00	0.50	0.13	0.00	1.00	0.25	0.50	0.50
0.25	0.00	0.00	0.50	0.13	0.00	0.25	1.00	0.13	0.13
0.13	0.00	0.00	0.25	0.06	0.00	0.50	0.13	1.00	0.25
0.13	0.25	0.25	0.25	0.19	0.50	0.50	0.13	0.25	1.00

AI

1.83	0.50	0.00	-0.67	-1.00	-0.00	-0.00	0.00	0.00	0.00
0.50	2.00	0.50	0.00	-1.00	-1.00	-0.00	0.00	0.00	0.00
0.00	0.50	1.50	0.00	0.00	-1.00	0.00	0.00	0.00	0.00
-0.67	0.00	0.00	2.00	-0.00	-0.00	-0.67	-0.67	-0.00	0.00
-1.00	-1.00	0.00	0.00	2.00	0.00	0.00	0.00	-0.00	-0.00
-0.00	-1.00	-1.00	0.00	0.00	2.50	0.50	0.00	-0.00	-1.00
-0.00	-0.00	0.00	-0.67	0.00	0.50	2.17	0.00	-0.67	-1.00
0.00	0.00	0.00	-0.67	0.00	0.00	0.00	1.33	0.00	0.00
0.00	0.00	0.00	0.00	-0.00	-0.00	-0.67	0.00	1.33	0.00
0.00	0.00	0.00	0.00	-0.00	-1.00	-1.00	0.00	0.00	2.00

RHS
4.38
12.50
0.72
0.80
0.00
0.00
4.38
3.13
3.75
5.63
0.00
0.00
0.00
0.00
0.00
0.00
0.00
0.00
0.00
0.00
0.40
0.32
0.30
0.50

