Multiple trait model

1- Traits with similar models

2- Traits with different models

Multiple Trait Animal Model (Similar Model)

$$y1 = Xb1 + Zu1 + e1$$
$$y2 = Xb2 + Zu2 + e2$$

LHS

```
[X1`*R11*X1 X1`*R12*X2 X1`*R11*Z1         X1`*R12*Z2         ][b1]
[X2`*R21*X1 X2`*R22*X2 X2`*R21*Z1         X2`*R22*Z2         ][b2]
[Z1`*R11*X1 Z1`*R12*X2 Z1`*R11*Z1+G11*Ai  Z1`*R12*Z2+G12*Ai][u1]
[Z2`*R21*X1 Z2`*R22*X2 Z2`*R21*Z1+G21*Ai  Z2`*R22*Z2+G22*Ai][u2]
```

RHS

```
[X1`*R11*y1+X1`*R12*y2]
[X2`*R21*y1+X2`*R22*y2]
[Z1`*R11*y1+Z1`*R12*y2]
[Z2`*R21*y1+Z2`*R22*y2]
```

Data Description

Data file:

id	sex	bw(kg)	yeargain (kg)
2	1	40	1.9
3	2	50	1.8
4	2	45	2.0
5	2	50	2.2

Pedigree file:

anim	s	d
1	0	0
2	0	0
3	1	2
4	1	2
5	4	0

```
Va1 =  5 Va2 = 0.2 Cov(a1,a2) = 2
Ve1 = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5;
```

Start computing

```
PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        0 1};
  /* Use same sex as cg for both trait */
  X2 = X1;

  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 1};

  Z2 = Z1;

  y1 = {40,50,45,50};
  y2 = {1.9,1.8,2.0,2.2};
```

```
A = { 1   0   .5  .5  .25,
      0   1   .5  .5  .25,
      .5  .5   1  .5  .25,
      .5  .5  .5   1  .5,
      .25 .25 .25 .5   1};
```

```
Ai = inv(A);

Va1 =  5; Va2 = 0.2; Va12 = 2;
Ve1 = 10; Ve2 = 0.5; Ve12 = -0.5

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);
```

```
Gi = INV(G);
Ri = INV(R);

G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];

R11 = Ri[1,1];
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2];
```

MME Setup

```
X1PX1 = X1`*R11*X1;
X1PX2 = X1`*R12*X2;
X1PZ1 = X1`*R11*Z1;
X1PZ2 = X1`*R12*Z2;
X2PX2 = X2`*R22*X2;
X2PZ1 = X2`*R21*Z1;
X2PZ2 = X2`*R22*Z2;
Z1PZ1 = Z1`*R11*Z1;
Z1PZ2 = Z1`*R12*Z2;
Z2PZ2 = Z2`*R22*Z2;
```

```
lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2       )//
      (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2       )//
      (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
      (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);

rhs = (X1`*R11*y1+X1`*R12*y2)//
      (X2`*R21*y1+X2`*R22*y2)//
      (Z1`*R11*y1+Z1`*R12*y2)//
      (Z2`*R21*y1+Z2`*R22*y2);

sol = GINV(lhs)*rhs;
```

Output

```
t1     b1    40.277
t1     b2    48.423
t2     b1     1.971
t2     b2     2.044
t1     u1    -0.277
t1     u2    -0.277
t1     u3    -0.290
t1     u4    -0.541
t1     u5     0.560
t2     u1    -0.071
t2     u2    -0.071
t2     u3     0.052
t2     u4    -0.264
t2     u5     0.080
```

Multiple Trait Animal Model (Different Model)

$$y1 = Xb1 + Zu1 + Wpe1 + e1$$
$$y2 = Xb2 + Zu2 + e2$$

LHS

```
[X1`R11X1 X1`R12X2 X1`R11Z1        X1`R12Z2        X1`R11W1        ][b1 ]
[X2`R21X1 X2`R22X2 X2`R21Z1        X2`R22Z2        X2`R21W1        ][b2 ] =
[Z1`R11X1 Z1`R12X2 Z1`R11Z1+G11*Ai Z1`R12Z2+G12*Ai Z1`R11W1        ][u1 ]
[Z2`R21X1 Z2`R22X2 Z2`R21Z1+G21*Ai Z2`R22Z2+G22*Ai Z2`R21W1        ][u2 ]
[W1`R11X1 W1`R12X2 W1`R11Z1        W1`R12Z2        W1`R11W1+1/Vpe*I][pe1]
```

RHS

```
[X1`R11y1+X1`R12y2]
[X2`R21y1+X2`R22y2]
[Z1`R11y1+Z1`R12y2]
[Z2`R21y1+Z2`R22y2]
[W1`R11y1+W1`R12y2]
```

