

3.8 Solved Problems

	Charged Rod	Charged Ring	Charged disk
Figure			
(2) Express dq in terms of charge density	$dq = \lambda dx'$	$dq = \lambda dl$	$dq = \sigma dA$
(3) Substitute dq into expression for dV	$dV = k_e \frac{\lambda dx'}{r}$	$dV = k_e \frac{\lambda dl}{r}$	$dV = k_e \frac{\sigma dA}{r}$
(4) Rewrite r and the differential element in terms of the appropriate coordinates	dx' $r = \sqrt{x'^2 + y^2}$	$dl = R d\phi'$ $r = \sqrt{R^2 + z^2}$	$dA = 2\pi r' dr'$ $r = \sqrt{r'^2 + z^2}$
(5) Rewrite dV	$dV = k_e \frac{\lambda dx'}{(x'^2 + y^2)^{1/2}}$	$dV = k_e \frac{\lambda R d\phi'}{(R^2 + z^2)^{1/2}}$	$dV = k_e \frac{2\pi\sigma r' dr'}{(r'^2 + z^2)^{1/2}}$
(6) Integrate to get V	$V = \frac{\lambda}{4\pi\epsilon_0} \int_{-\ell/2}^{\ell/2} \frac{dx'}{\sqrt{x'^2 + y^2}}$ $= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{(\ell/2) + \sqrt{(\ell/2)^2 + y^2}}{-(\ell/2) + \sqrt{(\ell/2)^2 + y^2}} \right]$	$V = k_e \frac{R\lambda}{(R^2 + z^2)^{1/2}} \int d\phi'$ $= k_e \frac{(2\pi R\lambda)}{\sqrt{R^2 + z^2}}$ $= k_e \frac{Q}{\sqrt{R^2 + z^2}}$	$V = k_e 2\pi\sigma \int_0^R \frac{r' dr'}{(r'^2 + z^2)^{1/2}}$ $= 2k_e \pi\sigma \left(\sqrt{z^2 + R^2} - z \right)$ $= \frac{2k_e Q}{R^2} \left(\sqrt{z^2 + R^2} - z \right)$
Derive E from V	$E_y = -\frac{\partial V}{\partial y}$ $= \frac{\lambda}{2\pi\epsilon_0 y} \frac{\ell/2}{\sqrt{(\ell/2)^2 + y^2}}$	$E_z = -\frac{\partial V}{\partial z} = \frac{k_e Qz}{(R^2 + z^2)^{3/2}}$	$E_z = -\frac{\partial V}{\partial z} = \frac{2k_e Q}{R^2} \left(\frac{z}{ z } - \frac{z}{\sqrt{z^2 + R^2}} \right)$
Point-charge limit for E	$E_y \approx \frac{k_e Q}{y^2} \quad y \gg \ell$	$E_z \approx \frac{k_e Q}{z^2} \quad z \gg R$	$E_z \approx \frac{k_e Q}{z^2} \quad z \gg R$