G		R	
40	5	80	0
5	20	0	50

GI		RI	
0.026	-0.006	0.013	0.000
-0.006	0.052	0.000	0.020

LABELT	SOL		DI	ACC
t1	b1	330.149	117.701	.
t1	b2	334.312	44.366	.
t2	h1	19.887	37.472	.
t2	h2	19.575	37.065	.
t1	u1	-19.777	37.708	0.239
t1	u2	13.606	37.699	0.240
t1	u3	19.851	37.701	0.240
t1	u4	-30.186	32.707	0.427
t1	u5	-9.331	34.371	0.375
t1	u6	36.580	32.676	0.428
t1	u7	-14.978	38.069	0.220
t1	u8	-15.208	38.069	0.220
t1	u9	-7.718	39.415	0.121
t1	u10	10.943	39.106	0.150
t2	u1	-2.472	19.964	0.042
t2	u2	1.988	19.773	0.106
t2	u3	2.769	19.774	0.106
t2	u4	-3.773	19.886	0.075
t2	u5	-1.022	19.863	0.083
t2	u6	5.148	19.125	0.209
t2	u7	-1.425	18.241	0.297
t2	u8	-2.348	18.241	0.297
t2	u9	-1.604	17.832	0.329
t2	u10	2.455	17.828	0.330

```
/* --------------------------------------- */
/* Multiple Trait Animal Model              */
/* (Canonical Transformation)               */
/* y1 = Xb1 + Zu1 + e1                      */
/* y2 = Xb2 + Zu2 + e2                      */
/* Written by Monchai Duangjinda            */
/* --------------------------------------- */

*================*
 Data Description
*================*
Data file:
id     sex     bw(kg)   yeargain (kg)
2      1       40      0.9
3      2       50      0.8
4      2       45      1.0
5      1       50      1.2

Pedigree file:
anim    s       d
1       0       0
2       0       0
3       2       1
4       2       1
5       0       4

Where Va1 =  5 Va2 = 0.2 Cov(a1,a2) = 2
      Ve1 = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5;

*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=4.2];
%LET f5 = [FORMAT=8.4];

PROC IML;
  /* Transform y1 and y2 to newy1 and newy2 where newy = Q*y
     which has independent R */

  Va1 =  5; Va2 = .2; Va12 = 2;
  Ve1 = 10; Ve2 = .5; Ve12 = -.5;

  G  = (Va1  || Va12)//
       (Va12 || Va2);

  R  = (Ve1  || Ve12)//
       (Ve12 || Ve2);


  /* Define Q */
  U = EIGVEC(R);
  D = DIAG(EIGVAL(R));
  P = U*SQRT(INV(D))*U`;
```

```
L = EIGVEC(P*G*P`);
W = DIAG(EIGVAL(P*G*P`));
Wi= INV(W);

Q = L`*P;
Qi = INV(Q);

TestR = Q*R*Q`;
TestG = Q*G*Q`;

y1 = {40,50,45,50};
y2 = {0.9,0.8,1.0,1.2};
y  = y1 || y2;
newy = y`;

/* Transfrom newy = Q*y by looping */
DO i = 1 TO 4;                         /* N records = 4 */
  newy[,i]  = Q*y[i,]`;
END;

newy = newy`;
newy1 = newy[,1];
newy2 = newy[,2];

X = {1 0,
     0 1,
     0 1,
     1 0};

Z = {0 1 0 0 0,
     0 0 1 0 0,
     0 0 0 1 0,
     0 0 0 0 1};

A = { 1   0  .5  .5 .25,
      0   1  .5  .5 .25,
     .5  .5   1  .5 .25,
     .5  .5  .5   1 .5,
     .25 .25 .25 .5  1};

Ai = inv(A);

* ------------------- *
  MME Setup for Trait1
* ------------------- *;
  alpha = 1/W[1,1];                          /* Use transformed alpha1 */

  XPX  = X`*X;
  XPZ  = X`*Z;
  ZPZ  = Z`*Z;
  ZPZ2 = Z`*Z+alpha#Ai;

  lhs = (X`*X || X`*Z          )//
        (Z`*X || Z`*Z+alpha#Ai );

  rhs = X`*newy1 // Z`*newy1;                 /* Use transformed y1 */
  sol1 = GINV(lhs)*rhs;

* --------------------- *
  Set Label for printing
* --------------------- *;
  label1 = 'b1':'b2';                       /* The levels for fix eff is 2 (sex) */
  label2 = 'u1':'u5';                       /* The levels for animal eff is 5 */
```

```
    label  = label1 || label2;

  * ------ *
    Print
  * ------ *;
    PRINT 'Multiple Trait Animal Model',
          'Cannonical Transformation (y = Qy)',,
          '[X`*X  X`*Z         ][b]   [X`*y]',
          '[Z`*X  Z`*Z+alpha#Ai][u] = [Z`*y]';
    PRINT G&f2 R&f2, y&f2 newy&f2,
          L&f3 U&f3 D&f3 P&f3,
          Q&f3 W&f3 Wi&f3, TestG&f3 TestR&f3;
    PRINT 'Analysis of Trait I';
    PRINT X&f1, Z&f1;                     /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,ZPZ&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2;
    PRINT rhs&f2;
    PRINT ALPHA&f3;
    PRINT sol1&f3 [rowname=label];

  * ------------------- *
    MME Setup for Trait2
  * ------------------- *;
    alpha = 1/W[2,2];                     /* Use transformed alpha2 */
    XPX  = X`*X;
    XPZ  = X`*Z;
    ZPZ  = Z`*Z;
    ZPZ2 = Z`*Z+alpha#Ai;
    lhs = (X`*X || X`*Z          )//
          (Z`*X || Z`*Z+alpha#Ai );
    rhs = X`*newy2 // Z`*newy2;           /* Use transformed y2 */
    sol2 = GINV(lhs)*rhs;

  * ------ *
    Print
  * ------ *;
    PRINT 'Analysis of Trait II';
    PRINT X&f1, Z&f1;                     /* Print matrix using f1 format */
    PRINT XPX&f1,XPZ&f1,ZPZ&f1;
    PRINT A&f2,Ai&f2,ZPZ2&f2;
    PRINT rhs&f2;
    PRINT ALPHA&f3;
    PRINT sol2&f3 [rowname=label];

  /* BACK Transform SOL to original value*/
  sol  = sol1 || sol2;
  origsol = sol`;

  /* Transfrom newy = Qi*sol by looping */
  DO i = 1 TO 7;                          /* Nsol = 2 sex + 5 animals */
    origsol[,i]  = Qi*sol[i,]`;
  END;
  origsol  = origsol`;
  origsol1 = origsol[,1];
  origsol2 = origsol[,2];

  * ------ *
    Print
  * ------ *;
    PRINT 'Back Solutions for Trait I and II';
    PRINT origsol1&f3 [rowname=label] origsol2&f3;
QUIT;
```