Data Description

Data file:

id	cg1	cg2	dam	ww(kg)	yeargain (kg)
2	1	1	0	400	1.9
3	2	1	2	500	1.8
4	2	2	2	450	2.0
5	2	3	0	500	2.2

Pedigree file:

anim	s	d
1	0	0
2	0	0
3	1	2
4	1	2
5	4	0

```
Va1  = 5 Va2 = 0.2 Cov(a1,a2) = 2
Vpe1 = 2
Ve1  = 8 Ve2 = 0.5 Cov(e1,e2) = -0.5
```

Start computing

```
PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        0 1};
  /* Use different cg for each trait */
  X2 = {1 0 0,
        1 0 0,
        0 1 0,
        0 0 1};

  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 1};

  Z2 = Z1;

  W1 = {0 0 0 0 0,
        0 1 0 0 0,
        0 1 0 0 0,
        0 0 0 0 0};

  y1 = {400,500,450,500};
  y2 = {1.9,1.8,2.0,2.2};

  A = { 1   0   .5  .5 .25,
        0   1   .5  .5 .25,
       .5  .5    1  .5 .25,
       .5  .5   .5   1 .5,
       .25 .25 .25 .5  1};
```

```
Ai = inv(A);

Va1  = 5; Va2  = 0.2; Va12  = 2;
Vpe1 = 2;
Ve1  = 8; Ve2  = 0.5; Ve12  = -0.5;

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);
```

```
Gi = INV(G);
Ri = INV(R);

G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];

R11 = Ri[1,1];
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2];
```

MME Setup

```
X1PX1 = X1`*R11*X1;
X1PX2 = X1`*R12*X2;
X1PZ1 = X1`*R11*Z1;
X1PZ2 = X1`*R12*Z2;
X1PW1 = X1`*R11*W1;

X2PX2 = X2`*R22*X2;
X2PZ1 = X2`*R21*Z1;
X2PZ2 = X2`*R22*Z2;
X2PW1 = X2`*R21*W1;

Z1PZ1 = Z1`*R11*Z1;
Z1PZ2 = Z1`*R12*Z2;
Z1PW1 = Z1`*R11*W1;
Z2PZ2 = Z2`*R22*Z2;
Z2PW1 = Z2`*R21*W1;
W1PW1 = W1`*R11*W1;
```

MME Setup

```
    lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1        || X1`*R12*Z2        ||
X1`*R11*W1)//
          (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1        || X2`*R22*Z2        ||
X2`*R21*W1)//
          (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai  ||
Z1`*R11*W1)//
          (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai  ||
Z2`*R21*W1)//
          (W1`*R11*X1 || W1`*R12*X2 || W1`*R11*Z1        || W1`*R12*Z2        ||
W1`*R11*W1+1/Vpe1#I(5));

     rhs = (X1`*R11*y1+X1`*R12*y2)//
           (X2`*R21*y1+X2`*R22*y2)//
           (Z1`*R11*y1+Z1`*R12*y2)//
           (Z2`*R21*y1+Z2`*R22*y2)//
           (W1`*R11*y1+W1`*R12*y2);

              sol = GINV(lhs)*rhs;
```

Output

```
t1      b1    401.429
t1      b2    486.770
t2      b1      2.146
t2      b2      3.611
t2      b3      2.342
t1      u1     -2.650
t1      u2     -1.022
t1      u3      1.734
t1      u4     -8.055
t1      u5      1.480
t2      u1     -0.816
t2      u2     -0.653
t2      u3      0.938
t2      u4     -3.222
t2      u5      0.592
t1      pe1     0.000
t1      pe2    -2.937
t1      pe3     0.000
t1      pe4     0.000
t1      pe5     0.000
```

Multiple trait model

3- Sex-limited traits

4- Missing records

Multiple Trait Animal Model (Sex limited Model)

$$y1 = Xb1 + Zu1 + e1$$
$$y2 = Xb2 + Zu2 + e2$$

LHS

```
[X1`*R11*X1      0      X1`*R11*Z1             0            ][b1]
[     0      X2`*R22*X2      0            X2`*R22*Z2       ][b2] =
[Z1`*R11*X1      0      Z1`*R11*Z1+G11*Ai     G12*Ai        ][u1]
[     0      Z2`*R22*X2     G21*Ai        Z2`*R22*Z2+G22*Ai][u2]
```

RHS

```
[X1`*R11*y1]
[X2`*R22*y2]
[Z1`*R11*y1]
[Z2`*R22*y2]
```

Data Description

Data file: (Dual purpose records)

id	sex	herd	ww(kg)	milk(kg)
3	m	1	350	-
4	m	2	250	-
5	m	2	300	-
6	m	2	450	-
7	f	1	-	20
8	f	1	-	16
9	f	2	-	15
10	f	2	-	25