▪ *Multi-trait Animal Model-Canonical Transformation (OUTPUT)*

```
                    The SAS System
             Multiple Trait Animal Model
          Cannonical Transformation (y = Qy)

            [X`*X  X`*Z         ][b]   [X`*y]
            [Z`*X  Z`*Z+alpha#Ai][u] = [Z`*y]

                   G             R
                5.00   2.00  10.00  -0.50
                2.00   0.20  -0.50   0.50

                   Y          NEWY
               40.00   0.90  11.35   7.00
               50.00   0.80  13.82   9.05
               45.00   1.00  12.75   7.88
               50.00   1.20  14.27   8.68

        L                 U                  D
    0.673    0.740    0.999    0.052   10.026    0.000
    0.740   -0.673   -0.052    0.999    0.000    0.474

                          P
                      0.319    0.060
                      0.060    1.450

        Q                 W                 WI
    0.259    1.112    1.733    0.000    0.577    0.000
    0.196   -0.932    0.000   -0.364    0.000   -2.744

             TESTG             TESTR
             1.733    0.000    1.000    0.000
             0.000   -0.364    0.000    1.000

                 Analysis of Trait I
                          X
                          1  0
                          0  1
                          0  1
                          1  0

                     Z
                     0  1  0  0  0
                     0  0  1  0  0
                     0  0  0  1  0
                     0  0  0  0  1

                         XPX
                          2  0
                          0  2

                    XPZ
                     0  1  0  0  1
                     0  0  1  1  0

                    ZPZ
                     0  0  0  0  0
                     0  1  0  0  0
                     0  0  1  0  0
                     0  0  0  1  0
                     0  0  0  0  1
```

A

1.00	0.00	0.50	0.50	0.25
0.00	1.00	0.50	0.50	0.25
0.50	0.50	1.00	0.50	0.25
0.50	0.50	0.50	1.00	0.50
0.25	0.25	0.25	0.50	1.00

AI

2.00	1.00	-1.00	-1.00	0.00
1.00	2.00	-1.00	-1.00	0.00
-1.00	-1.00	2.00	0.00	0.00
-1.00	-1.00	0.00	2.33	-0.67
0.00	0.00	0.00	-0.67	1.33

ZPZ2

1.15	0.58	-0.58	-0.58	0.00
0.58	2.15	-0.58	-0.58	0.00
-0.58	-0.58	2.15	0.00	0.00
-0.58	-0.58	0.00	2.35	-0.38
0.00	0.00	0.00	-0.38	1.77

RHS
25.61
26.57
0.00
11.35
13.82
12.75
14.27

ALPHA
0.577

SOL1

b1	12.886
b2	13.432
u1	0.290
u2	-0.870
u3	0.026
u4	-0.316
u5	0.712

Analysis of Trait II

X

1	0
0	1
0	1
1	0

Z

0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

XPX

2	0
0	2

XPZ

0	1	0	0	1
0	0	1	1	0

```
          ZPZ
           0   0   0   0   0
           0   1   0   0   0
           0   0   1   0   0
           0   0   0   1   0
           0   0   0   0   1

     A
  1.00   0.00   0.50   0.50   0.25
  0.00   1.00   0.50   0.50   0.25
  0.50   0.50   1.00   0.50   0.25
  0.50   0.50   0.50   1.00   0.50
  0.25   0.25   0.25   0.50   1.00

    AI
  2.00   1.00  -1.00  -1.00   0.00
  1.00   2.00  -1.00  -1.00   0.00
 -1.00  -1.00   2.00   0.00   0.00
 -1.00  -1.00   0.00   2.33  -0.67
  0.00   0.00   0.00  -0.67   1.33

  ZPZ2
 -5.49  -2.74   2.74   2.74   0.00
 -2.74  -4.49   2.74   2.74   0.00
  2.74   2.74  -4.49   0.00   0.00
  2.74   2.74   0.00  -5.40   1.83
  0.00   0.00   0.00   1.83  -2.66

                 RHS
               15.67
               16.93
                0.00
                7.00
                9.05
                7.88
                8.68

                ALPHA
               -2.744

           SOL2
           b1      7.807
           b2      8.415
           u1     -0.102
           u2      0.305
           u3     -0.017
           u4      0.119
           u5     -0.245

 Back Solutions for Trait I and II

    ORIGSOL1          ORIGSOL2
    b1          45.088    1.100
    b2          47.671    0.991
    u1           0.342    0.181
    u2          -1.027   -0.543
    u3           0.011    0.021
    u4          -0.354   -0.202
    u5           0.850    0.442
```

Part F

```
/* ---------------------------------------------------- */
/* Animal Model with Random Regression Testday          */
/* y = HTD + Xibi + Ziui + Wipi + e                     */
/* LeGendre function: f(dim) = L1 + L2 + L3             */
/* L1 = 1, L2 = sqrt(3)*L,L3 = sqrt(5/4)*(3*L^2-1)      */
/* L = -1+2*(dim-1)/(305-1)                             */
/* Written by Monchai Duangjinda                        */
/* ---------------------------------------------------- */

*================*
 Data Description
*================*
Data file:
cow htd dim   L      L1   L2      L3     milk (kg/d)
1   1   10   -0.941  1  -1.629   1.851    8
1   2   40   -0.743  1  -1.288   0.736   20
1   3   70   -0.546  1  -0.946  -0.118   18
2   4   12   -0.928  1  -1.607   1.768   10
2   2   42   -0.730  1  -1.265   0.671   25
2   3   72   -0.533  1  -0.923  -0.166   19
3   4   60   -0.612  1  -1.060   0.138   16
3   5   120  -0.217  1  -0.376  -0.960   12
3   6   150  -0.020  1  -0.034  -1.117    6

Pedigree file:
anim    s       d
1       0       0
2       0       1
3       0       2

where G  = {4.0 1.0 1.0,
            1.0 3.0 2.0,
            1.0 2.0 2.0}

      Pe = {4.0 0.5 0.1,
            0.5 1.5 0.1,
            0.1 0.1 1.0}
      Ve =  6.25;
*================*
 Start computing
*================*;
OPTIONS PS=500 NODATE NONUMBER;
%LET f1 = [FORMAT=2.0];
%LET f2 = [FORMAT=6.2];
%LET f3 = [FORMAT=8.3];
%LET f4 = [FORMAT=6.3];
PROC IML;
  /* Fixed effects for HTD */
  X1 = {1 0 0 0 0 0,
        0 1 0 0 0 0,
        0 0 1 0 0 0,
        0 0 0 1 0 0,
        0 1 0 0 0 0,
        0 0 1 0 0 0,
        0 0 0 1 0 0,
        0 0 0 0 1 0,
        0 0 0 0 0 1};
```

```
/* Fixed effects for DIM */
X2 = {1  -1.629   1.851,
      1  -1.288   0.736,
      1  -0.946  -0.118,
      1  -1.607   1.768,
      1  -1.265   0.671,
      1  -0.923  -0.166,
      1  -1.060   0.138,
      1  -0.376  -0.960,
      1  -0.034  -1.117};

/* Combined fixed effects */
X  = X1 || X2;

Z1 = {1 0 0,
      1 0 0,
      1 0 0,
      0 1 0,
      0 1 0,
      0 1 0,
      0 0 1,
      0 0 1,
      0 0 1};

/* Kronecor product Z with dim function which has 3 column of parameters */
Z = (X2#Z1[,1])||(X2#Z1[,2])||(X2#Z1[,3]);

/* Include random regression on PE effects */
W = Z;

y = {8,20,18,10,25,19,16,12,6};

A = { 1  .5   .5 ,
     .5   1   .25,
     .5  .25   1};

Ai = inv(A);

G   = {4.0 1.0 1.0,
       1.0 3.0 2.0,
       1.0 2.0 2.0};

Pe  = {4.0 0.5 0.1,
       0.5 1.5 0.1,
       0.1 0.1 1.0};

Ve  = 6.25;

Gi = INV(G);
Pei = INV(Pe);
Ri = 1/Ve;

* --------- *
  MME Setup
* --------- *;
  XPX  = X`*Ri*X;
  XPZ  = X`*Ri*Z;
  XPW  = X`*Ri*W;
  ZPZ  = Z`*Ri*Z;
  ZPW  = Z`*Ri*W;
  WPW  = W`*Ri*W;
  ZPZ2 = Z`*Ri*Z+Gi@Ai;
  WPW2 = W`*Ri*W+Pei@I(3);
```

```
  lhs = (X`*Ri*X || X`*Ri*Z       || X`*Ri*W         )//
        (Z`*Ri*X || Z`*Ri*Z+Gi@Ai || Z`*Ri*W         )//
        (W`*Ri*X || W`*Ri*Z       || W`*Ri*W+Pei@I(3));

  rhs = X`*Ri*y // Z`*Ri*y // W`*Ri*y;
  sol = GINV(lhs)*rhs;

* --------------------- *
  Set Label for printing
* --------------------- *;
  label1 = 'htd1':'htd6';                      /* The levels for HTD is 6 */
  label2 = 'b1':'b3';                          /* The levels for dim regression is 3 */
  label3 = ('u11':'u13')||('u21':'u23')||('u31':'u33');
                                               /* The levels for animal eff is 3*3 */
  label4 = ('pe11':'pe13')||('pe21':'pe23')||('pe31':'pe33');
                                               /* The levels for PE eff is 3 */
  label  = label1 || label2 || label3 || label4;

* ------ *
  Print
* ------ *;
  PRINT 'Random Regression Testday Model',