Pedigree file:

anim	s	d
1	0	0
2	0	0
3	0	0
4	1	2
5	3	2
6	1	5
7	3	4
8	1	7
9	1	0
10	1	0

Va1 = 40 Va2 = 20 Cov(a1,a2) = 10
Ve1 = 80 Ve2 = 50 Cov(e1,e2) = 0

Cov(e1,e2) need to set to zero since data are from different environment

Start computing

```
PROC IML;
  /* Use herd as fixed effects */
  X1 = {1 0,                    Z1 = {0 0 1 0 0 0 0 0 0 0,
        0 1,                          0 0 0 1 0 0 0 0 0 0,
        0 1,                          0 0 0 0 1 0 0 0 0 0,
        0 1,                          0 0 0 0 0 1 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        0 0};                         0 0 0 0 0 0 0 0 0 0};

  X2 = {0 0,                    Z2 = {0 0 0 0 0 0 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        0 0,                          0 0 0 0 0 0 0 0 0 0,
        1 0,                          0 0 0 0 0 0 1 0 0 0,
        1 0,                          0 0 0 0 0 0 0 1 0 0,
        0 1,                          0 0 0 0 0 0 0 0 1 0,
        0 1};                         0 0 0 0 0 0 0 0 0 1};
  y1 = {350,250,300,450,0,0,0,0};
  y2 = {0,0,0,0,20,16,15,25};
```

```
A = {1    0    0   .5    0   .5   .25   .625  .5   .5,
     0    1    0   .5   .5   .25  .25   .125  0    0,
     0   .5    1    0   .5   .25  .5    .25   0    0,
    .5   .5    0    1   .25  .375 .5    .5    .25  .25,
     0   .5   .5   .25   1   .5   .375  .188  0    0,
    .5   .25  .25  .375 .5    1   .313  .406  .25  .25,
    .25  .25  .5   .5   .375 .313  1    .625  .125 .125,
    .625 .125 .25  .5   .188 .406 .625 1.125  .313 .313,
    .5    0    0   .25   0   .25  .125  .313  1    0,
    .5    0    0   .25   0   .25  .125  .313  0    1};
Ai = inv(A);
```

```
Va1 = 40; Va2 = 20; Va12 = 10;
Ve1 = 80; Ve2 = 50; Ve12 = 0;

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);
```

```
Gi = INV(G);
Ri = INV(R);

G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];

R11 = Ri[1,1];
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2];
```

MME Setup

```
X1PX1 = X1`*R11*X1;
X1PX2 = X1`*R12*X2;
X1PZ1 = X1`*R11*Z1;
X1PZ2 = X1`*R12*Z2;
X2PX2 = X2`*R22*X2;
X2PZ1 = X2`*R21*Z1;
X2PZ2 = X2`*R22*Z2;
Z1PZ1 = Z1`*R11*Z1;
Z1PZ2 = Z1`*R12*Z2;
Z2PZ2 = Z2`*R22*Z2;
```

```
lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1       || X1`*R12*Z2       )//
      (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1       || X2`*R22*Z2       )//
      (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
      (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);
```

```
rhs = (X1`*R11*y1+X1`*R12*y2)//
      (X2`*R21*y1+X2`*R22*y2)//
      (Z1`*R11*y1+Z1`*R12*y2)//
      (Z2`*R21*y1+Z2`*R22*y2);
```

```
sol = GINV(lhs)*rhs;
```

Output

```
t1    b1    345.867
t1    b2    330.082
t2    b1     18.862
t2    b2     19.236
t1    u1      7.412
t1    u2    -11.544
t1    u3      4.133
t1    u4    -17.620
t1    u5      0.303
t1    u6     27.070
t1    u7     -6.597
t1    u8      0.130
t1    u9      2.991
t1    u10     4.420
t2    u1      1.530
t2    u2     -2.778
t2    u3      1.249
t2    u4     -4.405
t2    u5      0.237
t2    u6      6.687
t2    u7     -1.326
t2    u8     -0.399
t2    u9     -0.664
t2    u10     2.193
```

Multiple Trait Animal Model
(Similar Model with Missing Records)

$$y1 = Xb1 + Zu1 + e1$$
$$y2 = Xb2 + Zu2 + e2$$

LHS

```
[X1`*R11*X1 X1`*R12*X2 X1`*R11*Z1       X1`*R12*Z2       ][b1]
[X2`*R21*X1 X2`*R22*X2 X2`*R21*Z1       X2`*R22*Z2       ][b2] =
[Z1`*R11*X1 Z1`*R12*X2 Z1`*R11*Z1+G11*Ai Z1`*R12*Z2+G12*Ai][u1]
[Z2`*R21*X1 Z2`*R22*X2 Z2`*R21*Z1+G21*Ai Z2`*R22*Z2+G22*Ai][u2]
```