        'LeGendre Model: f(dim)=L1+L2+L3',
        'L1=1, L2=sqrt(3)*L, L3=sqrt(5/4)*(3*L^2-1)',
        'L = -1+2*(dim-1)/(305-1)',,

        '[X`*Ri*X  X`*Ri*Z        X`*Ri*W      ][b ]   [X`*Ri*y]',
        '[Z`*Ri*X  Z`*Ri*Z+Gi@Ai  Z`*Ri*W      ][u ] = [Z`*Ri*y]',
        '[W`*Ri*X  W`*Ri*Z        W`*Ri*W+Pei@I][pe]   [W`*Ri*y]';

  PRINT X&f4, Z&f4, W&f4;                    /* Print matrix using f1 format */
  PRINT XPX&f4,XPZ&f4,XPW&f4,ZPZ&f4,ZPW&f4,WPW&f4;
  PRINT A&f4,Ai&f4,ZPZ2&f4,WPW2&f4;
  PRINT rhs&f4;
  PRINT G&f3 Gi&f3, Pe&f3 Ve&f3;
  PRINT sol&f3 [rowname=label];

* ----------------------------------- *
  Calculate total EBV and Persistency
* ----------------------------------- *;
  /* first animal start at row 10 of sol */
  /* Initialize EBVt, 3 rows, 1 col with zero value */
  /* Initialize Persistency, 3 rows, 1 col with zero value */
  init    = 10;
  EBVt    = J(3,1,0);
  Persist = J(3,1,0);

  DO j = 1 TO 3;                             /* j for number of animals */
    DO i = 1 TO 305;                         /* i for dim */
      dim  = i;
      L    = -1+2*(dim-1)/(305-1);
      L1   = 1;
      L2   = sqrt(3)*L;
      L3   = sqrt(5/4)*(3*L*L-1);
      DVEC = L1 || L2 || L3;
      UVEC = sol[init:init+2,];            /* 2 is number of function parameters-1 */
      EBVt[j,] = EBVt[j,]+DVEC*UVEC;
    END;
    init = init+3;
  END;
```

```
      /* Aproximate Persistency can be calculated from area under curve
         from d60-d280 */
    DO j = 1 TO 3;                            /* j for number of animals */
      DO i = 60 TO 280;                       /* i for dim */
        dim  = i;
        L    = -1+2*(dim-1)/(305-1);
        L1   = 1;
        L2   = sqrt(3)*L;
        L3   = sqrt(5/4)*(3*L*L-1);
        DVEC = L1 || L2 || L3;
        UVEC = sol[init:init+2,];             /* 2 is number of function parameters-1 */
        Persist[j,] = Persist[j,]+DVEC*UVEC;
      END;
      init = init+3;
    END;
   label5 = 'a1':'a3';
   PRINT EBVt&f3 [rowname=label5] Persist&f3 ;
  * --------------------------------- *
    Analysis of genetic lactation curve
  * --------------------------------- *;
   CREATE GMdata VAR{mean dim};
   DO i = 1 TO 300 BY 10;                     /* i for dim */
     dim  = i;
     L    = -1+2*(dim-1)/(305-1);
     L1   = 1;
     L2   = sqrt(3)*L;
     L3   = sqrt(5/4)*(3*L*L-1);
     DVEC = L1 || L2 || L3;
     BVEC = sol[7:9,];                        /* b1,b2,b3 for dim is row 7-9 of sol */
     MEAN = DVEC*BVEC;                        /* predict milk for each testday */
     APPEND;
   END;
/* Create SAS dataset named ANIMdata has 3 variables */
   CREATE ANIMdata VAR{anim dim EBV MEAN PROD};
   init = 10;                                 /* first animal start at row 10 of sol */
   DO j = 1 TO 3;                             /* j for number of animals */
     anim = j;
     DO i = 1 TO 300 BY 10;                   /* i for dim */
       dim  = i;
       L    = -1+2*(dim-1)/(305-1);
       L1   = 1;
       L2   = sqrt(3)*L;
       L3   = sqrt(5/4)*(3*L*L-1);
       DVEC = L1 || L2 || L3;
       BVEC = sol[7:9,];                      /* b1,b2,b3 for dim is row 7-9 of sol */
       UVEC = sol[init:init+2,];              /* 2 is function parameters-1 */
       MEAN = DVEC*BVEC;
       EBV  = DVEC*UVEC;
       PROD = MEAN+EBV;
       APPEND;
     END;
     init = init+3;
   END;
QUIT;
/* PLOT Genetic Lactation Curve */
PROC GPLOT DATA=ANIMdata;
  SYMBOL1 I=JOIN;
  PLOT EBV*dim = anim /HAXIS = 0 TO 300 BY 30;
  PLOT MEAN*dim = anim /HAXIS = 0 TO 300 BY 30;
  PLOT PROD*dim = anim /HAXIS = 0 TO 300 BY 30;
RUN;
```

▪ *Random Regression Testday Model (OUTPUT)*

```
                              The SAS System

                      Random Regression Testday Model
                      LeGendre Model: f(dim)=L1+L2+L3
                L1=1, L2=sqrt(3)*L, L3=sqrt(5/4)*(3*L^2-1)
                         L = -1+2*(dim-1)/(305-1)

         [X`*Ri*X  X`*Ri*Z        X`*Ri*W      ][b ]   [X`*Ri*y]
         [Z`*Ri*X  Z`*Ri*Z+Gi@Ai  Z`*Ri*W      ][u ] = [Z`*Ri*y]
         [W`*Ri*X  W`*Ri*Z        W`*Ri*W+Pei@I][pe]   [W`*Ri*y]

          X
      1.000  0.000  0.000  0.000  0.000  0.000  1.000 -1.629  1.851
      0.000  1.000  0.000  0.000  0.000  0.000  1.000 -1.288  0.736
      0.000  0.000  1.000  0.000  0.000  0.000  1.000 -0.946 -0.118
      0.000  0.000  0.000  1.000  0.000  0.000  1.000 -1.607  1.768
      0.000  1.000  0.000  0.000  0.000  0.000  1.000 -1.265  0.671
      0.000  0.000  1.000  0.000  0.000  0.000  1.000 -0.923 -0.166
      0.000  0.000  0.000  1.000  0.000  0.000  1.000 -1.060  0.138
      0.000  0.000  0.000  0.000  1.000  0.000  1.000 -0.376 -0.960
      0.000  0.000  0.000  0.000  0.000  1.000  1.000 -0.034 -1.117

          Z
      1.000 -1.629  1.851  0.000  0.000  0.000  0.000  0.000  0.000
      1.000 -1.288  0.736  0.000  0.000  0.000  0.000  0.000  0.000
      1.000 -0.946 -0.118  0.000  0.000  0.000  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -1.607  1.768  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -1.265  0.671  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -0.923 -0.166  0.000  0.000  0.000
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -1.060  0.138
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -0.376 -0.960
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -0.034 -1.117

          W
      1.000 -1.629  1.851  0.000  0.000  0.000  0.000  0.000  0.000
      1.000 -1.288  0.736  0.000  0.000  0.000  0.000  0.000  0.000
      1.000 -0.946 -0.118  0.000  0.000  0.000  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -1.607  1.768  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -1.265  0.671  0.000  0.000  0.000
      0.000  0.000  0.000  1.000 -0.923 -0.166  0.000  0.000  0.000
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -1.060  0.138
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -0.376 -0.960
      0.000  0.000  0.000  0.000  0.000  0.000  1.000 -0.034 -1.117


        XPX
      0.160  0.000  0.000  0.000  0.000  0.000  0.160 -0.261  0.296
      0.000  0.320  0.000  0.000  0.000  0.000  0.320 -0.408  0.225
      0.000  0.000  0.320  0.000  0.000  0.000  0.320 -0.299 -0.045
      0.000  0.000  0.000  0.320  0.000  0.000  0.320 -0.427  0.305
      0.000  0.000  0.000  0.000  0.160  0.000  0.160 -0.060 -0.154
      0.000  0.000  0.000  0.000  0.000  0.160  0.160 -0.005 -0.179
      0.160  0.320  0.320  0.320  0.160  0.160  1.440 -1.460  0.448
     -0.261 -0.408 -0.299 -0.427 -0.060 -0.005 -1.460  1.841 -1.142
      0.296  0.225 -0.045  0.305 -0.154 -0.179  0.448 -1.142  1.564
```