RHS

```
[X1`*R11*y1+X1`*R12*y2]
[X2`*R21*y1+X2`*R22*y2]
[Z1`*R11*y1+Z1`*R12*y2]
[Z2`*R21*y1+Z2`*R22*y2]
```

Data Description

Data file:

id	sex	bw(kg)	yeargain (kg)
2	1	40	1.9
3	2	50	1.8
4	2	45	.
5	2	.	2.2

Pedigree file:

anim	s	d
1	0	0
2	0	0
3	1	2
4	1	2
5	4	0

```
Va1 =  5 Va2 = 0.2 Cov(a1,a2) = 2
Ve1 = 10 Ve2 = 0.5 Cov(e1,e2) = -0.5
```

Start computing

```
y1 = {40,50,45,0};
y2 = {1.9,1.8,0,2.2};

A = { 1   0   .5  .5 .25,
      0   1   .5  .5 .25,
     .5  .5   1   .5 .25,
     .5  .5  .5    1 .5,
     .25 .25 .25 .5  1};
```

```
PROC IML;
  X1 = {1 0,
        0 1,
        0 1,
        0 0};

  X2 = {1 0,
        0 1,
        0 0,
        0 1};

  Z1 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 1 0,
        0 0 0 0 0};

  Z2 = {0 1 0 0 0,
        0 0 1 0 0,
        0 0 0 0 0,
        0 0 0 0 1};
```

```
Ai = inv(A);

Va1 =  5; Va2 = 0.2; Va12 = 2;
Ve1 = 10; Ve2 = 0.5; Ve12 = -0.5;

G  = (Va1  || Va12)//
     (Va12 || Va2);

R  = (Ve1  || Ve12)//
     (Ve12 || Ve2);
```

```
Gi = INV(G);
Ri = INV(R);

G11 = Gi[1,1];
G12 = Gi[1,2];
G21 = G12;
G22 = Gi[2,2];
```

Create Residual variance structure for 4 records

```
R11 = Ri[1,1]*I(4);
R12 = Ri[1,2];
R21 = R12;
R22 = Ri[2,2]*I(4);
```

Change some residual variance with missing trait residual variance

Inverse of Residual variance for missing traits

```
Ri0 = I(2);
Ri0[1,1] = 1/Ve1;
Ri0[2,2] = 1/Ve2;
```

records 4 missing trait I

```
R11[4,4] = 1/Ve1;
```

records 3 missing trait II

```
R22[3,3] = 1/Ve2;
```

MME Setup

```
X1PX1 = X1`*R11*X1;
X1PX2 = X1`*R12*X2;
X1PZ1 = X1`*R11*Z1;
X1PZ2 = X1`*R12*Z2;
X2PX2 = X2`*R22*X2;
X2PZ1 = X2`*R21*Z1;
X2PZ2 = X2`*R22*Z2;
Z1PZ1 = Z1`*R11*Z1;
Z1PZ2 = Z1`*R12*Z2;
Z2PZ2 = Z2`*R22*Z2;
```

MME Setup

```
lhs = (X1`*R11*X1 || X1`*R12*X2 || X1`*R11*Z1         || X1`*R12*Z2       )//
      (X2`*R21*X1 || X2`*R22*X2 || X2`*R21*Z1         || X2`*R22*Z2       )//
      (Z1`*R11*X1 || Z1`*R12*X2 || Z1`*R11*Z1+G11*Ai || Z1`*R12*Z2+G12*Ai)//
      (Z2`*R21*X1 || Z2`*R22*X2 || Z2`*R21*Z1+G21*Ai || Z2`*R22*Z2+G22*Ai);

rhs = (X1`*R11*y1+X1`*R12*y2)//
      (X2`*R21*y1+X2`*R22*y2)//
      (Z1`*R11*y1+Z1`*R12*y2)//
      (Z2`*R21*y1+Z2`*R22*y2);


                sol = GINV(lhs)*rhs;
```

Output

t1	b1	40.245
t1	b2	47.689
t2	b1	1.924
t2	b2	2.006
t1	u1	-0.245
t1	u2	-0.245
t1	u3	-0.174
t1	u4	-0.560
t1	u5	0.454
t2	u1	-0.024
t2	u2	-0.024
t2	u3	0.151
t2	u4	-0.224
t2	u5	-0.039