XPZ

0.160	-0.261	0.296	0.000	0.000	0.000	0.000	0.000	0.000
0.160	-0.206	0.118	0.160	-0.202	0.107	0.000	0.000	0.000
0.160	-0.151	-0.019	0.160	-0.148	-0.027	0.000	0.000	0.000
0.000	0.000	0.000	0.160	-0.257	0.283	0.160	-0.170	0.022
0.000	0.000	0.000	0.000	0.000	0.000	0.160	-0.060	-0.154
0.000	0.000	0.000	0.000	0.000	0.000	0.160	-0.005	-0.179
0.480	-0.618	0.395	0.480	-0.607	0.364	0.480	-0.235	-0.310
-0.618	0.833	-0.616	-0.607	0.806	-0.566	-0.235	0.203	0.040
0.395	-0.616	0.637	0.364	-0.566	0.577	-0.310	0.040	0.350

XPW

0.160	-0.261	0.296	0.000	0.000	0.000	0.000	0.000	0.000
0.160	-0.206	0.118	0.160	-0.202	0.107	0.000	0.000	0.000
0.160	-0.151	-0.019	0.160	-0.148	-0.027	0.000	0.000	0.000
0.000	0.000	0.000	0.160	-0.257	0.283	0.160	-0.170	0.022
0.000	0.000	0.000	0.000	0.000	0.000	0.160	-0.060	-0.154
0.000	0.000	0.000	0.000	0.000	0.000	0.160	-0.005	-0.179
0.480	-0.618	0.395	0.480	-0.607	0.364	0.480	-0.235	-0.310
-0.618	0.833	-0.616	-0.607	0.806	-0.566	-0.235	0.203	0.040
0.395	-0.616	0.637	0.364	-0.566	0.577	-0.310	0.040	0.350

ZPZ

0.480	-0.618	0.395	0.000	0.000	0.000	0.000	0.000	0.000
-0.618	0.833	-0.616	0.000	0.000	0.000	0.000	0.000	0.000
0.395	-0.616	0.637	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.480	-0.607	0.364	0.000	0.000	0.000
0.000	0.000	0.000	-0.607	0.806	-0.566	0.000	0.000	0.000
0.000	0.000	0.000	0.364	-0.566	0.577	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.480	-0.235	-0.310
0.000	0.000	0.000	0.000	0.000	0.000	-0.235	0.203	0.040
0.000	0.000	0.000	0.000	0.000	0.000	-0.310	0.040	0.350

ZPW

0.480	-0.618	0.395	0.000	0.000	0.000	0.000	0.000	0.000
-0.618	0.833	-0.616	0.000	0.000	0.000	0.000	0.000	0.000
0.395	-0.616	0.637	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.480	-0.607	0.364	0.000	0.000	0.000
0.000	0.000	0.000	-0.607	0.806	-0.566	0.000	0.000	0.000
0.000	0.000	0.000	0.364	-0.566	0.577	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.480	-0.235	-0.310
0.000	0.000	0.000	0.000	0.000	0.000	-0.235	0.203	0.040
0.000	0.000	0.000	0.000	0.000	0.000	-0.310	0.040	0.350

WPW

0.480	-0.618	0.395	0.000	0.000	0.000	0.000	0.000	0.000
-0.618	0.833	-0.616	0.000	0.000	0.000	0.000	0.000	0.000
0.395	-0.616	0.637	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.480	-0.607	0.364	0.000	0.000	0.000
0.000	0.000	0.000	-0.607	0.806	-0.566	0.000	0.000	0.000
0.000	0.000	0.000	0.364	-0.566	0.577	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.480	-0.235	-0.310
0.000	0.000	0.000	0.000	0.000	0.000	-0.235	0.203	0.040
0.000	0.000	0.000	0.000	0.000	0.000	-0.310	0.040	0.350

A

1.000	0.500	0.500
0.500	1.000	0.250
0.500	0.250	1.000

AI

1.667	-0.667	-0.667
-0.667	1.333	0.000
-0.667	0.000	1.333

ZPZ2

0.956	-0.809	0.205	0.000	0.000	0.000	-0.238	0.095	0.095
-0.809	1.214	-0.616	0.000	0.000	0.000	0.095	-0.190	0.000
0.205	-0.616	1.018	0.000	0.000	0.000	0.095	0.000	-0.190
0.000	-0.000	-0.000	2.147	-1.274	-0.303	-1.667	0.667	0.667
-0.000	0.000	0.000	-1.274	2.139	-0.566	0.667	-1.333	0.000
-0.000	0.000	0.000	-0.303	-0.566	1.910	0.667	0.000	-1.333
-0.238	0.095	0.095	-1.667	0.667	0.667	3.099	-1.283	-1.358
0.095	-0.190	0.000	0.667	-1.333	0.000	-1.283	2.298	0.040
0.095	0.000	-0.190	0.667	0.000	-1.333	-1.358	0.040	2.445

WPW2

0.741	-0.618	0.395	-0.086	0.000	0.000	-0.018	0.000	0.000
-0.618	1.094	-0.616	0.000	-0.086	0.000	0.000	-0.018	0.000
0.395	-0.616	0.898	0.000	0.000	-0.086	0.000	0.000	-0.018
-0.086	0.000	0.000	1.179	-0.607	0.364	-0.061	0.000	0.000
0.000	-0.086	0.000	-0.607	1.505	-0.566	0.000	-0.061	0.000
0.000	0.000	-0.086	0.364	-0.566	1.276	0.000	0.000	-0.061
-0.018	0.000	0.000	-0.061	0.000	0.000	1.488	-0.235	-0.310
0.000	-0.018	0.000	0.000	-0.061	0.000	-0.235	1.210	0.040
0.000	0.000	-0.018	0.000	0.000	-0.061	-0.310	0.040	1.358

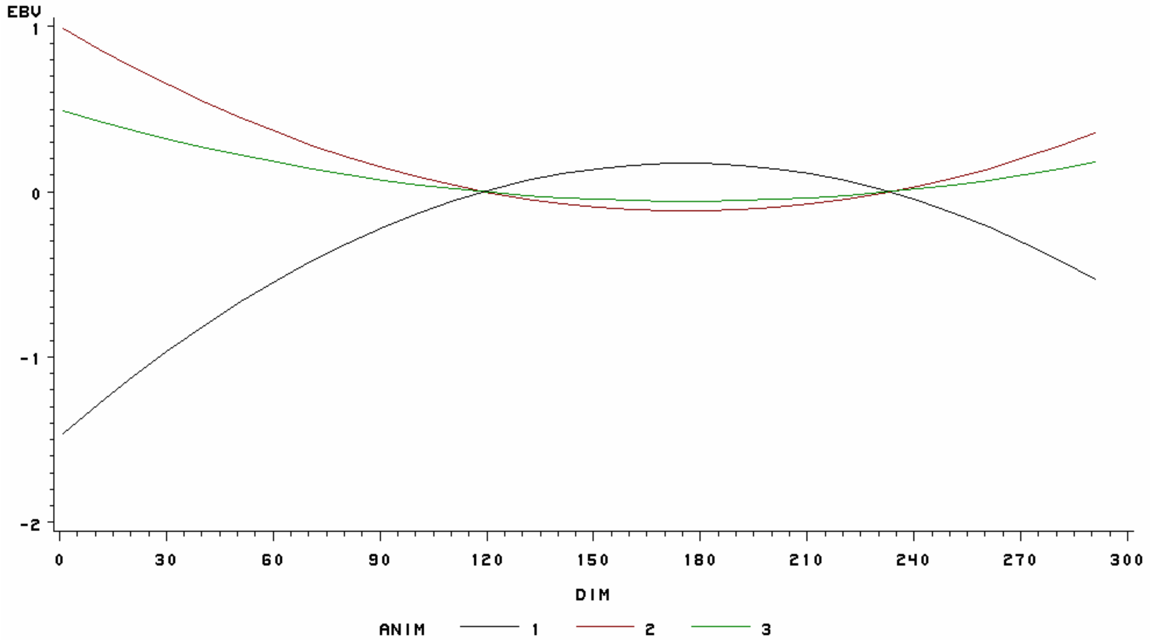
RHS
1.280
7.200
5.920
4.160
1.920
0.960
21.440
-22.84
6.831
7.360
-8.931
4.385
8.640
-10.44
5.008
5.440
-3.468
-2.562
7.360
-8.931
4.385
8.640
-10.44
5.008
5.440
-3.468
-2.562

G			GI		
4.000	1.000	1.000	0.286	0.000	-0.143
1.000	3.000	2.000	0.000	1.000	-1.000
1.000	2.000	2.000	-0.143	-1.000	1.571

PE			PEI			VE
4.000	0.500	0.100	0.261	-0.086	-0.018	6.250
0.500	1.500	0.100	-0.086	0.699	-0.061	
0.100	0.100	1.000	-0.018	-0.061	1.008	

SOL	
htd1	-22.454
htd2	-19.235
htd3	-21.002
htd4	-26.165
htd5	-0.999
htd6	23.459
b1	-66.395
b2	-107.563
b3	-40.580
u11	-0.267
u12	0.216
u13	-0.368
u21	0.180
u22	-0.147
u23	0.247
u31	0.089
u32	-0.072
u33	0.123
pe11	-0.398
pe12	0.542
pe13	-0.365
pe21	0.121
pe22	-0.162
pe23	0.105
pe31	-0.004
pe32	0.005
pe33	-0.001

EBVT		PERSIST
a1	-82.122	-25.629
a2	55.542	8.493
a3	27.374	-0.696



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